# Recitation 5 - EECS 451, Winter 2009

Feb 4th, 2009

## OUTLINE

- Review of important concepts (Lecture 7-8)
- Practice problems

## Concepts: 1-sided z-transforms

1. Definition of 1-sided z-transform
   - For a given DT signal $x(n)$, $X^+(z) := \sum_{n=0}^{\infty} x(n)z^{-n}$, where $z$ is complex valued.
   - Unique for causal signals
   - One sided z-transform of $x(n)$ is identical to the 2-sided z-transform of $x(n)u(n)$
   - For causal signals, $X^+(z)$ and $X(z)$ are the same
   - *ROC* of $X^+(z)$ is always exterior of a circle
2. Properties of 1-sided $z$-transform: We have $x(n) \stackrel{Z^+}{\leftrightarrow} X^+(z)$
   - Almost all properties of the two-sided $z$-transform carry over to the one-sided $z$-transform
   - Time delay: $x(n-k) \stackrel{Z^+}{\leftrightarrow} z^{-k}[X^+(z) + \sum_{n=1}^{k} x(-n)z^n]$
   - Time advance: $x(n+k) \stackrel{Z^+}{\leftrightarrow} z^k[X^+(z) - \sum_{n=0}^{k-1} x(n)z^{-n}]$
   - Final value Theorem: $\lim_{n\to\infty} x(n) = \lim_{z\to 1}(z-1)X^+(z)$

## Concepts: Frequency analysis of signals

1. Complex exponential functions are eigen-functions of LTI systems
2. Fourier series for periodic signals
   - For a continuous periodic signal $x(t)$

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi kF_0 t}, \qquad c_k = \frac{1}{T_p}\int_{T_p} x(t)e^{-j2\pi kF_0 t}dt$$

- For a discrete periodic signal $x(n)$

$$x(n) = \sum_{k=0}^{N-1} c_k e^{j2\pi \frac{kn}{N}}, \qquad c_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j2\pi \frac{kn}{N}}$$

3. Fourier transform for aperiodic signals

   - For a continuous aperiodic signal $x(t)$

$$x(t) = \int_{-\infty}^{\infty} X(F) e^{j2\pi Ft} dF, \qquad X(F) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi Ft} dt$$

   - For a discrete aperiodic signal $x(n)$

$$x(n) = \frac{1}{2\pi} \int_{2\pi} X(\omega) e^{j\omega n} d\omega, \qquad X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

4. Density spectrums

   - For a periodic signal, power density spectrum is defined as $|c_k|^2$
   - For an aperiodic signal, energy density spectrum is defined as $|X(F)|^2$ (continuous signal) or $|X(\omega)|^2$(discrete signal)

5. Relationship of DTFT to the $z$-transform

   - If the *ROC* of $X(z)$ includes the unit circle, DTFT is an evaluation of $X(z)$ on the unit circle, i.e. $X(\omega) = X(z)|_{z=e^{j\omega}}$

## Problems

1. Use the one-sided z-transform to determine the step response for the system $y(n) = 0.25y(n-2) + x(n)$. Also, $y(-1) = 0, y(-2) = 1$.

2. Determine the impulse response and the step response of the causal system $y(n) = y(n-1) - 0.5y(n-2) + x(n) + x(n-1)$. Plot the pole-zero pattern and determine whether the system is stable.

3. Consider the system

$$H(z) = \frac{1 + \frac{1}{2}z^{-1}}{1 - \frac{3}{5}z^{-1}}$$

   Determine:

   (a) The impulse response

   (b) The zero-state step response

   (c) The step response if $y(-1) = 1$