1a. **Reverse** the second sequence: $\{4,3,2,2\} \rightarrow \{2,2,3,4\}$, repeat the cycle, and shift:

y(0): $\begin{matrix}\{1,2,3,1,\underline{1},2,3,1\}\\ \{4,2,2,3,\underline{4},2,2,3\}\end{matrix} \rightarrow (1)(4)+(2)(2)+(3)(2)+(1)(3)=17.$

y(1): $\begin{matrix}\{1,2,3,1,\underline{1},2,3,1\}\\ \{3,4,2,2,\underline{3},4,2,2\}\end{matrix} \rightarrow (1)(3)+(2)(4)+(3)(2)+(1)(2)=19.$

y(2): $\begin{matrix}\{1,2,3,1,\underline{1},2,3,1\}\\ \{2,3,4,2,\underline{2},3,4,2\}\end{matrix} \rightarrow (1)(2)+(2)(3)+(3)(4)+(1)(2)=22.$

y(3): $\begin{matrix}\{1,2,3,1,\underline{1},2,3,1\}\\ \{2,2,3,4,\underline{2},2,3,4\}\end{matrix} \rightarrow (1)(2)+(2)(2)+(3)(3)+(1)(4)=19.$ $\{\underline{17},19,22,19\}$.

1b. $X_1(0)=1+2+3+1=07.$ $X_1(1)=1+2(-j)+3(-1)+1(j)=-2-j.$

$X_1(2)=1-2+3-1=01.$ $X_1(3)=1+2(j)+3(-1)+1(-j)=-2+j.$

$X_2(0)=4+3+2+2=11.$ $X_2(1)=4+3(-j)+2(-1)+2(j)=+2-j.$

$X_2(2)=4-3+2-2=01.$ $X_2(3)=4+3(j)+2(-1)+2(-j)=+2+j.$

Multiply: $Y(0)=(7)(11)=77.$ $Y(1)=(-2-j)(2-j)=-5.$ $Y(2)=(1)(1)=1.$

Multiply: $Y(3)=(-2+j)(2+j)=-5$ or conjugate symmetry: $Y(3)=Y(1)^*=-5.$

$y(0)=\frac{1}{4}[77-5+1-5]=17.$ $y(1)=\frac{1}{4}[77-5(j)+1(-1)-5(-j)]=19.$
$y(2)=\frac{1}{4}[77+5+1+5]=22.$ $y(3)=\frac{1}{4}[77-5(-j)+1(-1)-5(j)]=19.$

2a. $X(k)=\sum_{n=0}^{N/2-1}x(n)e^{-j2\pi nk/N}+\sum_{n=N/2}^{N-1}x(n)e^{j2\pi nk/N}$. Let $n\rightarrow n-\frac{N}{2}$:
$X(k)=\sum_{n=0}^{N/2-1}[x(n)+x(n+\frac{N}{2})e^{-j\pi k}]e^{-j2\pi nk/N}=0$ if $e^{j\pi k}=1\rightarrow k$ even.

2b. k odd $\rightarrow e^{j\pi k}=-1\rightarrow X(k)=\sum_{n=0}^{N/2-1}2x(n)e^{-j2\pi nk/N}=DFT_{N/2}\{2x(n)e^{-j2\pi n/N}\}$
since $k=2m+1\rightarrow e^{-j2\pi nk/N}=e^{-j2\pi\frac{nm}{N/2}}e^{-j2\pi n/N}=W_{N/2}^{nm}e^{-j2\pi n/N}$.

3. $$\begin{bmatrix}x(0)\\x(1)\\x(2)\\x(3)\end{bmatrix}=\frac{1}{4}\begin{bmatrix}1&1&1&1\\1&j&-1&-j\\1&-1&1&-1\\1&-j&-1&j\end{bmatrix}\begin{bmatrix}X(0)\\X(1)\\X(2)\\X(3)\end{bmatrix}\rightarrow\begin{bmatrix}X(0)\\X(1)\\X(2)\\X(3)\end{bmatrix}=\begin{bmatrix}7\\-2-j\\1\\-2+j\end{bmatrix}.$$ Agrees #1b.

4. $x(n)=\{\underline{1},1,1,1,1,1,1\}$ becomes *aliased* to $\{\underline{2},2,1,1,1\}$.
$X(k)=1+1e^{\frac{-j2\pi k}{5}}+1e^{\frac{-j4\pi k}{5}}+1e^{\frac{-j6\pi k}{5}}+1e^{\frac{-j8\pi k}{5}}+1e^{\frac{j-10\pi k}{5}}+1e^{\frac{-j12\pi k}{5}}$
$X(k)=2+2e^{\frac{-j2\pi k}{5}}+1e^{\frac{-j4\pi k}{5}}+1e^{\frac{-j6\pi k}{5}}+1e^{\frac{-j8\pi k}{5}}.$ $DFT^{-1}\rightarrow\{\underline{2},2,1,1,1\}$.

5. Let $h(n)=h_e(n)+h_o(n)$ and $u(n)=u_e(n)+u_o(n)$ (break into even and odd parts).
Let $A(k)=DFT\{h(n)+ju(n)\}=\underbrace{DFT\{h_e(n)+ju_o(n)\}}_{REAL}+\underbrace{DFT\{h_o(n)+ju_e(n)\}}_{IMAGINARY}$

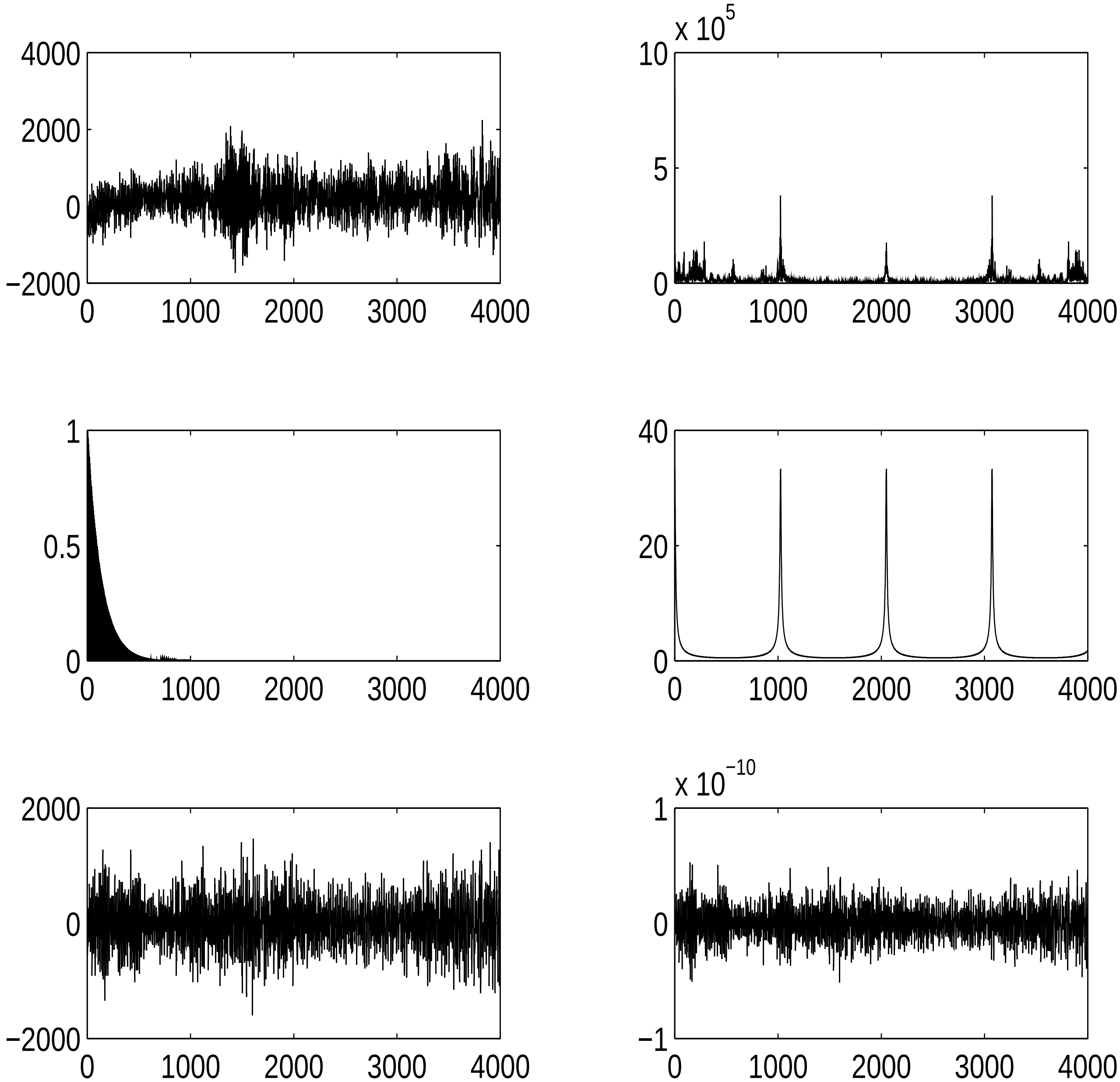
$=[H_e(k)-U_o(k)]+j[H_o(k)+U_e(k)]; H_e(k)=DFT\{h_e(n)\}; H_o(k)=\frac{1}{j}DFT\{h_o(n)\}.$

$2H(k)=Re[A(k)+A(N-k)]+jIm[A(k)-A(N-k)].$

$2U(k)=Im[A(k)+A(N-k)]-jRe[A(k)-A(N-k)].$

Easy to confirm you get the right answer when either $h(n)=0$ or $u(n)=0$.

6. 4 spikes are caused by $H(e^{j\omega})$, so $H(z)$ has poles $\{0.97,0.97e^{j\pi/2},0.97e^{j\pi},0.97e^{j3\pi/2}\}$.
$H(z)=\frac{z^4}{z^4-0.97^4}$ and $y[n]=x[n]-0.97^4x[n-4]+0.97^8x[n-8]+\ldots$ and $x[n]=y[n]-0.97^4y[n-4]$.



4000
2000
0
−2000
0
1000
2000
3000
4000
x 10^5
10
5
0
0.5
1
40
20
x 10^−10
−1