EECS 216 FEEDBACK CONTROL

Given: Fixed LTI system with impulse response $h(t) = e^{2t}u(t)$ (unstable!)
Goal: Somehow connect a *controller* to the system to make system stable.

Open Loop: Use *cascade* controller (compensator) $C(s)$ in *series* with given system $H(s)=\mathcal{L}\{e^{2t}u(t)\}=\frac{1}{s-2}=\rightarrow C(s)=\frac{1}{H(s)}$=s–2$\rightarrow c(t)*x(t)=\frac{dx}{dt}-2x(t)$.
But: This is a **BAD** idea: *Roundoff* error→won't cancel pole $\{2\}$ *exactly*!

Closed Loop: Use a *feedback* controller $G(s) = K$ (constant gain) in *feedback loop*:

$x(t) \rightarrow \bigoplus \rightarrow \overline{\underline{|h(t)|}} \rightarrow \bullet \rightarrow y(t)$ equivalent to $\frac{Y(s)}{X(s)} = \frac{H(s)}{1-G(s)H(s)}$.
$\nwarrow \overline{\underline{|g(t)|}} \swarrow$ (feedback connection)

Then: $H_{\substack{\text{CLOSED}\\ \text{LOOP}}}(s)=H_{\text{CL}}(s)=\frac{Y(s)}{X(s)}=\frac{H(s)}{1-KH(s)}=[\frac{1}{s-2}]/[1-K\frac{1}{s-2}]=\frac{1}{s-2-K}$.
Pole: K+2<0 if K<–2. In fact, K=–102$\rightarrow h_{\substack{\text{CLOSED}\\ \text{LOOP}}}(t)=h_{\text{CL}}(t)=e^{-100t}u(t)$!
Note: We need K<0→*negative feedback* to stabilize an unstable system.

Given: Mechanical *actuator* has a very slow step response s(t)=(1–e^{-t})u(t).
Goal: Use feedback to speed up step response so output tracks input rapidly.

Have: S(s)=$\frac{1}{s}-\frac{1}{s+1}=\frac{1}{s(s+1)}\rightarrow H(s)=\frac{1}{s+1}\rightarrow H_{\text{CL}}(s)=[\frac{1}{s+1}]/[1-\text{K}\frac{1}{s+1}]=\frac{1}{s+1-K}$
Then: Using K=–99$\rightarrow H_{\text{CL}}(s)=\frac{1}{s+100}\rightarrow S_{\text{CL}}(s)=\frac{1}{s(s+100)}=\frac{0.01}{s}-\frac{0.01}{s+100}$
And: $s_{CL}(t)$=.01u(t)–.01$e^{-100t}u(t)$. Much faster! Also need amplify $\times 100$.

Given: h(t)=cos(t)u(t)→H(s)=$\frac{s}{s^2+1}\rightarrow H_{\text{CL}}(s)=[\frac{s}{s^2+1}]/[1-K\frac{s}{s^2+1}]=\frac{s}{s^2-Ks+1}$.

Poles: $\frac{K}{2}\pm\sqrt{(\frac{K}{2})^2-1}$. Need $K<0$ (negative feedback again) to stabilize.

$|K|<2$: Poles: $\frac{K}{2}\pm j\sqrt{1-(\frac{K}{2})^2}$ both lie on the unit circle at $e^{\pm j\cos^{-1}(K/2)}$.

$|K|>2$: Poles: $\frac{K}{2}\pm\sqrt{(\frac{K}{2})^2-1}$ both lie on the real axis (their midpoint is $\frac{K}{2}$).

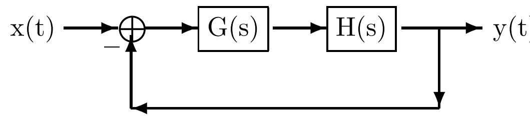
K=–2: Double pole at –1$\rightarrow H_{\text{CL}}(s)=\frac{s}{(s+1)^2}\rightarrow h_{CL}(t)$=(1–t)$e^{-t}$u(t) (try $\frac{s-1}{s^2}$)

Note: H(s)=$\frac{N(s)}{D(s)}\rightarrow H_{\text{CL}}(s)=\frac{N(s)/D(s)}{1-KN(s)/D(s)}=\frac{N(s)}{D(s)-KN(s)}$=polynomial ratio.
So: Zeros unchanged; Poles move from roots of D(s)=0 to D(s)–KN(s)=0

Problem: What choices of poles? Need:plot of roots of D(s)–KN(s)=0 as K varies
Solution: This is called a *root locus*. Learn about it in EECS 460. Above example:
EX: K decreases from 0 to $-\infty$: Poles move along unit circle from $\pm j$ to -1; *coalesce* at –1; split apart; move in opposite directions along real axis. One pole becomes "fast," other becomes "slow." Fastest for both=–1.

PD (PROPORTIONAL+DERIVATIVE COMPENSATORS

Now: *Compenator* $G(s)$ in series with $H(s)$
and: Negative feedback using gain K=1.
Then: $H_{CL}(s) = \frac{G(s)H(s)}{1+G(s)H(s)}$ (right figure).



Let: $H(s) = \frac{1}{s^2-as+b}$ with $a > 0$. Many systems have this transfer function.
Then: **Unstable** since poles$=\frac{a}{2} \pm \frac{1}{2}\sqrt{a^2-4b}$ have real parts in right $\frac{1}{2}$-plane.
Goals: (1) Stabilize system (2) Make *step response* $s(t) \approx u(t)$=step function.

Try: $G(s)=\frac{1}{H(s)}=s^2-as+\mathrm{b} \rightarrow g(t)*x(t)=\frac{d^2x}{dt^2}-a\frac{dx}{dt}+bx(t)$ ($G(s)$ cancels $H(s)$).
But: If a, b are even *slightly* wrong, then the overall system is still unstable!
Note: (1) May not know a, b exactly–want *insensitive* to a, b; roundoff error.

Try: $G(s)=K+Ds$: Called Proportional+Derivative (PD) compensator.
Then: $H_{CL}(s)=\frac{(K+Ds)/(s^2-as+b)}{1+(K+Ds)/(s^2-as+b)}=\frac{K+Ds}{s^2+s(D-a)+(K+b)}$. Can choose D, K.

Stable: Choose $D > a \rightarrow$poles in left half-plane$\rightarrow$stable (real part $-\frac{1}{2}(D-a)$).
Step: Compute step response $Y(s)=S(s)$ to $X(s)=U(s)=\frac{1}{s}$ either of 2 ways:
1. $S(s)=\frac{1}{s}\frac{K+Ds}{s^2+s(D-a)+(K+b)}=\frac{H_{CL}(0)}{s}+\frac{\textbf{MESS}}{s^2+s(D-a)+(K+b)}$ (using $^{\text{PARTIAL}}_{\text{FRACTION}}$).

Then: $s(t) \simeq H_{CL}(0)u(t)=\frac{K}{K+b}$ as $t \rightarrow \infty$ since the 2^{nd} term is strictly proper.

2. $\lim_{t\rightarrow\infty} s(t)=\lim_{s\rightarrow 0} sS(s)=H(0)=\frac{K}{K+b}$ using the Final Value Theorem.

PROCEDURE FOR CHOOSING CONSTANTS K AND D:

1. Choose $D > a$ to stabilize closed-loop system (real part $-\frac{1}{2}(D-a)$).

Then: Larger $D \rightarrow$ real parts of poles more negative$\rightarrow$ faster response.

2. Choose K so $(K+b)=\frac{1}{4}(D-a)^2 \rightarrow$have double pole at $-\frac{1}{2}(D-a)$.

Note: Else get ringing (underdamped) or slower response (overdamped).

3. Check if steady-state step response error $1-\frac{K}{K+b}=\frac{b}{K+b}$ is small enough.

Else: Increase K and D. **But:** There may be physical limits on their sizes.

SIMPLE NUMERICAL EXAMPLE OF PD COMPENSATION

Given: Open loop $H(s)=\frac{1}{s^2-2s+5}$. NOTE: Poles$=1 \pm j2 \rightarrow$system unstable.
Goals: Stabilize system; fastest step response; make steady-state error=5%.

Soln: **Given:** a=2 and b=5 and design spec $0.05=\frac{b}{K+b}=\frac{5}{K+5} \rightarrow K=95$.
Then: $95+5=K+b=\frac{1}{4}(D-a)^2=\frac{1}{4}(D-2)^2 \rightarrow D=22$ for critical damping.

Get: Double pole at –10 and $H_{CL}(s) = \frac{95+22s}{(s+10)^2}$. The system is now stable.
So that: Natural response for any input decays as $te^{-10t}u(t)$ (critical damping).
And: Forced response to step input u(t) is 0.95u(t) (5% steady-state error).