Recitation 6 - EECS 451, Winter 2009

Feb 18th, 2009

OUTLINE

- Review of important concepts (Lecture 8,9 and part of 10)
- Practice problems

Concepts: Frequency analysis of signals

1. Frequency analysis of discrete signals
   - Discrete Time Fourier Series (DTFS): For a discrete periodic signal $x(n)$

$$x(n) = \sum_{k=0}^{N-1} c_k e^{j2\pi \frac{kn}{N}}, \qquad c_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j2\pi \frac{kn}{N}}$$

   - Discrete Time Fourier Transform (DTFT): For a discrete aperiodic signal $x(n)$

$$x(n) = \frac{1}{2\pi} \int_{2\pi} X(\omega) e^{j\omega n} d\omega, \qquad X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

2. Useful transform pair with a complex exponential

$$e^{j\omega_0 n} \overset{DTFT}{\leftrightarrow} 2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - \omega_0 - 2\pi k)$$

3. Symmetry properties of DTFT: For a discrete signal $x(n)$ with Fourier transform $X(\omega)$
   - If $x(n)$ is real, $X(\omega)$ is conjugate symmetric i.e. $X^*(\omega) = X(-\omega)$
   - If $x(n)$ is real and even, $X(\omega)$ is real and even
   - If $x(n)$ is real and odd, $X(\omega)$ is imaginary and odd
   - If $x(n)$ is imaginary and even, $X(\omega)$ is imaginary and even
   - If $x(n)$ is imaginary and odd, $X(\omega)$ is real and odd
4. Properties of DTFT: Many follow from the fact $X(\omega) = X(z)|_{z=e^{j\omega}}$

- Linearity: $a_1 x_1(n) + a_2 x_2(n) \overset{DTFT}{\leftrightarrow} a_1 X_1(\omega) + a_2 X_2(\omega)$
- Time shifting: $x(n-k) \overset{DTFT}{\leftrightarrow} e^{-j\omega k} X(\omega)$
- Time Reversal: $x(-n) \overset{DTFT}{\leftrightarrow} X(-\omega)$
- Frequency shifting: $e^{j\omega_0 n} x(n) \overset{DTFT}{\leftrightarrow} X(\omega - \omega_0)$
- Convolution: $x_1(n) * x_2(n) \overset{DTFT}{\leftrightarrow} X_1(\omega) X_2(\omega)$
- Multiplication: $x_1(n) x_2(n) \overset{DTFT}{\leftrightarrow} \frac{1}{2\pi} \int_{-\pi}^{\pi} X_1(\omega) X_2(\omega - \lambda) d\lambda$
- Differentiation in the frequency domain: $n x(n) \overset{DTFT}{\leftrightarrow} j \frac{dX(\omega)}{d\omega}$
- Conjugation: $x^*(n) \overset{DTFT}{\leftrightarrow} X^*(-\omega)$

5. Parseval's relation

   - The conservation of average power or energy in the time and frequency domains
   - For a discrete time periodic signal, we have average power conserved

$$P_x = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2 = \sum_{k=0}^{N-1} |c_k|^2$$

   - For a discrete time aperiodic signal, we have the total energy conserved

$$E_x = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(\omega)|^2 d\omega$$

Problems

1. Consider the signal

$$x(n) = 2 + 2\cos(\frac{\pi n}{4}) + \cos(\frac{\pi n}{2}) + \frac{1}{2}\cos(\frac{3\pi n}{4})$$

   (a) Determine and sketch its power density spectrum

   (b) Evaluate the power of the signal

2. Consider the signal $x(n) = \{1, 0, -1, \underline{2}, 3\}$ with Fourier transform $X(\omega) = X_r(\omega) + jX_i(\omega)$. Determine the signal $y(n)$ with Fourier transform $Y(\omega) = X_i(\omega) + X_r(\omega) e^{j2\omega}$.

3. The center of gravity of a signal $x(n)$ is defined as

$$c = \frac{\sum_{n=-\infty}^{\infty} n x(n)}{\sum_{n=-\infty}^{\infty} x(n)}$$

   which provides a measure of "time delay" of the signal. Express c in terms of $X(\omega)$.

4. Express the Fourier transform of the following signals in terms of $X(\omega)$, where $x(n) \overset{DTFT}{\leftrightarrow} X(\omega)$

   (a) $y(n) = \sum_{k=-\infty}^{n} x(k)$

   (b) $y(n) = x(n/2)$ for n even and 0 for n odd