# Recitation 8 - EECS 451, Winter 2009

March 11th, 2009

## OUTLINE

- Review of important concepts (Lecture 12 – 13)
- Practice problems

## Concepts: Discrete Fourier Transform

1. For a time limited signal $x(n)$ of length $N$, the $N$-point DFT is defined by

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi k}{N}n}$$

where $k = 0, \cdots, N-1$ and the inverse DFT formula is given by

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi n}{N}k}$$

where $n = 0, \cdots, N-1$

   (a) Note that the indices range from 0 to $N-1$ for $x(n)$ and $X(k)$
   (b) The DFT is efficiently implemented by the Fast Fourier transform (FFT)
   (c) The DFT matrix $W$ whose size is $N \times N$ is an orthogonal transformation

2. Relationship to other transformations

   (a) If $x_p(n)$ is the $N$-periodic superposition of $x(n)$ (a time-limited signal of length $N$), then the DTFS of $x_p(n)$ is obtained by

$$c_k = \frac{1}{N}\sum_{n=0}^{N-1} x_p(n) e^{-j\frac{2\pi k}{N}n},$$

   thus leading to $X(k) = Nc_k$

(b) The DFT of $x(n)$ has the following relationship with $X(\omega)$ (DTFT) and $X(z)$ (z-transformation)

$$X(k) = X(\omega)|_{\omega=\frac{2\pi k}{N}}, \qquad X(k) = X(z)|_{z=e^{j\frac{2\pi k}{N}}}$$

where $x(n)$ is a time-limited $N$-point signal.

3. Circular Symmetries

(a) $N$-point circular even is defined by $x(N-n) = x(n)$ for $1 \leq n \leq N-1$

(b) $N$-point circular odd is defined by $x(N-n) = -x(n)$ for $1 \leq n \leq N-1$

(c) If $x(n)$ is real, we have $X(k) = X^*((-k) \mod N)$ (circular Hermitian symmetry)

4. Properties of the DFT (Suppose $x(n) \overset{N-DFT}{\longleftrightarrow} X(k)$)

(a) Linearity: $a_1x_1(n) + a_2x_2(n) \overset{N-DFT}{\longleftrightarrow} a_1X_1(k) + a_2X_2(k)$

(b) Circular time shift: $x((n-l) \mod N) \overset{N-DFT}{\longleftrightarrow} e^{-j2\pi lk/N}X(k)$

(c) Time reversal: $x((-n) \mod N) \overset{N-DFT}{\longleftrightarrow} X((-k) \mod N)$

(d) Circular frequency shift: $x(n)e^{j2\pi k_0 n/N} \overset{N-DFT}{\longleftrightarrow} X((k-k_0) \mod N)$

(e) Complex conjugate: $x^*(n) \overset{N-DFT}{\longleftrightarrow} X^*((-k) \mod N)$ and $x^*((-n) \mod N) \overset{N-DFT}{\longleftrightarrow} X^*(k)$

(f) Convolution: $x_1(n) \; Ⓝ \; x_2(n) \overset{N-DFT}{\longleftrightarrow} X_1(k)X_2(k)$

(g) Multiplication: $x_1(n)x_2(n) \overset{N-DFT}{\longleftrightarrow} \frac{1}{N}X_1(k) \; Ⓝ \; X_2(k)$, where Ⓝ denotes $N$-point circular convolution

(h) Parseval's theorem: $\sum_{n=0}^{N-1} |x(n)|^2 \overset{N-DFT}{\longleftrightarrow} \frac{1}{N}\sum_{k=0}^{N-1} |X(k)|^2$

# Problems

1. (a) Determine the Fourier transform $X(\omega)$ of the signal $x(n) = \{1, 2, \underline{3}, 2, 1, 0\}$

(b) Compute the 6-point DFT $V(k)$ of the signal $v(n) = \{\underline{3}, 2, 1, 0, 1, 2\}$

(c) Is there any relation between $X(\omega)$ and $V(k)$?

2. If $X(k)$ is the DFT of the sequence $x(n)$, determine the $N$-point DFTs of the sequences

$$x_c(n) = x(n)\cos(\frac{2\pi k_0 n}{N}), \quad 0 \leq n \leq N-1$$
$$x_s(n) = x(n)\sin(\frac{2\pi k_0 n}{N}), \quad 0 \leq n \leq N-1$$

in terms of $X(k)$.

3. An LTI system with frequency response $H(\omega)$ is excited with the periodic input

$$x(n) = \sum_{k=-\infty}^{\infty} \delta(n - kN)$$

Suppose that we compute the N-point DFT $Y(k)$ of the samples $y(n)$ for $0 \leq n \leq N-1$ of the output sequence. How is $Y(k)$ related to $H(\omega)$? Assume $h(n)$ is an N-point signal for $0 \leq n \leq N-1$.