1a. From handout: $\mathcal{Z}\{u(n)\} = U(z) = \frac{1}{1-z^{-1}}$; $\mathcal{Z}\{nu(n)\} = -z\frac{dU}{dz} = \frac{z^{-1}}{(1-z^{-1})^2}$
$\rightarrow \mathcal{Z}\{(1+n)u(n)\} = \frac{1}{(1-z^{-1})^2}$. ROC: $|z| > 1$.

1b. From handout: $\mathcal{Z}\{(a^n + a^{-n})u(n)\} = \frac{1}{1-az^{-1}} + \frac{1}{1-\frac{1}{a}z^{-1}} = \frac{2-(a+\frac{1}{a})z^{-1}}{(1-az^{-1})(1-\frac{1}{a}z^{-1})}$.
ROC: $|z| > max[|a|, \frac{1}{|a|}]$. NOTE: $a = e^{j\omega_o} \rightarrow \mathcal{Z}\{2\cos(\omega_o n)\}$ agrees with handout.

1c. $\mathcal{Z}\{(-1)^n 2^{-n} u(n)\} = \mathcal{Z}\{(-\frac{1}{2})^n u(n)\} = \frac{1}{1+\frac{1}{2}z^{-1}}$. ROC: $|z| > \frac{1}{2}$.

1d. $\mathcal{Z}\{na^n \sin(\omega_o n)u(n)\} = -z\frac{d}{dz}\left[\frac{az^{-1}\sin(\omega_o)}{1-2az^{-1}\cos\omega_o + a^2z^{-2}}\right] = \frac{az^{-1}\sin\omega_o - a^3z^{-3}\sin\omega_o}{(1-2az^{-1}\cos\omega_o+a^2z^{-2})^2}.|z| > a.$

2a. $\mathcal{Z}\{(\frac{1}{3})^n u(n) + 2^n u(-n-1)\} = \frac{1}{1-\frac{1}{3}z^{-1}} - \frac{1}{1-2z^{-1}} = \frac{-\frac{5}{3}z^{-1}}{(1-\frac{1}{3}z^{-1})(1-2z^{-1})}$. ROC: $\frac{1}{3} < |z| < 2$.

2b. $\mathcal{Z}\{(\frac{1}{3})^n u(n) - 2^n u(n)\} = \frac{1}{1-\frac{1}{3}z^{-1}} - \frac{1}{1-2z^{-1}} = \frac{-\frac{5}{3}z^{-1}}{(1-\frac{1}{3}z^{-1})(1-2z^{-1})}$. ROC: $|z| > 2$.
Note that the expressions for #2a and #2b are identical, but the ROCs differ!

2c. $\mathcal{Z}\{x_1(n+4)\} = z^4 X_1(z) = \frac{-\frac{5}{3}z^3}{(1-\frac{1}{3}z^{-1})(1-2z^{-1})}$. ROC: $\frac{1}{3} < |z| < 2$ (same as (a)).

2d. $\mathcal{Z}\{x_1(-n)\} = X_1(\frac{1}{z}) = \frac{-\frac{5}{3}z}{(1-\frac{1}{3}z)(1-2z)}$. ROC: $\frac{1}{3} < \frac{1}{|z|} < 2 \rightarrow \frac{1}{2} < |z| < 3$.

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3. $Y(z) = X_1(z)X_2(z) = \frac{-\frac{5}{3}z^{-1}}{(1-\frac{1}{3}z^{-1})(1-2z^{-1})}\frac{1}{1-\frac{1}{2}z^{-1}} = \frac{-2}{1-\frac{1}{3}z^{-1}} + \frac{10/3}{1-\frac{1}{2}z^{-1}} + \frac{-4/3}{1-2z^{-1}}$
$\rightarrow y(n) = -2(\frac{1}{3})^n u(n) + \frac{10}{3}(\frac{1}{2})^n u(n) + \frac{4}{3}(2)^n u(-n-1)$. NOTE: 2-sided.

4a. $\frac{1+3z^{-1}}{1+3z^{-1}+2z^{-2}} = \frac{2}{1+z^{-1}} - \frac{1}{1+2z^{-1}} \rightarrow 2(-1)^n u(n) - (-2)^n u(n)$. NOTE: unstable.

4b. $\frac{1}{1-z^{-1}+\frac{1}{2}z^{-2}} = \frac{(1-j)/2}{1-\frac{1+j}{2}z^{-1}} + \frac{(1+j)/2}{1-\frac{1-j}{2}z^{-1}}$. $\frac{1\pm j}{2} = \frac{\sqrt{2}}{2}e^{j\pm\pi/4} \rightarrow 2\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{2})^n \cos(\frac{\pi}{4}n - \frac{\pi}{4})u(n)$
$= (\frac{\sqrt{2}}{2})^n[\cos(\frac{\pi}{4}n) + \sin(\frac{\pi}{4}n)]u(n)$ using $Ap^n + A^*(p^*)^n = 2|A||p|^n\cos(\omega_o n + \theta)$.

4c. $\mathcal{Z}\{u(n)\} = \frac{1}{1-z^{-1}} \rightarrow u(n-6) + u(n-7)$. Rewrite as: $\{0, n \le 5; 1, n = 6; 2, n \ge 7\}$.

4d. $\frac{1+2z^{-2}}{1+z^{-2}} = 2 - \frac{1}{1+z^{-2}} \rightarrow 2\delta(n) - \cos(\frac{\pi}{2}n)u(n)$.

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5. $\frac{5z^{-1}}{(1-2z^{-1})(3-z^{-1})} = \frac{1}{1-2z^{-1}} - \frac{1}{1-\frac{1}{3}z^{-1}}$. 2 poles $(2, \frac{1}{3})$ →2+1=3 different ROCs:

ROC: $|z| > 2 \rightarrow 2^n u(n) - (\frac{1}{3})^n u(n)$. Strictly causal. Note: unstable.

ROC: $\frac{1}{3} < |z| < 2 \rightarrow -2^n u(-n-1) - (\frac{1}{3})^n u(n)$. Two-sided (causal and anticausal).

ROC: $|z| < \frac{1}{3} \rightarrow -2^n u(-n-1) + (\frac{1}{3})^n u(-n-1)$. Strictly anticausal. Unstable.