

---

 LOW-PASS FILTER DESIGN USING POLES AND ZEROS
 

---

**Given:** Sum of two continuous-time signals: Call this sum  $x_1(t) + x_2(t)$ .

**such** Signal  $x_1(t)$  is bandlimited to 000-250 Hz (low pass signal).

**that** Signal  $x_2(t)$  is bandlimited to 250-500 Hz (highpass signal).

---

**Given:** A DSP system having a sampling rate=1 kHz= $1000 \frac{\text{SAMPLE}}{\text{SECOND}}$ .

**and:** A DSP chip having a total of 10 delays (storage registers).

**Goal:** Design a digital lowpass filter to pass  $x_1(t)$  and reject  $x_2(t)$ .

---

	contin. :	1000 Hz	500 Hz	250 Hz	
<b>Freqs:</b>	discrete :	$2\pi$	$\pi$	$\pi/2$	$H(\omega) = \begin{cases} 1 & \text{for } 0 \leq  \omega  < \pi/2 \\ 0 & \text{for } \pi/2 <  \omega  \leq \pi \end{cases}$
	identity :	sampling	Nyquist	cutoff	

---

**Ideal:**  $h[n] = \sin(\frac{\pi}{2}n)/(\pi n)$  (sinc function). Impulse response of IIR filter.

**Problems:** (1) Noncausal (2) Unstable (3) No difference equation implements.

---

**Zeros:**  $\{e^{\pm j\pi/2}, e^{\pm j3\pi/4}, e^{j\pi}\}$  to reject high frequencies (zeros *on* unit circle).

**Poles:**  $\{.6, .8e^{\pm j\pi/4}, .8e^{\pm j\pi/2}\}$  to pass low frequencies (.6 works better here).

**Note:** Need all the poles inside the unit circle ( $|p_n| < 1$ ) for BIBO stability.

---

**H(z):**  $H(z) = \frac{(z - e^{j\pi/2})(z - e^{-j\pi/2})(z - e^{j3\pi/4})(z - e^{-j3\pi/4})(z - e^{j\pi})}{(z - .8e^{j\pi/2})(z - .8e^{-j\pi/2})(z - .8e^{j\pi/4})(z - .8e^{-j\pi/4})(z - .6)}$ .

---

**using:**  $(z - e^{j\pi/2})(z - e^{-j\pi/2})(z - e^{j\pi}) = (z - j)(z + j)(z + 1) = z^3 + z^2 + z + 1$

**and:**  $(z - e^{j3\pi/4})(z - e^{-j3\pi/4}) = z^2 - 2\cos(\frac{3\pi}{4})z + 1 = z^2 + \sqrt{2}z + 1$

**and:** Performing similar computations for the denominator factors.

---

**H(z):**  $H(z) = \frac{z^5 + 2.414z^4 + 3.414z^3 + 3.414z^2 + 2.414z + 1}{z^5 - 1.73z^4 + 1.96z^3 - 1.49z^2 + 0.84z - 0.25} = \frac{Y(z)}{X(z)}$ . Cross-multiply:

---

$y[n] - 1.73y[n-1] + 1.96y[n-2] - 1.49y[n-3] + 0.84y[n-4] - 0.25y[n-5]$   
 $= x[n] + 2.414x[n-1] + 3.414x[n-2] + 3.414x[n-3] + 2.414x[n-4] + x[n-5]$

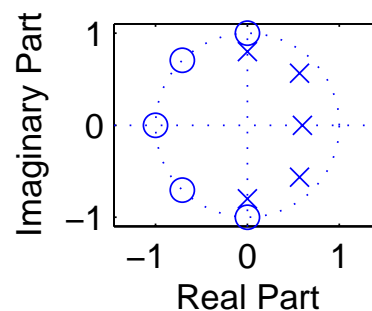
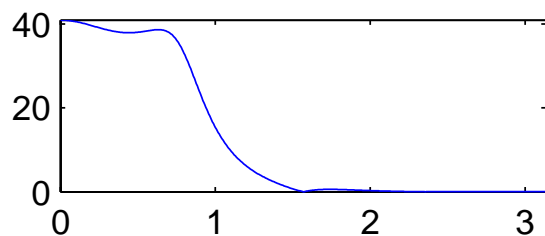
---

$Z = [\exp(j\pi/2), \exp(-j\pi/2), \exp(3j\pi/4), \exp(-3j\pi/4), \exp(j\pi)]$

$P = [.8 * \exp(j\pi/2), .8 * \exp(-j\pi/2), .8 * \exp(j\pi/4), .8 * \exp(-j\pi/4), .6]$

$B = \text{poly}(Z); A = \text{poly}(P); [H, W] = \text{freqz}(B, A); \text{plot}(W, \text{abs}(H)), \text{zplane}(B, A)$

---



---

**COMB FILTER DESIGN USING POLES AND ZEROS**


---

**Given:** A 48 Hz sinusoidal signal plus interference (from a motor) at 60 Hz.

**But:** Interference *not* sinusoidal, just periodic: 4 harmonics up to 240 Hz.

**Given:** DSP system having: (1) sampling rate of 480 Hz; and (2) 14 delays.

**Goal:** Filter out 60 Hz periodic interference, while keeping 48 Hz sinusoid.

**DSP:**  $x(t) \rightarrow \left| \begin{array}{c} \text{ANTI-} \\ \text{ALIAS} \end{array} \right| \rightarrow \left| \begin{array}{c} \text{SAMPLE} \\ @480\text{Hz} \end{array} \right| \rightarrow \left| \begin{array}{c} \text{COMB} \\ \text{FILTER} \end{array} \right| \rightarrow \left| \begin{array}{c} \text{INTERP-} \\ \text{OLATOR} \end{array} \right| \rightarrow y(t)$

**x[n]:** Discretized sinusoidal signal has  $\omega = 2\pi \frac{48 \text{ Hz}}{480 \text{ Hz}} = 0.2\pi$ . Want to keep.

**x[n]:** Interference has  $\omega = 2\pi \frac{60 \text{ Hz}}{480 \text{ Hz}} = 0.25\pi$  and harmonics at  $0.5\pi, 0.75\pi, \pi$ .

**Goal:** Reject  $\omega = 0.25\pi, 0.5\pi, 0.75\pi, \pi$  from interference; keep  $0.2\pi$  sinusoid.

**Zeros:**  $\{e^{\pm j\pi/4}, e^{\pm j\pi/2}, e^{\pm j3\pi/4}, e^{j\pi}\}$  to reject harmonics of the interference.

**Poles:**  $\{.9e^{\pm j\pi/4}, .9e^{\pm j\pi/2}, .9e^{\pm j3\pi/4}, .9e^{j\pi}\}$  to keep all the other frequencies.

**Note:** Poles inside unit circle ( $|p_n| < 1$ ) for BIBO stability.  $h[n] \approx (.9)^n u[n]$ .

**H(z):** 
$$\frac{(z - e^{j\pi/4})(z - e^{-j\pi/4})(z - e^{j\pi/2})(z - e^{-j\pi/2})(z - e^{j3\pi/4})(z - e^{-j3\pi/4})(z - e^{j\pi})}{(z - .9e^{j\pi/4})(z - .9e^{-j\pi/4})(z - .9e^{j\pi/2})(z - .9e^{-j\pi/2})(z - .9e^{j3\pi/4})(z - .9e^{-j3\pi/4})(z - .9e^{j\pi})}$$

**using:**  $(z - e^{j\pi/4}) \dots (z - e^{j8\pi/4}) = z^8 - 1$  (the eight 8<sup>th</sup> roots of unity)

**and:**  $(z - e^{j\pi/4}) \dots (z - e^{j7\pi/4}) = \frac{z^8 - 1}{z - 1} = z^7 + z^6 + z^5 + z^4 + z^3 + z^2 + z + 1$

**H(z):** 
$$\frac{z^7 + z^6 + z^5 + z^4 + z^3 + z^2 + z + 1}{z^7 + .9z^6 + .9^2z^5 + .9^3z^4 + .9^4z^3 + .9^5z^2 + .9^6z + .9^7} = H(z) = \frac{Y(z)}{X(z)}$$
. Cross-multiply:

**difference equation**  $y[n] + (.9)y[n - 1] + (.9)^2y[n - 2] + \dots + (.9)^6y[n - 6] + (.9)^7y[n - 7]$   
 $= x[n] + x[n - 1] + x[n - 2] + \dots + x[n - 5] + x[n - 6] + x[n - 7]$

`E=exp(j*pi/4); Z=[E,E^2,E^3,E^4,E^5,E^6,E^7];`

`P=.9*Z; N=0:79; S=[3,1,4,1,5,-9,2,-7]/2; 0-mean interference`

`X=1+cos(.2*pi*N)+[S S S S S S S S S S]; M=60:79; repeat period`

`subplot(421),stem(M,X(M)); B=poly(Z); A=poly(P); start at 60`

`subplot(424),zplane(B,A); [H,W]=freqz(B,A); to allow transient`

`subplot(422),plot(W,abs(H)); Y=filter(B,A,X); to decay to 0`

`subplot(423),stem(M,real(Y(M))); Interference rejected`

