**ASSIGNED:** Never. **READ:** Sections 10.1-10.4 (p. 782-792) (1996 ed.)
**DUE DATE:** Never. **TOPICS:** Multirate filtering, IIR filtering design.

**NOTE: This problem set will not be collected or graded.**

[10] 1. *Using multirate filtering to alter the pitch of musical notes:*
You have a snippet of music at note E. You want to change note E to note A.
Note A has frequency 2/3 that of note E. Design a multirate system to do this.

[30] 2. *Drill on multirate filtering and aliasing:*
A 400 Hertz sinusoid is input to the DSP systems below at $1000\frac{\text{SAMPLE}}{\text{SECOND}}$.
The output of each one is one or more sinusoids. Compute their frequencies.

[10] a. 400 Hz→**[A/D]** →**[↑ 2]** →**[↓ 3]** →**[D/A]**→?
[10] b. 400 Hz→**[A/D]** →**[↓ 3]** →**[↑ 2]** →**[D/A]**→?
[10] c. 400 Hz→**[A/D]** →**[↓ 4]** →**[↑ 2]** →**[D/A]**→?
Try checking your answers using Matlab (see my lecture notes for Matlab commands).

[20] 3. *Using multirate filtering to ease digital-to-analog conversion:*
A signal bandlimited to 500 Hertz is input into a DSP system at $1500\frac{\text{SAMPLE}}{\text{SECOND}}$.
Consider the analog lowpass filter used to reconstruct the signal from its samples.
[05] a. What are the widths of the passband and transition bands, in Hertz?
[05] b. How many inductors or capacitors are needed for stopband gain≤0.001?
[10] c. Suppose we upsample by 11 and then use a digital lowpass filter.
How many inductors or capacitors are needed for stopband gain≤0.001?
**Hint:** From EECS 215, we get 6 dB/octave/capacitor or inductor. 0.1=–20 dB.
**Note:** In (b) and (c) neglect the gain drop at the analog filter corner frequency.

[40] 4. *IIR filter design using analog Butterworth filters:*
An analog $N^{th}$-order Butterworth filter has no zeros, and poles at
$\{\Omega_o e^{j\pi k/N}, N-\frac{N-1}{2}\leq k\leq N+\frac{N-1}{2}\}$. **Example:** N=5→ $\{e^{\pm j\frac{3\pi}{5}}, e^{\pm j\frac{4\pi}{5}}, e^{j\pi}\}$.
Note these poles are all on a circle of radius $\Omega_o$ in the left half-plane.

Assume throughout $N$ is an odd integer and the DC gain $|H_a(0)| = 1$.
We use bilinear transformation with $T = 2$ to design a digital filter $H(z)$.

[10] a. Show that $H_a(s)H_a(-s) = -\frac{\Omega_o^{2N}}{s^{2N}-\Omega_o^{2N}}$ and $|H_a(j\Omega)|^2 = 1/[1+(\frac{\Omega}{\Omega_o})^{2N}]$.

[10] b. Show that $H(z)H(1/z) = -\frac{\Omega_o^{2N}(z+1)^{2N}}{(z-1)^{2N}-\Omega_o^{2N}(z+1)^{2N}}$.

[10] c. Show that $|H(e^{j\omega})|^2 = 1/[1+(\frac{\tan(\omega/2)}{\Omega_o})^{2N}]$. Compare this to result of (a).

[10] d. Describe the gain functions $|H_a(j\Omega)|$ and $|H(e^{j\omega})|$ as $N\to\infty$.