BASIC CONCEPT BEHIND s-PLANE CIRCUIT ANALYSIS

Given: A circuit with several inductors and capacitors and different sources.
Also: Non-zero initial conditions: capacitors charged and inductors "juiced."
Goal: Compute any circuit voltage or current as a function of time t for $t > 0$.

Soln: Solve constant-coefficient differential equation with initial conditions.
But: Lots of algebra, even using Laplace transform (have to *obtain* diff. eqn.)
But: Is there easier way, like phasors but for any source and initial conds.?

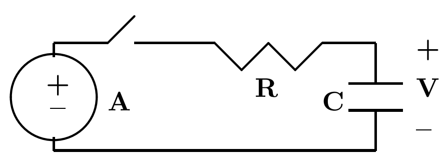
Idea: Take Laplace transform of *entire circuit*. Using $\mathcal{L}\{\frac{dx}{dt}\} = sX(s) - x(0^+)$,

Device Name :	**Resistor**	**Inductor**	**Capacitor**
Its Formula :	$v = Ri$	$v(t) = L\frac{di}{dt}$	$i(t) = C\frac{dv}{dt}$
After $\mathcal{L}$xform :	$V = RI$	$V = L(sI - i(0))$	$I = C(sV - v(0))$
Impedance Z :	R	sL	$\frac{1}{sC}$
Device Model :	$-\boxed{R}-$	$-\boxed{sL}-\boxed{-\ Li_L(0)\ +}-$	$-\boxed{\frac{1}{sC}}-\boxed{+\ \frac{v_c(0)}{s}\ -}-$

1. **Sources** have their values replaced with their Laplace transforms. Note that a **constant source** A volts or amps is replaced with $\frac{A}{s}$. Dependent sources now depend on some circuit variable $V(s)$ or $I(s)$.
2. **Capacitor:** $I = C(sV - v(0)) \to V = \frac{1}{sC}I + \frac{v(0)}{s}$ (note the + sign). Can also use the **Norton equivalent:** $(\frac{1}{sC})||$(current source $= Cv(0)$).
3. Note **sign difference:** Voltage sources for L and C initial conditions.
4. Now straightforward circuit analysis. Like phasors except s, not $j\omega$.
5. After compute desired V(s)=$\mathcal{L}\{v(t)\}$ or I(s)=$\mathcal{L}\{i(t)\}$, compute $\mathcal{L}^{-1}$. Note $V(s)$ or $I(s)$ will be a rational function, so use partial fractions.

EXAMPLE: RC CIRCUIT DRIVEN BY STEP FUNCTION

Given: Series RC circuit driven by step voltage source $A \cdot 1(t)$
Goal: Compute capacitor voltage
With: Initial capacitor voltage=B



Soln: **KVL** $\to \frac{A}{s} - I(s)R - I(s)\frac{1}{sC} - \frac{B}{s} = 0 \to I(s) = \frac{A/s - B/s}{R + 1/(sC)} = \frac{(A-B)/R}{s + 1/(RC)}$.

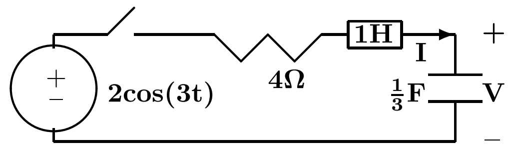
Then: $V(s) = \frac{B}{s} + \frac{1}{sC}I(s) = \frac{B}{s} + \frac{(A-B)/(RC)}{s(s+1/(RC))}$. Compute partial fractions:

Then: $V(s) = \frac{A}{s} + \frac{B-A}{s+1/(RC)} \to v(t) = \underbrace{A}_{\text{SS}} + \underbrace{(B-A)e^{-t/(RC)}}_{\text{TRANSIENT}}$ for $t > 0$.

Also: Can rewrite this as $v(t) = \underbrace{A(1 - e^{-t/(RC)})}_{\text{ZSR}} + \underbrace{Be^{-t/(RC)}}_{\text{ZIR}}$ for $t > 0$.

REDO PREVIOUS 2^{nd}-ORDER CIRCUIT EXAMPLE:

Given: Circuit shown at right:
Initially: $i(0) = 0$ and $v(0) = -1$
Then: Throw switch at $t = 0$.
Goal: Compute $i(t)$ for $t > 0$



KVL: $\frac{2s}{s^2+9} - 4I - s1I + 0 - \frac{1}{s\frac{1}{3}}I - \frac{-1}{s} = 0 \rightarrow I = [\frac{2s}{s^2+9} + \frac{1}{s}]/[4 + s + \frac{3}{s}]$.

Solve: $I = \frac{3s^2+9}{(s^2+9)(s^2+4s+3)} = \frac{0.223e^{j27^o}}{s+j3} + \frac{0.223e^{-j27^o}}{s-j3} + \frac{0.6}{s+1} + \frac{-1}{s+3}$ PARTIAL FRACTION

$\mathcal{L}^{-1}$: $i(t) = 0.223e^{-j3t+j27^o} + 0.223e^{j3t-j27^o} + 0.6e^{-t} - e^{-3t}$ for $t > 0$

Then: Simplifies to: $i(t) = 0.447\cos(3t - 27^o) + 0.6e^{-t} - e^{-3t}$ for $t > 0$.

1. This agrees with previous handout result using differential equations.
2. Note characteristic equation appears in denominator of $I(s)$ expression.
3. Partial fraction shows where transient and steady-state responses arise.

Given: The circuit shown at right:
Initially: Switch *closed* for long time.
Then: Switch is *opened* at $t = 0$.
Goal: Compute $v_C(t)$ for $t > 0$.

$\mathbf{t = 0^-}$: $C \rightarrow$ open and $L \rightarrow$ short $\rightarrow i_L(0^-) = \frac{10}{50} = 0.2$ and $v_C(0^-) = 0$.

$\mathbf{t = 0^+}$: We don't need rest of the messy computation from previous handout!

Now: With the swich open, this is now just a voltage divider for $V_C(s)$:

$$V_C(s) = \frac{1/(0.02s)}{1/(0.02s)+10+5s}(-5(0.2)) = \frac{-10}{s^2+2s+10} = \frac{(-3.33)(3)}{(s+1)^2+(3)^2} \quad \text{so that:}$$

$\mathcal{L}^{-1}$: $v_c(t) = -3.33e^{-t}\sin(3t)$ for $t > 0$. Agrees with the previous handout.

1. We don't need to go through previous mess of finding $\frac{dv_C}{dt}(0^+)$ (ugh).
2. We can compute inverse Laplace transform without partial fractions.
3. Note characteristic equation appears in denominator of $I(s)$ expression.

Given: Series RLC circuit (resistor+inductor+capacitor connected together).
Initial: Capacitor is charged up to $v(0)$ and inductor is "juiced up" to $i(0)$.
t=0: Close or throw the switch at $t = 0$. Recall $i(t)$ and $v(t)$ don't jump.
Goal: Compute current through inductor (and everything else) $i(t)$ for $t > 0$.

KVL: $Li_L(0) - sLI - RI - \frac{1}{sC}I - \frac{v_c(0)}{s} = 0 \rightarrow I(s) = \frac{Li(0)-v(0)/s}{sL+R+1/(sC)}$. Easy!

Soln: $i(t) = \mathcal{L}^{-1}$ of $I(s) = \frac{si(0)-v(0)/L}{s^2+\frac{R}{L}s+\frac{1}{LC}}$. Note the characteristic equation.