TYPE	APPLICATION	CONTINUOUS – TIME	DISCRETE – TIME
2 – sided	Atlanta Airport	$\mathcal{L}\{x(t)\} = \int_{-\infty}^{\infty} x(t)e^{-st}dt$	$\mathcal{Z}\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$
Example	Inverse Rational	$\mathcal{L}\{e^{at}u(t)\} = 1/(s-a)$	$\mathcal{Z}\{a^n u[n]\} = z/(z-a)$
EX : ROC	Noncausal Stability	$\{s : Re[s] > Re[a]\}$	$\{z : \|z\| > \|a\|\}$
1 – sided	Initial Condition	$\mathcal{L}\{x(t)\} = \int_{0}^{\infty} x(t)e^{-st}dt$	$\mathcal{Z}\{x[n]\} = \sum_{n=0}^{\infty} x[n]z^{-n}$
Formula	Differ. Equations	$\mathcal{L}\{\frac{dx}{dt}\} = sX(s) - x(0)$	$\mathcal{Z}\{x[n-1]\} = z^{-1}X(z) + x(-1)$
Relation	Fourier – Laplace – Z	$s = j\omega$	$z = e^{j\omega}$
Fourier	Frequency Response	$\mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$	$\mathrm{DTFT}\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$
Example	$dy/dt = ay(t) + x(t)$ $y[n] = ay[n-1] + x[n]$	$H(\omega) = 1/(j\omega - a)$	$H(e^{j\omega}) = 1/(1 - ae^{-j\omega})$
Inverse	Filter δ Response	$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$	$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$
Example	$\begin{cases} 1 & \text{for } \|\omega\| < \omega_o \\ 0 & \text{for otherwise} \end{cases}$	$h(t) = \sin(\omega_o t)/(\pi t)$	$h[n] = \sin(\omega_o n)/(\pi n)$
Series	Periodic functions	$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j2\pi kt/T}$	$\mathrm{DTFS} : x[n] = \sum_{k=0}^{N-1} X_k e^{j2\pi nk/N}$
Coefficients	Line Spectrum	$X_k = \frac{1}{T}\int_0^T x(t)e^{-j2\pi kt/T}dt$	$X_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-j2\pi nk/N}$
DFT	Discrete Spectrum	$X_k = \sum_{n=0}^{N-1} x[n]e^{-j2\pi nk/N}$	$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{j2\pi nk/N}$
Relation	DFT – DTFT – Z	Compute using FFT	$\omega = 2\pi k/N \quad z = e^{j2\pi k/N}$
BIBO Stability	Causal systems	Poles in LHP : $Re[p_n] < 0$	Poles inside unit circle : $\|z_n\| < 1$

- DTFT $X(e^{j\omega})$ of $x[n]$ is periodic in ω with period=2π.
- DFT X_k of $x[n]$ is *single period* of periodic in k with period=N.
- IDFT $x[n]$ of X_k is *single period* of periodic in n with period=N.
- If $x[n]$ is real, all Fourier transforms are *conjugate symmetric*:
- $X(e^{j\omega})^* = X(e^{j(2\pi-\omega)})$ and $X_k^* = X_{N-k}$;
- $X(e^{j\omega}) = \sum_{k=0}^{\infty} a_k \cos(k\omega) + j\sum_{k=1}^{\infty} b_k \sin(k\omega)$
- $|X(e^{j\omega})|$ and $|X_k|$ are real, non-negative, and even in k;
- Real$[X(e^{j\omega})]$ and Real$[X_k]$ are real and even in k;
- Arg$[X(e^{j\omega})]$ and Arg$[X_k]$ are real and imaginary in k;
- Imag$[X(e^{j\omega})]$ and Imag$[X_k]$ are real and odd in k.

CONTINUOUS AND DISCRETE FOURIER SERIES

CONCEPT	CONTINUOUS	DISCRETE
Name	Fourier Series	DTFS
Periodic	$x(t) = x(t+T)$	$x[n] = x[n+N]$
Expansion	$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j2\pi kt/T}$	$x[n] = \sum_{k=0}^{N-1} X_k e^{j2\pi kn/N}$
Coefficient	$X_k = \frac{1}{T}\int_0^T x(t)e^{-j2\pi kt/T}dt$	$X_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}$
Parseval	$\frac{1}{T}\int_0^T \lvert x(t)\rvert^2 dt = \sum_{k=-\infty}^{\infty} \lvert X_k\rvert^2$	$\frac{1}{N}\sum_{n=0}^{N-1} \lvert x[n]\rvert^2 = \sum_{k=0}^{N-1} \lvert X_k\rvert^2$
Parseval	$\frac{1}{T}\int_0^T x(t)y(t)^* dt = \sum_{k=-\infty}^{\infty} X_k Y_k^*$	$\frac{1}{N}\sum_{n=0}^{N-1} x[n]y[n]^* = \sum_{k=0}^{N-1} X_k Y_k^*$

EX: $x[n] = \{\ldots 7,5,3,1,\underline{7},5,3,1,7,5,3,1\ldots\}$. Compute DTFS.

Soln: Period=N=4 $\rightarrow X_k = \frac{1}{4}\sum_{n=0}^{3} x[n]e^{-j2\pi nk/4} = \frac{1}{4}\sum_{n=0}^{3} x[n](-j)^{nk}$

Coeff: $X_0 = \frac{1}{4}(7+5+3+1) = 4$. $X_1 = \frac{1}{4}(7\text{–j}5\text{–}3\text{+j}1) = 1\text{–j} = \sqrt{2}e^{-j\pi/4}$.

Coeff: $X_2 = \frac{1}{4}(7\text{–}5 + 3\text{–}1) = 1$. $X_3 = \frac{1}{4}(7\text{+j}5\text{–}3\text{–j}1) = 1\text{+j} = \sqrt{2}e^{+j\pi/4}$.

DTFS $x[n] = 4e^{j0n} + (1\text{–j})e^{j2\pi n/4} + 1e^{j4\pi n/4} + (1\text{+j})e^{j6\pi n/4} = 4 + \sqrt{2}\cos(\frac{\pi}{4}n - \frac{\pi}{2}) + \cos(\pi n)$

since $e^{j6\pi n/4} = e^{-2j\pi n/4} = (-j)^n$ and $e^{j4\pi n/4} = e^{-j4\pi n/4} = (-1)^n = \cos(\pi n)$

and $Ap^n + A^*(p^*)^n = 2\lvert A\rvert \cdot \lvert p\rvert^n \cos(\omega n + \theta)$ where $A = \lvert A\rvert e^{j\theta}$ and $p = \lvert p\rvert e^{j\omega}$.

Also: Parseval: $\frac{1}{4}(7^2+5^2+3^2+1^2) = \lvert 4\rvert^2 + \lvert 1-j\rvert^2 + \lvert 1\rvert^2 + \lvert 1+j\rvert^2 = 21$.

CONTINUOUS AND DISCRETE FOURIER TRANSFORMS

CONCEPT	CONTINUOUS	DISCRETE
Forward	$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$	$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$
Inverse	$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$	$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$
Delay	$\mathcal{F}\{x(t-T)\} = X(\omega)e^{-j\omega T}$	$\text{DTFT}\{x[n-N]\} = X(e^{j\omega})e^{-j\omega N}$
Pulse	$\mathcal{F}\left\{\begin{cases} 1 & \lvert t\rvert < \frac{T}{2} \\ 0 & \lvert t\rvert > \frac{T}{2}\end{cases}\right\} = \frac{\sin(\omega T/2)}{\omega/2}$	$\text{DTFT}\left\{\begin{cases} 1 & \lvert n\rvert \le \frac{N-1}{2} \\ 0 & \lvert n\rvert > \frac{N-1}{2}\end{cases}\right\} = \frac{\sin(\omega N/2)}{\sin(\omega/2)}$
Lowpass	$\mathcal{F}^{-1}\left\{\begin{cases} 1 & \lvert\omega\rvert < W \\ 0 & \lvert\omega\rvert > W\end{cases}\right\} = \frac{\sin(Wt)}{\pi t}$	$\text{DTFT}^{-1}\left\{\begin{cases} 1 & \lvert\omega\rvert < W \\ 0 & \text{otherwise}\end{cases}\right\} = \frac{\sin(Wn)}{\pi n}$
Relation	$\mathcal{F}\{x(t)\} = \mathcal{L}\{x(t)\}\rvert_{s=j\omega}$	$\text{DTFT}\{x[n]\} = \mathcal{Z}\{x[n]\}\rvert_{z=e^{j\omega}}$