PRINT YOUR NAME HERE:

HONOR CODE PLEDGE: "I have neither given nor received aid on this exam, nor have I concealed any violations of the honor code." Open book; SHOW ALL OF YOUR WORK!

SIGN YOUR NAME HERE:

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(30) 1. At 6:00 PM, EECS 501 students start arriving for their EECS 501 exam.
ESE students arrive in a Poisson process with average arrival rate λ=5/minute.
CSE students arrive in a Poisson process with average arrival rate λ=4/minute.
EES students arrive in a Poisson process with average arrival rate λ=3/minute.
All students arrive independently of each other.

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(05) a. Compute Pr[3 ESE and 2 CSE students arrive between 6:00 and 6:02]. Simplify.
(05) b. Compute pdf for t=#*minutes* after 6:00 that the first student *of any type* arrives.
(05) c. Compute Pr[exactly 3 of the first 5 students to arrive are ESE students].
(05) d. Compute Pr[the 2^{nd} EES student to arrive arrives after 6:05]. Use Poisson pmf.
(05) e. Compute Pr[the 2^{nd} EES student to arrive arrives after 6:05]. Use Erlang pdf.
You may leave your answer to (e) in the form of a definite integral.
(05) f. $x(n)$=discrete-time random walk with $p = \frac{1}{2}$. Compute $Pr[x(10) = 0]$.

WRITE YOUR ANSWERS HERE:

(a): $Pr =$ **(b):** $f_t(T) =$ **(c):** $Pr =$

(d): $Pr =$ **(e):** $Pr =$ **(f):** $Pr =$

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(30) 2. Zero-mean white Gaussian WSS random process $u(t)$ has PSD $S_u(\omega) = 4$.
$u(t)$ is input into an LTI system with $h(t) = 3e^{-2t}, t \geq 0$. The output is $x(t)$.

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(05) a. Compute the power spectral density $S_x(\omega)$ of the output $x(t)$.
(05) b. Compute the covariance $K_x(t,s)$ and variance $\sigma^2_{x(t)}$ of $x(t)$.
(05) c. Compute $Pr[x(7) < 6]$. HINT: Recall table of $erf(x) = \Phi(x) - \frac{1}{2}$ on p.62.
(05) d. Compute the joint pdf $f_{x(3),x(7),x(9)}(X_3, X_7, X_9)$. You may use $\mathcal{N}$ notation.
(05) e. Compute $\hat{x}(7)_{LSE}$ given the observation that $x(5) = 6$.
(05) f. Compute $\hat{x}(7)_{LSE}$ given the observations that $x(5) = 0$ **and** $x(3) = 4$.
HINT: THINK first. EXPLAIN YOUR ANSWER FULLY, or you get no credit.

WRITE YOUR ANSWERS HERE:

(a): $S_x(\omega) =$ **(b):** $K_x(t,s) =$ $\sigma^2_{x(t)} =$

(c): $Pr =$ **(e):** $\hat{x}(7) =$ **(f):** $\hat{x}(7) =$

(d): $f_{x(3),x(7),x(9)}(X_3, X_7, X_9) =$

#1:

#2:

#3:

$\sum$:

(40) 3. $x(n)$ is a 0-mean stationary Independent Increments (II) random process.

(10) a. Compute $\hat{x}(i)_{LLSE}$ based on $\{x(j), x(k)\}$ for any times $i > j > k > 0$.
Use the formula $\hat{x}_{LLSE}(y) = K_{xy}K_y^{-1}y$ where $x = x(i)$ and $y = [x(j)\ x(k)]'$.

(05) b. Your answer to (a) should disregard $x(k)$! Explain why this makes sense.

(10) c. If $f_{x(n)}(X) = \frac{2^n X^{n-1} e^{-2X}}{(n-1)!}, X \geq 0$, compute $f_{x(i),x(j)}(X_i, X_j)$ for any $i > j > 0$.

Let $b(n)$ be a Bernoulli process with $p \neq 1$, and let $y(n) = \prod_{i=1}^{n} b(i)$.

(05) d. Prove that $y(n) \to 0$ *in probability.*

(05) e. Prove that $y(n) \to 0$ *in mean square.*

(05) f. Prove that $y(n) \to 0$ *with probability one.* Use the Borel-Cantelli-based condition.

WRITE YOUR ANSWERS HERE:

(a): $\hat{x}(i)[x(j), x(k)] =$ **(c):** $f_{x(i),x(j)}(X_i, X_j) =$

(b):

(d):

(e):

(f):