Recall: $z_o^n \rightarrow \boxed{H(z)} \rightarrow H(z_o)z_o^n$. Eigenfunction of LTI.
Now: $z_o^n = e^{+j\omega_o n} \rightarrow \boxed{H(z)} \rightarrow H(e^{+j\omega_o})e^{+j\omega_o n}$.
and: $z_o^n = e^{-j\omega_o n} \rightarrow \boxed{H(z)} \rightarrow H(e^{-j\omega_o})e^{-j\omega_o n}$.
$\cos(\omega_o n) \rightarrow \boxed{H(z)} \rightarrow |H(e^{j\omega_o})| \cos(\omega_o n + arg[H(e^{j\omega_o})])$.

Gain: Amplitude increases by factor of $|H(e^{j\omega_o})|$.
Phase: Shift by $arg[H(e^{j\omega_o})] = \tan^{-1} \frac{Im[H(e^{j\omega_o})]}{Re[H(e^{j\omega_o})]}$.
DTFT: $H(e^{j\omega_o}) = \sum h(n)e^{-j\omega_o n} = DTFT[h(n)]$.
Zero: $H(z)$ has a zero at $e^{\pm j\omega_o} \rightarrow y(n) = 0$ in steady-state.
Pole: $H(z)$ has a pole at $e^{\pm j\omega_o} \rightarrow y(n) \rightarrow \infty$ blows up.

EX #1: $h(n) = (\frac{1}{2})^n u(n)$ and $x(n) = \{\ldots -1, 0, 1, 0, -1 \ldots\} = \cos(\frac{\pi n}{2})$.
$H(e^{j\omega}) = 1/(1 - \frac{1}{2}e^{-j\omega}) = 1/(1 + \frac{j}{2}) = 0.89e^{-j26.6^o}$ at $\omega = \frac{\pi}{2}$
$y(n) = 0.89\cos(\frac{\pi n}{2} - 26.6^o) = \{\ldots 0.8, 0.4, -0.8, -0.4, 0.8, 0.4 \ldots\}$.

EX #2: $h(n) = (\frac{1}{2})^n u(n)$ and $x(n) = \{\ldots -1, 1, -1, 1, -1 \ldots\} = \cos(\pi n)$.
$H(e^{j\omega}) = 1/(1 + \frac{1}{2}) = \frac{2}{3}$ at $\omega = \pi \rightarrow y(n) = \frac{2}{3}\cos(\pi n) = \frac{2}{3}(-1)^n$.
Why $\frac{2}{3}$? $y(n) = h(n) * x(n) = \sum(\frac{1}{2})^i(-1)^{n-i} = (-1)^n \sum(-\frac{1}{2})^i = (-1)^n \frac{1}{1-(-\frac{1}{2})}$.

EX #3a: $h(n) = \{\frac{1}{2}, +\frac{1}{2}\} \rightarrow H(e^{j\omega}) = \frac{1}{2}(1 + e^{-j\omega}) = \cos(\frac{\omega}{2})e^{-j\omega/2}$. Lowpass.
EX #3b: $h(n) = \{\frac{1}{2}, -\frac{1}{2}\} \rightarrow H(e^{j\omega}) = \frac{1}{2}(1 - e^{-j\omega}) = \sin(\frac{\omega}{2})e^{j(\pi-\omega)/2}$. High.

Notch: $H(z) = (z - e^{j\omega_o})(z - e^{-j\omega_o})\frac{1}{z} = z - 2\cos(\omega_o) + z^{-1}$.
filter: $H(e^{j\omega}) = 2\cos(\omega) - 2\cos(\omega_o)$. $h(n) = \{1, -2\cos(\omega_o), 1\}$.

Comb: $H(z) = \frac{1}{2M+1}\sum_{k=-M}^{M} z^{-Lk} \rightarrow$ zeros at $z = e^{\frac{j2\pi k}{L(2M+1)}}$
for $k = \pm 1 \ldots \pm LM$ unless L divides k. See p. 349.

Reson- $H(z) = Bz^2/[(z - re^{j\omega_o})(z - re^{-j\omega_o})] = Bz^2/[z^2 - 2r\cos(\omega_o)z + r^2]$.
ator: On unit circle $|z| = 1$, $H(e^{j\omega})$ peaks at $\omega = \pm\cos^{-1}[\frac{1+r^2}{2r}\cos\omega_o]$.
$r \rightarrow 1$: Resonant freq.$\simeq \omega_o$; 3 dB bandwidth$\simeq 2(1 - r)$. See p. 342.

All- $H(z) = z^D \prod \frac{z^{-1} - z_k^*}{1 - z_k z^{-1}}$ or $\frac{A(1/z)}{z^N A(z)} \rightarrow H(z)H(1/z) = 1 \rightarrow |H(e^{j\omega})| = 1$.
pass: $|z_i| < 1 \rightarrow$stable and causal; $A(z) = \mathcal{Z}\{causal\ signal\}$.

Signal: $x(n) = \{\ldots 4, 0, 1, 0, 1, 0, \underline{4}, 0, 1, 0, 1, 0, 4 \ldots\}$ has period=N=6.
System: $y(n) = \frac{1}{2}(x(n) + x(n-1))$ (average the 2 most recent inputs).
Goal: Compute Discrete Time Fourier Series (DTFS) of input $x(n)$.
Goal: Compute Discrete Time Fourier Series (DTFS) of output $y(n)$.

DTFS: $x(n) = \sum_{k=0}^{N-1} X_k e^{j2\pi nk/N}$ for all n where N=period of $x(n)$.

Compute: $X_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N}$ for $k = 0 \ldots N-1$. Use `fft/N`

Note: Like Fourier series, except for finite number N of harmonics.
Compare: $x(t) = \sum_{-\infty}^{\infty} X_k e^{j2\pi kt/T}$ where $X_k = \frac{1}{T}\int_0^T x(t) e^{-j2\pi kt/T} dt$.

Compute: `fft([4 0 1 0 1 0],6)/6=[1 .5 .5 1 .5 .5]`
Note: $x(n)$ real and even $\rightarrow$ DTFS coefficients real and even (extensions).
Note: $x(n) = 0$ for odd n $\rightarrow$ DTFS coefficients repeat (the last 3=first 3).

DTFS: $x(n) = 1 + .5e^{j2\pi n/6} + .5e^{j4\pi n/6} + e^{j6\pi n/6} + .5e^{j8\pi n/6} + .5e^{j10\pi n/6}$
DTFS: $x(n) = 1 + \cos(\frac{\pi}{3}n) + \cos(\frac{2\pi}{3}n) + \cos(\pi n)$ after simplifying above.

Average $\frac{1}{6}(4^2 + 0^2 + 1^2 + 0^2 + 1^2 + 0^2) = 3$ in time domain agrees with

Power: $1^2 + (\frac{1}{2})^2 + (\frac{1}{2})^2 + 1^2 + (\frac{1}{2})^2 + (\frac{1}{2})^2 = 3$ by Parseval's theorem.

DTFT: $y(n) = \frac{1}{2}(x(n) + x(n-1)) \rightarrow h(n) = \{\underline{\frac{1}{2}}, \frac{1}{2}\} \rightarrow H(e^{j\omega}) = \frac{1}{2} + \frac{1}{2}e^{-j\omega}$.
DTFT: $H(e^{j\omega}) = \frac{1}{2} + \frac{1}{2}e^{-j\omega} = (\cos\frac{\omega}{2})e^{-j\omega/2}$ after simplifying. Then have:

$x(n)$:	1	$\cos(\frac{\pi}{3}n)$	$\cos(\frac{2\pi}{3}n)$	$\cos(\pi n)$
ω :	0	$\pi/3$	$2\pi/3$	π
$H(e^{j\omega})$:	1	$0.866\angle-\frac{\pi}{6}$	$0.5\angle-\frac{\pi}{3}$	0
Gain :	1	0.866	0.5	0
Phase :	0	$-\pi/6$	$-\pi/3$	NA

Then: $y(n) = 1 + 0.866\cos(\frac{\pi}{3}n - \frac{\pi}{6}) + 0.5\cos(\frac{2\pi}{3}n - \frac{\pi}{3}) + 0$ simplifies to
$y(n) = \{\ldots 2, 2, .5, .5, .5, .5, \underline{2}, 2, .5, .5, .5, .5, 2, 2 \ldots\}$ Period still=6.

Note: Higher frequencies of $x(n)$ reduced in amplitude (attenuated) in $y(n)$.
Thus: This system smoothes the input signal (n) (running 2-point average).