Review Session III - EECS 451, Winter 2009

April 15th, 2009

OUTLINE

- Review of Important concepts

Concepts

1. Spectral Estimation and resolution techniques - Let $x(n)$ be L samples of the signal $x(t)$ and $X(k)$ be the N-pt DFT of the signal formed by zero padding $x(n)$ with $N - L$ zeros

   (a) Increasing the number of samples L improves the resolution of the spectrum $X(k)$

   (b) Increasing N (i.e. the number of zeros padded) smoothens the spectrum produced but does not increase the resolution of the spectrum

   (c) To resolve frequencies ω_1, ω_2, the number of samples should be atleast $L > \frac{2\pi}{|\omega_1 - \omega_2|}$

2. Design of digital IIR filters - Let $h_a(t)$ be an analog IIR filter with $H_a(s)$ as its s-transform and $H_a(s)|_{s=j\Omega}$ being its frequency response

   (a) Impulse invariance method - The digital IIR filter is constructed by sampling the continuous time impulse response, i.e.

   $$h(n) = T_s h_a(t)|_{t=nT_s}$$

   where T_s is the sampling interval. The frequency response of the digital IIR filter is related to the frequency response of the analog IIR filter by

   $$H(\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} H_a(\Omega - \frac{2\pi k}{T_s})$$

   where the digital frequency and the analog frequency are related by $\omega = \Omega T_s$

   (b) Some properties of Impulse invariance method

   i. A stable and causal analog filter $h_a(t)$ yields a stable and causal digital filter $h(n)$

   ii. A real analog filter $h_a(t)$ yields a real digital filter $h(n)$

(c) Bilinear Transformation - The s-domain of the analog IIR filter is mapped to the z-domain as follows

$$H(z) = H_a(s)\Big|_{s=\frac{2}{T_s}\frac{1-z^{-1}}{1+z^{-1}}}$$

and the inverse mapping is given by

$$H_a(s) = H(z)\Big|_{z=\frac{2/T_s+s}{2/T_s-s}}$$

(d) Some properties of the Bilinear Transformation (BLT)

i. When analog filter $H_a(s)$ has a rational form, BLT yields a $H(z)$ which has a rational form
ii. The entire **left half** of the s-plane is mapped **inside** the unit circle in the z-plane
iii. The entire **right half** of the s-plane is mapped **outside** the unit circle in the z-plane
iv. The **imaginary axis** in the s-plane is mapped **onto** the unit circle in the z-plane
v. A stable $H_a(s)$ yields a stable $H(z)$ by performing BLT since all poles in the left half s-plane is mapped inside the unit circle in the z-plane
vi. $H(\omega)$ depends only on $H_a(\Omega)$ since the imaginary axis in the s-plane is mapped onto the unit circle in the z-plane
vii. The relationship between ω and Ω is highly nonlinear (called "frequency warping") given by

$$\Omega = \frac{2}{T_s}\tan\frac{\omega}{2}$$

viii. Poles/zeros at $s = \infty$ map to poles/zeros at $z = -1$ by BLT
ix. Real poles/zeros remain real and complex conjugate pairs remain complex conjugate pairs after applying BLT

3. Some prominent analog IIR filters

(a) Butterworth filter

i. They have rational system function $H_a(s)$
ii. The frequency response is given by

$$|H_a(\Omega)|^2 = \frac{1}{1+(\frac{\Omega}{\Omega_c})^{2N}}$$

where N is the order of the filter
iii. The filter has N poles equally spaced on a circle of radius Ω_c in the left half plane
iv. Pro: Maximally flat in the pass band
v. Con : Not a sharp cut off

(b) Chebyshev filter (Type I)

i. They have a rational system function $H_a(s)$
ii. They have equi-ripples in the passband and monotonically decreasing in stopband

iii. They are all-pole analog filters and the frequency response is given by

$$|H_a(\Omega)|^2 = \frac{1}{1+\epsilon^2 C_N^2(\frac{\Omega}{\Omega_c})}$$

where C_N is the N-th order Chebyshev polynomial

iv. Pro : The cut-off is sharper than in Butterworth filters

v. Con : It has ripples in either the passband or the stopband

(c) Elliptic filter

i. The frequency response is given by

$$|H_a(\Omega)|^2 = \frac{1}{1+\epsilon^2 U_N^2(\frac{\Omega}{\Omega_c})}$$

where $U_N(x)$ is the Jacobian elliptic function (don't need not know)

ii. Pro: Sharpest transition for a given N

iii. Con: Ripple in both passband and stopband

4. Design of Linear phase FIR filters - Let $H_d(\omega)$ be the desired frequency response

(a) Window method

i. Step 1: Find the impulse response of the filter $h_d(n)$ using the inverse DTFS equation

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega)e^{j\omega n}d\omega$$

ii. Step 2: The obtained impulse response $h_d(n)$ can be infinite in length and hence should be truncated using a window function $w(n)$ to get the final impulse response $h(n)$

(b) Frequency-Sampling method

i. Step 1: Sample the given frequency response $H_d(\omega)$ to form $H(k)$

$$H(k) = H_d(\omega)|_{\omega=2\pi\frac{k}{M}}$$

where $k = 0, 1, \cdots, M-1$

ii. Step 2: Find the impulse response $h_d(n)$ using the M-pt inverse DFT equation

$$h_d(n) = \frac{1}{M}\sum_{k=0}^{M-1} H(k)e^{j2\pi kn/M}, n = 0, 1, \cdots, M-1$$

iii. Step 3: If the obtained $h_d(n)$ is not real, take the real part of $h_d(n)$ to get the final impulse response $h(n)$

5. Multirate Signal Processing

(a) Decimation by a factor D - Let $x(n)$ be a signal with spectrum $X(\omega)$, where $X(\omega)$ is non-zero in the frequency interval $0 \leq |\omega| \leq \pi$ or $|F| \leq F_x/2$

i. $x(n)$ is downsampled by D, where every Dth value of the signal $x(n)$ is retained

ii. The resulting signal will be an aliased version of $x(n)$ with a folding frequency of $F_x/2D$

iii. To avoid aliasing, the bandwidth of the signal $x(n)$ is reduced to $F_x/2D$ or π/D, using a low pass filter

iv. The signal can be downsampled after this pre-processing

v. The frequency spectrum of the output signal is given by

$$Y(\omega_y) = \frac{1}{D}\sum_{k=0}^{D-1} H_D(\frac{\omega_y - 2\pi k}{D})X(\frac{\omega_y - 2\pi k}{D})$$

where $\omega_y = D\omega_x$ and $H_D(\omega)$ is a low pass filter with cut-off at $|\omega| = \pi/D$

(b) Interpolation by a factor I - Let $x(n)$ be a signal with spectrum $X(\omega)$, where $X(\omega)$ is non-zero in the frequency interval $0 \le |\omega| \le \pi$ or $|F| \le F_x/2$

i. $x(n)$ is upsampled by I, by interpolating I-1 new samples between successive values of the signal

ii. The upsampling results in compression and replication of the frequency spectrum of $x(n)$

iii. The replicas are eliminated by passing the output through a low pass filter

iv. The frequency spectrum of the resultant signal is given by

$$Y(\omega_y) = \begin{cases} X(\omega_y I) & 0 \le |\omega_y| \le \pi/I \\ 0 & \text{otherwise} \end{cases}$$

6. Image Processing

(a) A median filter is a nonlinear filter which eliminates salt and pepper noise (isolated point noise) while preserving the edge information

(b) A low pass filter is a linear filter which eliminates white Gaussian noise and also blurs the image thereby disturbing the edge information

(c) An edge detector is a high pass filter which emphasizes image features such as edges

ALL THE BEST