1a. $c_k = \frac{1}{6}\sum_{n=0}^{5} x(n)e^{-j2\pi nk/6} = \frac{1}{6}(3+2e^{-j\frac{2\pi k}{6}}+1e^{-j\frac{4\pi k}{6}}+0e^{-j\frac{6\pi k}{6}}+1e^{-j\frac{8\pi k}{6}}+2e^{-j\frac{10\pi k}{6}})$

$c_k = \frac{3}{6}+\frac{4}{6}\cos(\pi k/3)+\frac{2}{6}\cos(2\pi k/3)$. $c_o = \frac{9}{6}; c_1 = c_5 = \frac{4}{6}; c_2 = c_4 = 0; c_3 = \frac{1}{6}$.

1b. Time domain: $\frac{1}{6}\sum_{n=0}^{5}|x(n)|^2 = \frac{1}{6}(3^2+2^2+1^2+0^2+1^2+2^2) = \frac{19}{6}$.

Freq domain: $\sum_{k=0}^{5}|c_k|^2 \quad = \frac{1}{6^2}(9^2+4^2+0^2+1^2+0^2+4^2) = \frac{19}{6}$

2a. $c_k = \cos\frac{k\pi}{4} \to x(n) = \sum_{k=0}^{7}\cos\frac{k\pi}{4}\cos\frac{2\pi kn}{8} = 4\delta(n-1)+4\delta(n+1)$.

$c_k = \sin\frac{3k\pi}{4} \to x(n) = j\sum_{k=0}^{7}\sin\frac{3k\pi}{4}\sin\frac{2\pi kn}{8} = j4\delta(n-3)-j4\delta(n+3)$.

Hence $x(n)$=sum of the above two signals, repeated with period=8.

2c. $x(n) = \sum_{k=-3}^{4} c_k e^{j\frac{2\pi nk}{8}} = 2+1e^{j\frac{2\pi n}{8}}+\frac{1}{2}e^{j\frac{4\pi n}{8}}+\frac{1}{4}e^{j\frac{6\pi n}{8}}+\frac{1}{4}e^{j\frac{10\pi n}{8}}+\frac{1}{2}e^{j\frac{12\pi n}{8}}+1e^{j\frac{14\pi n}{8}}$

$x(n) = 2+2\cos\frac{\pi n}{4}+1\cos\frac{\pi n}{2}+\frac{1}{2}\cos\frac{3\pi n}{4}$. Real, as expected.

3a. $X(z) = (1-z^{-6})\frac{1}{1-z^{-1}} \to X(e^{j\omega}) = \frac{1-e^{-j6\omega}}{1-e^{-j\omega}}$.

3b. $X(z) = \frac{1}{1-\frac{1}{2}z^{-1}}|_{z=\frac{1}{z}} = \frac{1}{1-\frac{1}{2}z} \to X(e^{j\omega}) = \frac{2}{2-e^{j\omega}}$.

3c. $X(z) = z^4\frac{4^4}{1-\frac{1}{4}z^{-1}} \to X(e^{j\omega}) = \frac{4^4e^{j4\omega}}{1-\frac{1}{4}e^{-j\omega}}$.

3d. $X(z) = \frac{a\sin w_o z^{-1}}{1-2a\cos w_o z^{-1}+a^2z^{-2}} \to X(e^{j\omega}) = \frac{a\sin w_o e^{-j\omega}}{1-2a\cos w_o e^{-j\omega}+a^2e^{-j2\omega}}$. Plot overleaf.

4a. $x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi}X(e^{j\omega})e^{j\omega n}d\omega = \delta(n)-\frac{1}{2\pi}\int_{-\omega_o}^{\omega_o}1e^{j\omega n}d\omega = \delta(n)-\frac{\sin(\omega_o n)}{\pi n}$.

4b. $X(e^{j\omega}) = \cos^2\omega = \frac{1}{2}(1+\cos(2\omega)) = \frac{1}{2}(1+\frac{1}{2}e^{j2\omega}+\frac{1}{2}e^{-j2\omega})$

$\to X(z) = \frac{1}{2}(1+\frac{1}{2}z^2+\frac{1}{2}z^{-2}) \to x(n) = \frac{1}{2}\delta(n)+\frac{1}{4}\delta(n+2)+\frac{1}{4}\delta(n-2)$.

4c. Using modulation principle, $x(n) = 2\cos(\omega_o n)\frac{\sin(n\delta\omega/2)}{\pi n}$.

5a. $X(0)$ means $X(e^{j0}) = \sum x(n) = -1+2-3+2-1 = -1$.

5b. $x(n)$ real and even (again!)$\to X(e^{j\omega})$ real and even$\to \angle X(e^{j\omega}) = \pi$ since $X(e^{j0}) < 0$.

5c. $\int_{-\pi}^{\pi}X(e^{j\omega})e^{j\omega 0}d\omega = 2\pi x(0) = 2\pi(-3) = -6\pi$.

5d. $X(\pi)$ means $X(e^{j\pi}) = X(-1) = \sum x(n)(-1)^n = -1-2-3-2-1 = -9$.

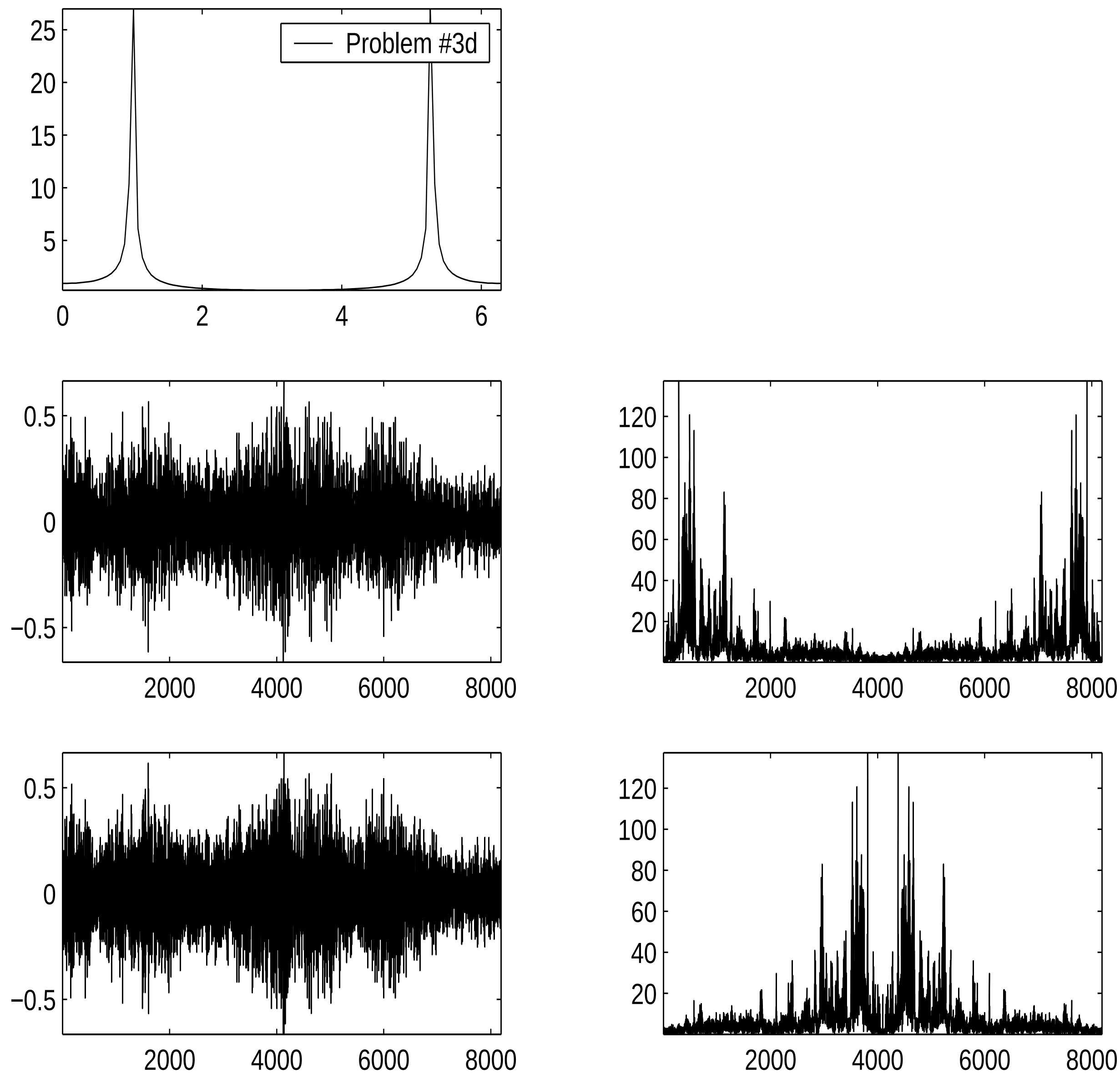
5e. $\int_{-\pi}^{\pi}|X(e^{j\omega})|^2 d\omega = 2\pi\sum|x(n)|^2 = 2\pi(1^2+2^2+3^2+2^2+1^2) = 38\pi$.

6a-c. (a) See plots overleaf. (b) Swapped upper and lower halves of spectrum. (c) No.

6d. $Y(e^{j\omega}) = X(e^{j(\omega+\pi)}) = X(e^{j\omega}e^{j\pi}) = X(-e^{j\omega}) \to y(n) = (-1)^n x(n)$.

Much easier than how they did it during World War II! (analog modulators).

6e. Repeating the same procedure used to scramble the signal will unscramble it.


Problem #3d