Recitation 10 - EECS 451, Winter 2009

April 1st, 2009

OUTLINE

- Review of Important Concepts
- Practice problems

Concepts: Design of Digital Filters

1. Design of digital IIR filters from analog filters
   (a) IIR filters can generally offer better approximation to a desired magnitude response $|H(\omega)|$ than FIR filters at the expense of linear phase
   (b) For a given analog filter, we can convert it into a digital filter using a transformation from the s-plane to the $z-$plane, e.g. bilinear transformation
2. Bilinear Transformation (BLT)
   (a) The mapping of BLT is defined as $s = \frac{2}{T_s}\frac{1-z^{-1}}{1+z^{-1}}$, equivalently $z = \frac{2/T_s+s}{2/T_s-s}$
   (b) When analog filter $H_a(s)$ has a rational form, BLT yields a $H(z)$ which has a rational form
   (c) The entire **left half** of the s-plane is mapped **inside** the unit circle in the z-plane
   (d) The entire **right half** of the s-plane is mapped **outside** the unit circle in the z-plane
   (e) The **imaginary axis** in the s-plane is mapped **onto** the unit circle in the z-plane
3. Bilinear Transformation (BLT)
   (a) A stable $H_a(s)$ yields a stable $H(z)$ by performing BLT since all poles in the left half s-plane is mapped inside the unit circle in the z-plane
   (b) $H(\omega)$ depends only on $H_a(\Omega)$ since the imaginary axis in the s-plane is mapped onto the unit circle in the z-plane
   (c) The relationship between ω and Ω is highly nonlinear (called "frequency warping") given by

$$\Omega = \frac{2}{T_s}\tan\frac{\omega}{2}$$

(d) Poles/zeros at $s = \infty$ map to poles/zeros at $z = -1$ by BLT

(e) Real poles/zeros remain real and complex conjugate pairs remain complex conjugate pairs after applying BLT

4. Butterworth filter

   (a) They have rational system function $H_a(s)$

   (b) The frequency response is given by

$$|H_a(\Omega)|^2 = \frac{1}{1 + (\frac{\Omega}{\Omega_c})^{2N}}$$

   where N is the order of the filter

   (c) The filter has N poles equally spaced on a circle of radius Ω_c in the left half plane

   (d) Pro: Maximally flat in the pass band

   (e) Con : Not a sharp cut off

5. Chebyshev filter (Type I)

   (a) They have a rational system function $H_a(s)$

   (b) They have equi-ripples in the passband and monotonically decreasing in stopband

   (c) They are all-pole analog filters and the frequency response is given by

$$|H_a(\Omega)|^2 = \frac{1}{1 + \epsilon^2 C_N^2(\frac{\Omega}{\Omega_c})}$$

   where C_N is the N-th order Chebyshev polynomial

6. Elliptic filter

   (a) The frequency response is given by

$$|H_a(\Omega)|^2 = \frac{1}{1 + \epsilon^2 U_N^2(\frac{\Omega}{\Omega_c})}$$

   where $U_N(x)$ is the Jacobian elliptic function (something which we need not know and MATLAB takes care of)

   (b) Pro: Sharpest transition for a given N

   (c) Con: Ripple in both passband and stopband

Problems

1. Use the bilinear transformation to convert the analog filter with system function

$$H_a(s) = \frac{s + 0.1}{(s + 0.1)^2 + 9}$$

   into a digital IIR filter. Select $T_s = 0.1$ and determine the location of the poles and zeros of $H(z)$

2. An ideal analog integrator is described by the system function $H_a(s) = \frac{1}{s}$. A digital integrator with system function $H(z)$ can be obtained by use of the bilinear transformation

   (a) Write the difference equation for the digital integrator relating the input $x(n)$ to the output $y(n)$

   (b) Roughly sketch the magnitude $|H_a(F)|$ and phase $\Theta(F)$ of the analog integrator

   (c) Determine the magnitude and phase response of $H(\omega)$

   (d) Compare the magnitude and phase characteristics obtained in part (b) and (c). How well does the digital integrator match the magnitude and phase characteristics of the analog integrator?

   (e) The digital integrator has a pole at $z = 1$. If you implement this filter on a digital computer, what restriction might you place on the input sequence $x(n)$ to avoid computational difficulties?

3. Suppose that we are given a continuous-time lowpass filter with the passband ripple δ_1, stopband ripple δ_2, passband edge frequency Ω_p and stop band edge frequency Ω_s. A set of discrete-time lowpass filters can be obtained from the above analog filter by bilinear transformation with T_s variable

   (a) Assuming that Ω_p is fixed, find the value of T_s such that the corresponding cut off frequency for the discrete time system is $\omega_p = \pi/2$

   (b) With Ω_p fixed, sketch ω_p as a function of T_s such that $0 < T_s < \infty$