1a. $1 = c\int_0^1 dX\,X\int_0^X dY\,Y = c\int_0^1 dX\,X\frac{X^2}{2} = c\frac{X^4}{8}|_0^1 \to c = 8.$

1b. x and y are NOT independent, since pdf has nonsquare support.

HARD WAY: Compute marginal pdfs $f_x(X) = \int f_{x,y}(X,Y)dY$ and $f_y(Y) = \int f_{x,y}(X,Y)dX$ and show that $f_{x,y}(X,Y) \neq f_x(X)f_y(Y)$.

1c. $f_x(X) = \int_0^X dY\,8XY = 4XY^2|_0^X = 4X^3, 0 < X < 1$; 0, otherwise.

1d. $f_{y|x}(Y|X) = \frac{8XY}{4X^3} = 2Y/X^2, 0 < Y < X < 1 = 8Y, 0 < Y < 1/2.$

1e. $F_z(Z) = Pr[z \leq Z] = Pr[(\frac{y}{x}) \leq Z] = Pr[y \leq xZ] = 8\int_0^1 dX\,X\int_0^{XZ} dY\,Y$

$= 4\int_0^1 dX\,X(XZ)^2 = Z^2, 0 < Z < 1 \to f_z(Z) = \frac{dF_z}{dZ} = 2Z, 0 < Z < 1.$

Note if $Z > 1$ then upper integral limit changes to X and $F_z(Z) = 1$.

This makes sense: $y \leq x \to z = y/x \leq 1 \to Pr[z \leq Z > 1] = 1$.

1f. $Pr[(x+y) < 1] = 8\int_0^{1/2} dY\,Y\int_Y^{1-Y} dX\,X$

$= 4\int_0^{1/2} dY\,Y((1-Y)^2 - Y^2) = 4(\frac{Y^2}{2} - \frac{2Y^3}{3})|_0^{1/2} = 1/6.$

2a. Pr[second coin heads]=$(\frac{2}{3})(\frac{3}{4}) + (\frac{1}{3})(\frac{4}{5}) = 23/30$.

2b. Pr[A heads|second coin heads]= $\frac{Pr[{coin\,A\,heads \atop coin\,B\,heads}]}{Pr[2^{nd}\,coin\,heads]} = \frac{(2/3)(3/4)}{23/30} = 15/23.$

2c. Pr[n flips of second coin heads]=$(\frac{2}{3})(\frac{3}{4})^n + (\frac{1}{3})(\frac{4}{5})^n$.

2d. Pr[A heads|n flips of 2^{nd} coin heads]= $(\frac{2}{3})(\frac{3}{4})^n/[(\frac{2}{3})(\frac{3}{4})^n + (\frac{1}{3})(\frac{4}{5})^n]$.

2e. $\underset{n\to\infty}{\text{LIM}}$ [answer to (d)]=0 since $(\frac{4}{5})^n$ dominates $(\frac{3}{4})^n$.

2f. Coin C is more likely than coin B to land heads indefinitely.

2g. $Pr[E] = Pr[E \cap F] + Pr[E \cap F'] \to Pr[E \cap F'] = Pr[E] - Pr[E \cap F]$

$= Pr[E] - Pr[E]Pr[F] = Pr[E](1 - Pr[F]) = Pr[E]Pr[F']$

$\to E, F'$ are independent. Q.E.D. All these from Problem Set #1.

3a. C,Y,0. **3b.** U,Y,1. **3c.** C,Y,0. **3d.** U,Y,0.