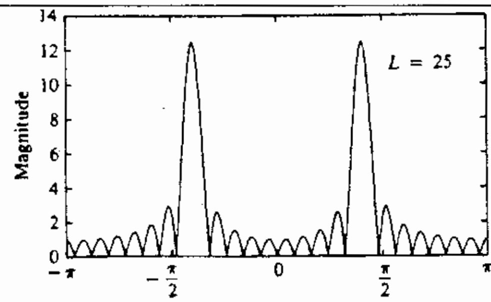
Given: $y(n) = A_1 \cos(\omega_1 n + \theta_1) + A_2 \cos(\omega_2 n + \theta_2), 0 \le n \le L-1$.
Goal: To determine the unknown ω_i from $\{y(n), 0 \le n \le L-1\}$.
Using: DFT, FFT, convolution, filtering, sinc functions, windowing.

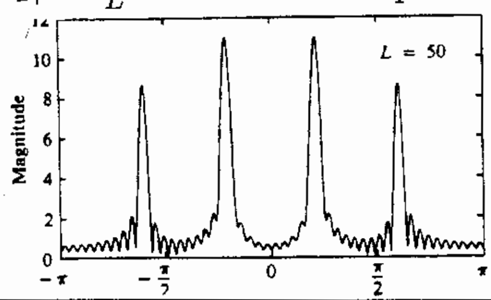
Easy? Compute $Y_k = \sum_{n=0}^{N-1} y(n) e^{-j2\pi nk/N}, 0 \le k \le N-1$, where $N >> L$. Look for peaks in $|Y_k|$ at $k = k_1, k_2$. Then $\omega_i = \frac{2\pi}{N} k_i \rightarrow$frequencies.

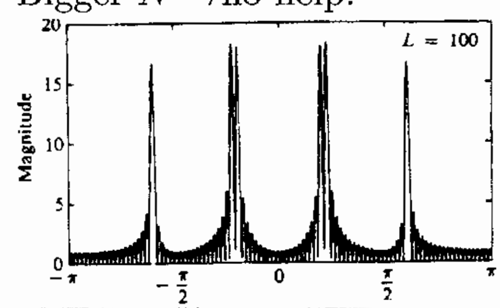
But: Consider $y(n) = \cos(\omega_o n)$ for $0 \le n \le L-1 \rightarrow y(n) = \cos(\omega_o n) w(n)$
$\rightarrow Y(e^{j\omega}) = \frac{1}{2}[W(e^{j(\omega-\omega_o)}) + W(e^{j(\omega+\omega_o)})]$ where $w(n) = 1, 0 \le n \le N-1$
and $W(e^{j\omega}) = DTFT\{w(n)\} = \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$ (previous HO).

EX: Data length is $L = 25$
0-pad: Zero-padded $N = 2048$
Does NOT increase the resolution; smoothes.
Note: *Sidelobes → leakage*
(from text p. 434-439).



Lobe: $W(e^{j\omega})$ has first zero crossing at $\omega = \frac{2\pi}{L}$. Well-known in optics:
Need: **Roughly** $|\omega_1 - \omega_2| > \frac{2\pi}{L}$ to resolve two peaks. Bigger $N \rightarrow$no help.
EX: $L = 50, 100$
DFT: $N = 2048$
Freqs: $0.2\pi, 0.22\pi, 0.6\pi$
L=50: Can't resolve
L=100: CAN resolve
(text p.435)





Windows (DSP kind, not the Microsoft kind)

Idea: Instead of *rectangular window* $w(n) = 1, 0 \le n \le L-1$
use some *smoother* window $w(n)$ to avoid sidelobes (ripple).
But: Reduce sidelobes→increase main lobe width→ *lose* resolution.
Why? Lowpass filter: Use $h(n) = \frac{\sin(\omega_c n)}{\pi n} w(n)$ for various $w(n)$:

Lowpass filters. Windows defined overleaf (p. 629)

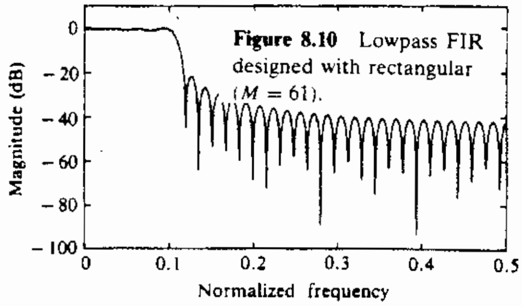


Figure 8.10 Lowpass FIR designed with rectangular ($M = 61$).

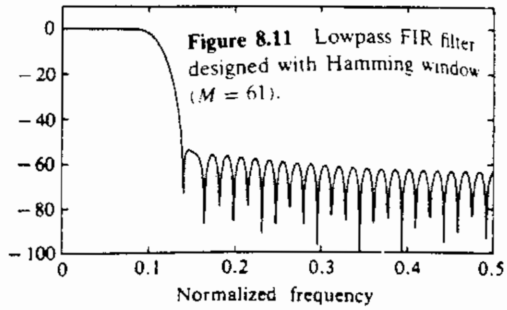


Figure 8.11 Lowpass FIR filter designed with Hamming window ($M = 61$).

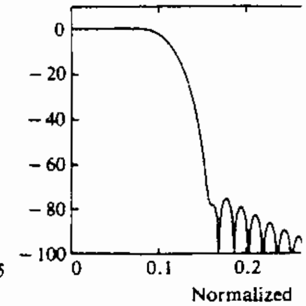


Figure 8.12 Lowpass FIR designed with Blackman ($M = 61$).

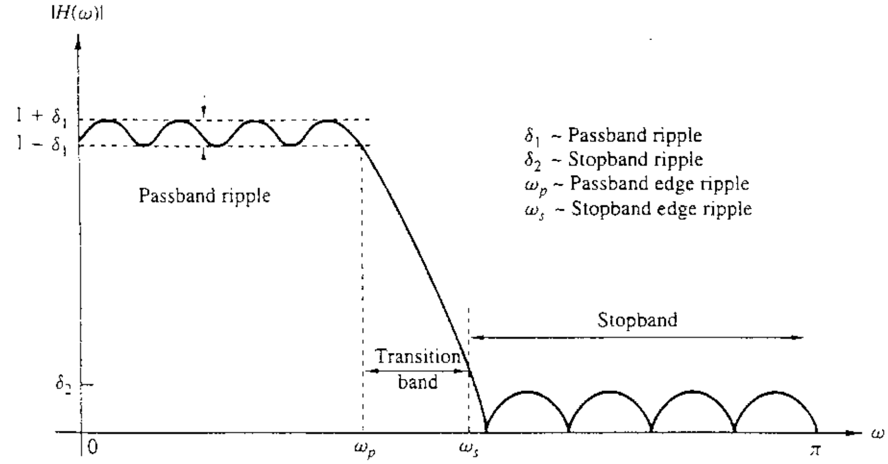


Figure 8.2 Magnitude characteristics of physically realizable filters.

TABLE 8.2 IMPORTANT FREQUENCY-DOMAIN CHARACTERISTICS OF SOME WINDOW FUNCTIONS

Type of window	Approximate transition width of main lobe	Peak sidelobe (dB)
Rectangular	$4\pi/M$	−13
Bartlett	$8\pi/M$	−27
Hanning	$8\pi/M$	−32
Hamming	$8\pi/M$	−43
Blackman	$12\pi/M$	−58

TABLE 8.1 WINDOW FUNCTIONS FOR FIR FILTER DESIGN

Name of window	Time-domain sequence, $h(n)$, $0 \le n \le M-1$
Bartlett (triangular)	$1 - \dfrac{2\left\lvert n - \frac{M-1}{2}\right\rvert}{M-1}$
Blackman	$0.42 - 0.5\cos\dfrac{2\pi n}{M-1} + 0.08\cos\dfrac{4\pi n}{M-1}$
Hamming	$0.54 - 0.46\cos\dfrac{2\pi n}{M-1}$
Hanning	$\frac{1}{2}\left(1-\cos\dfrac{2\pi n}{M-1}\right)$
Kaiser	$\dfrac{I_0\left[\alpha\sqrt{\left(\frac{M-1}{2}\right)^2 - \left(n-\frac{M-1}{2}\right)^2}\right]}{I_0\left[\alpha\left(\frac{M-1}{2}\right)\right]}$
Lanczos	$\left\{\dfrac{\sin\left[2\pi\left(n-\frac{M-1}{2}\right)\Big/(M-1)\right]}{2\pi\left(n-\frac{M-1}{2}\right)\Big/\left(\frac{M-1}{2}\right)}\right\}^L \quad L>0$
Tukey	$1, \quad \left\lvert n-\frac{M-1}{2}\right\rvert \le \alpha\frac{M-1}{2} \quad 0<\alpha<1$ $\frac{1}{2}\left[1+\cos\left(\dfrac{n-(1+\alpha)(M-1)/2}{(1-\alpha)(M-1)/2}\pi\right)\right]$ $\alpha(M-1)/2 \le \left\lvert n-\frac{M-1}{2}\right\rvert \le \frac{M-1}{2}$

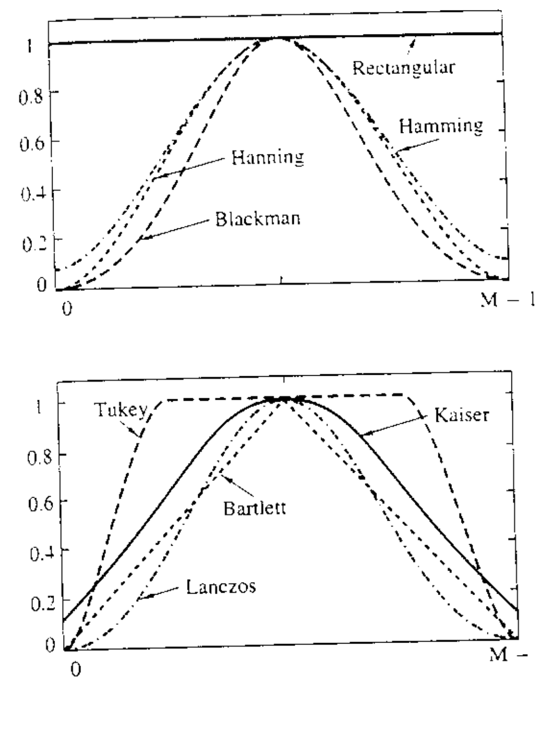


Figure 8.5 Shapes of several window

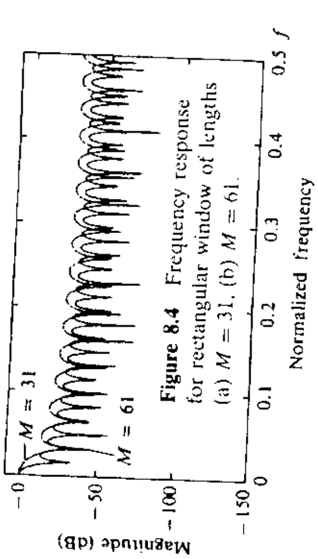
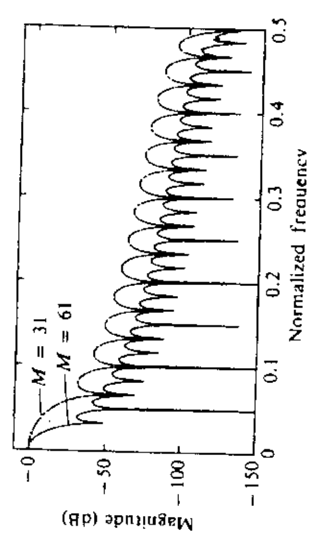


Figure 8.4 Frequency response for rectangular window of lengths (a) $M = 31$, (b) $M = 61$.



Normalized frequency

Figure 8.6 Frequency responses of Hanning window for (a) $M = 31$ and (b) $M = 61$.

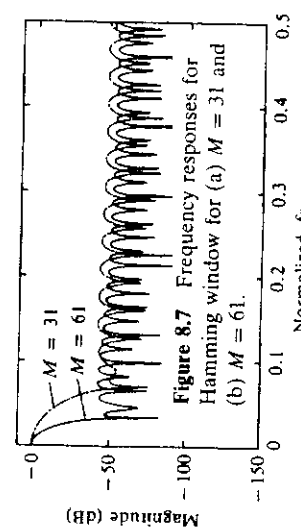


Figure 8.7 Frequency responses for Hamming window for (a) $M = 31$ and (b) $M = 61$.

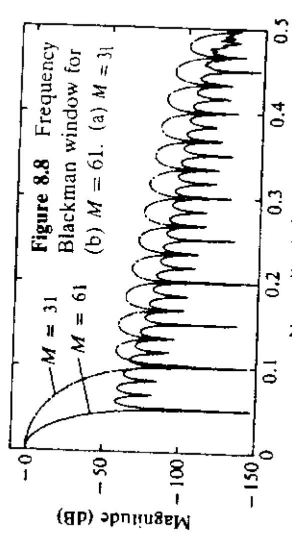


Figure 8.8 Frequency Blackman window for (a) $M = 31$ (b) $M = 61$.