USEFUL MATHEMATICAL METHODS AND FORMULAE

Useful Trigonometry Facts and Identities

1. $\sin(0) = \cos(\frac{\pi}{2}) = 0; \cos(0) = \sin(\frac{\pi}{2}) = 1; \cos(\frac{\pi}{4}) = \sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$.
2. $\cos(-x) = \cos(x)$ (even function); $\sin(x) = -\sin(-x)$ (odd function).
3. $\sin(x) = \cos(x - \frac{\pi}{2}); \cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x); \sin^2(x) + \cos^2(x) = 1$.
4. $\cos(x)\cos(y) = \frac{1}{2}\cos(x+y) + \frac{1}{2}\cos(x-y)$ (note $x = y \rightarrow$above).

5. $e^{jx} = \cos(x) + j\sin(x)$ makes rest of trig obsolete. For example:
   $e^{jx}e^{jy} = e^{j(x+y)} = \cos(x+y) + j\sin(x+y)$. But we also have:
   $e^{jx}e^{jy} = [\cos(x) + j\sin(x)][\cos(y) + j\sin(y)]$. Multiplying out:
   $= [\cos(x)\cos(y) - \sin(x)\sin(y)] + j[\sin(x)\cos(y) + \cos(x)\sin(y)]$.
   Equating real and imaginary parts$\rightarrow$ cos and sin addition formulae.

Means of Signals with Infinite Duration

EX: $x(t) = e^{-|t|}$ has support $(-\infty, \infty)$ and duration$\rightarrow \infty$.
Then: $M(x) = \frac{1}{\infty}\int_{-\infty}^{\infty} e^{-|t|}dt$. Huh? Now what do I do?

What: Compute the *Cauchy Principal Value* of this indefinite integral:
To Do: Compute $M(x)$ for finite support $[-T, T]$ and let $T \rightarrow \infty$:
Soln: $M(x) = \underset{T\rightarrow\infty}{\text{LIM}} \frac{1}{2T}\int_{-T}^{T} e^{-|t|}dt = \underset{T\rightarrow\infty}{\text{LIM}} \frac{2}{2T}\int_{0}^{T} e^{-t}dt$ (using symmetry)
$M(x) = \underset{T\rightarrow\infty}{\text{LIM}} \frac{1}{T}(1 - e^{-T}) = 0$.

Periodic: If $x(t) = x(t+T)$ for all t (NOT just for, say, $t > 0$)
$M(x) = \frac{1}{T}\int_0^T x(t)dt = \frac{1}{T}\int_{-T/2}^{T/2} x(t)dt$ (use 1 period).

Gaps: $x(t) = \begin{cases} t & \text{for } 0 < t < 1; \\ 2 & \text{for } 2 < t < 3; \\ 0 & \text{for otherwise} \end{cases} \rightarrow$ $E(x) = \int_0^1 t^2 dt + \int_1^2 0^2 dt + \int_2^3 2^2 dt = \frac{13}{3}$.
$MS(x) = E(x)/\text{duration(x)} = \frac{13/3}{3-0} = \frac{13}{9}$.

Useful Formulae for Discrete-Time Signals

1. $1 + r + r^2 + \ldots + r^{N-1} = \frac{1-r^N}{1-r}$ for any r.
2. $1 + r + r^2 + \ldots = \frac{1}{1-r}$ if $|r| < 1$.
3. For signals having infinite support $[0, \infty)$, use
   $M(x) = \underset{N\rightarrow\infty}{\text{LIM}} \frac{1}{N}\sum_{n=0}^{N-1} x[n]$ (compare to Cauchy Principal Value).
4. $x[n] = A\cos(\omega n + \theta)$ is periodic only if $\omega = 2\pi(\frac{\text{RATIONAL}}{\text{NUMBER}})$.
5. $|x| < a$ is equivalent to $-a < x < a$.