Recitation 3 - EECS 451, Winter 2009

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OUTLINE

- Review of important concepts (Lecture 3-4)
- Practise problems

Concepts: Discrete Time Systems

1. Classification of DT systems: For $y(n) = \mathcal{T}(x(n))$,
   - static (memoryless): $y(n)$ depends only on the present input
   - causal: $y(n)$ depends only on the present and past inputs
   - time-invariant: $\mathcal{T}(x(n-k)) = y(n-k)$, where k is an integer
   - linear: $\mathcal{T}(a_1 x_1(n) + a_2 x_2(n)) = a_1 \mathcal{T}(x_1(n)) + a_2 \mathcal{T}(x_2(n))$
   - stable (BIBO): every bounded input provides a bounded output
2. Linear Time Invariant (LTI) System: $y(n) = \mathcal{T}(x(n))$
   - completely characterized by the impulse response $h(n) = \mathcal{T}(\delta(n))$
   - output given by simple convolution operation, $y(n) := h(n) * x(n) = \sum_{k=-\infty}^{\infty} x(n-k)h(k)$
3. Properties of convolution
   - Commutative: $x(n) * h(n) = h(n) * x(n)$
   - Associative: $(x(n) * h_1(n)) * h_2(n) = x(n) * (h_1(n) * h_2(n))$
   - Distributive: $x(n) * (h_1(n) + h_2(n)) = x(n) * h_1(n) + x(n) * h_2(n)$
4. Classification of LTI systems by the impulse response
   - Causal: The given LTI system is causal if and only if $h(n) = 0 \ \forall n < 0$.
   - Stable (BIBO): The given LTI system is stable if and only if $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$
5. Two classes of LTI systems characterized by the impulse response
   - Finite Impulse Response (FIR) system: number of non-zero $h(n)$'s are finite.
   - Infinite Impulse Response (IIR) system: number of non-zero $h(n)$'s are infinite.

6. The system defined by linear constant coefficient difference equation $y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$,
   - is LTI and causal
   - can be implemented by direct form I or direct form II (more efficient).
   - is recursive and the impulse response is IIR if $N \geq 1$.
   - is non-recursive and the impulse response is FIR if $N = 0$. $h(n) = \{\underline{b_0}, b_1, \cdots, b_M\}$.

Concepts: 2-sided z-transform

1. Definition of z-transform
   - For a given DT signal $x(n)$, $X(z) := \sum_{n=-\infty}^{\infty} x(n) z^{-n}$, where z is complex valued.
   - If $x(n)r^{-n}$ is absolutely summable, then $X(z)$ has a finite value where $|z| = r$.
2. ROC (Region of Convergence)
   - The set of values of z for which the sequence $x(n)z^{-n}$ is absolutely summable, i.e. $\{z \in \mathbf{C} : \sum_{n=-\infty}^{\infty} |x(n)z^{-n}| < \infty\}$, where $\mathbf{C}$ is the set of complex numbers.
   - Simply put, ROC indicates the region of z where $X(z)$ is finite.
   - By definition, ROC cannot contain any poles.
3. The shape of ROCs:
   - The ROC of an anti-causal signal is of the form $|z| < |a|$.
   - The ROC of a causal signal is of the form $|z| > |a|$.
   - The ROC of a two sided signal is of the form $|a| < |z| < |b|$.
   - The ROC of a finite length signal is the entire $z-$space except for $z = 0$ and/or $z = \infty$.

Problems

1. Determine which of the following systems is static, linear, time-invariant, causal, stable
   (a) $y(n) = x^2(n+1)$
   (b)
   $$y(n) = \begin{cases} x(-n), & n < 0 \\ x(n), & n \geq 0. \end{cases}$$
   (c) $y(n) = \sum_{k=-\infty}^{\infty} x(n-k)p(k)$, where $p(n) = \{-10, \cdots, -1, \underline{0}, 1, \cdots, 10\}$
2. Compute the output of the following LTI systems
   (a) $x(n) = \{\underline{0}, 0, 1, 1, 1, 1\}, h(n) = \{1, \underline{-2}, 3\}$
   (b) $x(n) = \{\underline{1}, 1, 2\}, h(n) = u(n)$
3. Compute the z-transform and the associated ROC's of the following signals

(a) $x(n) = \frac{1}{5^n} u(n)$

(b) $x(n) = 2^n u(-n) + \frac{1}{3^n} u(n)$

(c) $x(n) = \{-1, \underline{0}, 1\}$