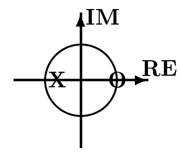
EECS 451 SOLUTIONS TO PROBLEM SET #6

1. Input $x(n)$ clearly has frequency components only at $\omega = 0$; $\omega = \pm\frac{\pi}{2}$; and $\omega = \pi$.
   Impulse resp. $h(n) = \frac{1}{2}(\delta(n) - \delta(n-2)) \rightarrow$transfer function $H(e^{j\omega}) = \frac{1}{2}(1 - e^{-2j\omega})$.
   $\omega = 0: \quad X(e^{j0}) = 5e^{j00^o}; \quad H(e^{j0}) = 0 \rightarrow Y(e^{j0}) = 0$. Recall EECS 210?
   $\omega = \frac{\pi}{2}: \quad X(e^{j\frac{\pi}{2}}) = 3e^{j60^o}; \quad H(e^{j\frac{\pi}{2}}) = 1 \rightarrow Y(e^{j\frac{\pi}{2}}) = 3e^{j60^o}; Y(e^{-j\frac{\pi}{2}}) = 3e^{-j60^o}$.
   $\omega = \pi: \quad X(e^{j\pi}) = 4e^{-j45^o}; \quad H(e^{j\pi}) = 0 \rightarrow Y(e^{j\pi}) = 0. \quad y(t) = 3\cos(\frac{\pi}{2}n + 60^o)$.

2a. $H(z) = C\frac{z-1}{z+0.9}$ for some constant C.
No constant signals pass$\rightarrow$zero at $z = 1$.
High-pass with 1 pole$\rightarrow$pole$=|p|e^{j\pi} = -0.9$.



2c. $1 = H(e^{j\pi}) = C\frac{e^{j\pi}-1}{e^{j\pi}+0.9} = \frac{-2}{-.1} \rightarrow C = \frac{1}{20}$.

2d. $H(z) = \frac{1}{20}\frac{1-z^{-1}}{1+0.9z^{-1}} = \frac{Y(z)}{X(z)}$. Cross-multiply:
$Y(z)(1 + 0.9z^{-1}) = \frac{1}{20}X(z)(1 - z^{-1}) \rightarrow y(n) + 0.9y(n-1) = \frac{1}{20}x(n) - \frac{1}{20}x(n-1)$.

2e. $X(e^{j\frac{\pi}{6}}) = 2e^{j45^o}; \quad H(e^{j\frac{\pi}{6}}) = \frac{1}{20}\frac{e^{j\frac{\pi}{6}}-1}{e^{j\frac{\pi}{6}}+0.9} = 0.014e^{j90^o} \rightarrow Y(e^{j\frac{\pi}{6}}) = 0.028e^{j135^o} \rightarrow$
$y(n) = 0.028\cos(\frac{\pi}{6}n + 135^o)$. NOTE: $H(e^{j\omega}) \approx \frac{1}{20}\frac{e^{j\omega}-1}{e^{j\omega}+1} = \frac{1}{20}j\tan\frac{\omega}{2}$.

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3. Linear phase$\rightarrow$ $b_2 = \pm b_0$. Choose $b_2 = b_0$; otherwise forced to $b_1 = 1$ below.
   $H(e^{j0}) = 1 \rightarrow b_0 + b_1 + b_0 = 1$. Zero at $\omega = \frac{2\pi}{3} \rightarrow b_0 + b_1e^{-j2\pi/3} + b_0e^{-j4\pi/3} = 0$.
   Solving$\rightarrow$ $b_0 = b_1 = b_2 = \frac{1}{3} \rightarrow H(e^{j\omega}) = \frac{1}{3}(1 + 2\cos\omega)e^{-j\omega}$. Meets all specs.

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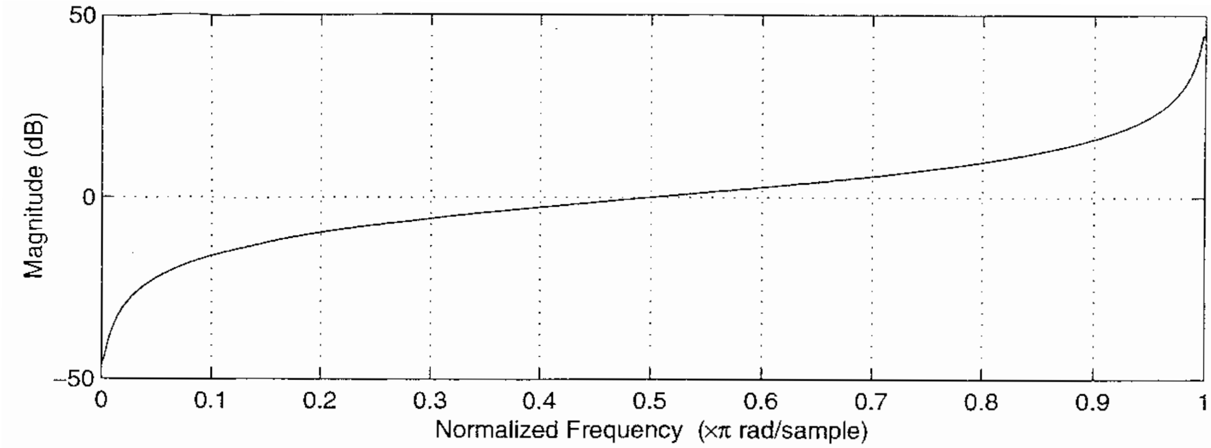
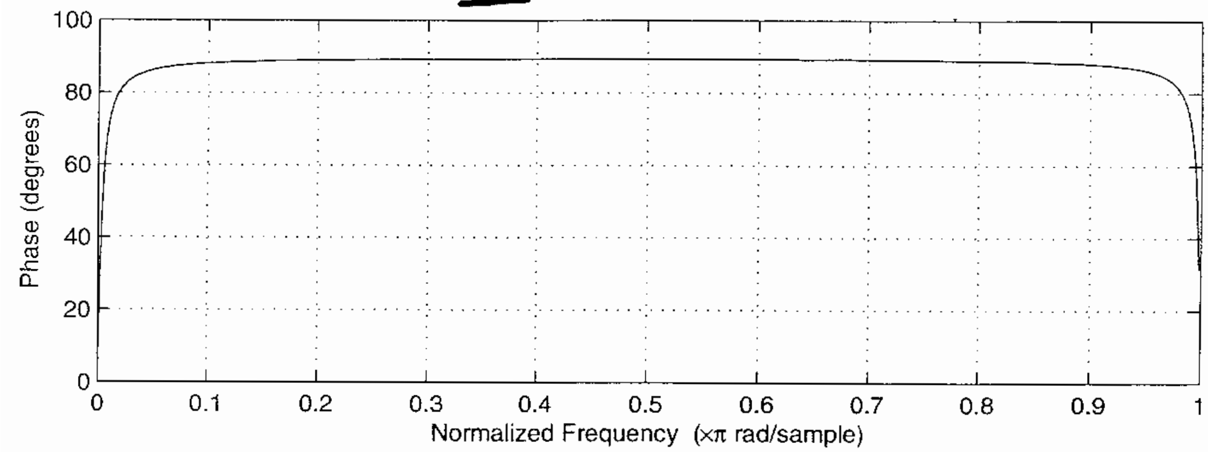
4. $|H(e^{j\omega})|^2 = \frac{5/4-\cos\omega}{10/9-(2/3)\cos\omega} \rightarrow H(z)H(z^{-1}) = \frac{\frac{5}{4}-\frac{1}{2}(z+z^{-1})}{\frac{10}{9}-\frac{1}{3}(z+z^{-1})} = \frac{1-\frac{1}{2}z^{-1}}{1-\frac{1}{3}z^{-1}}\frac{1-\frac{1}{2}z}{1-\frac{1}{3}z} \rightarrow$
   $H(z) = \frac{1-\frac{1}{2}z^{-1}}{1-\frac{1}{3}z^{-1}}$ is the minimum-phase factor. Note the nice numbers.

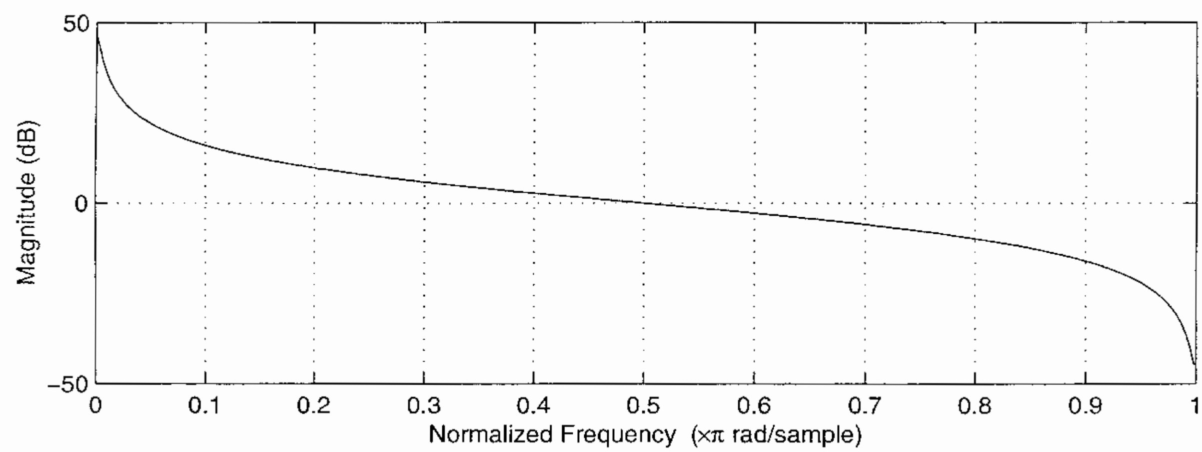
5a. You should hear Handel with a very annoying 2000 Hz tone overwhelming it.
5b. Missing constant$=-2\cos(2\pi\frac{2000}{8192})$ $\rightarrow$notch filter has zeros at $\pm$2000Hz.
Notch filter eliminates the 2000Hz tone; it also affects similar frequencies of Handel.
Can you hear any difference? I can't (but I'm tone-deaf).

6a. Actual tape hiss is a little softer than this, but this gives you some idea.
6b. The *preemphasis* filter boosts high frequencies of Handel, so it sounds tinny.
6c. Twice-filtered **Z** and original signal **X** agree to around 10^{-12}.
Z1 sounds *much* better than the noisy **W**: tape hiss is greatly reduced.
6d. The *preemphasis* and *deemphasis* filters are inverse filters to each other.
Output of **freqz** overleaf: Note the gains **(in dB)** sum to 0 dB=1 for all frequencies.
Boosting high frequencies of original signal$\rightarrow$overwhelm high-frequency tape hiss.



PRE-EMPHASIS





DE-EMPHASIS