1a. $\frac{(5\cdot2)^3e^{-5\cdot2}}{3!}\frac{(4\cdot2)^2e^{-4\cdot2}}{2!} = \frac{16000}{3}e^{-18} = 0.0000812.$

1b. Poisson process with $\lambda = 5+4+3 = 12$. $f_t(T) = 12e^{-12T}, T \geq 0.$

1c. $Pr[EE] = \frac{5}{5+4+3} = \frac{5}{12}$. $Pr = \binom{5}{3}(\frac{5}{12})^3(\frac{7}{12})^2 = 0.246$

1d. Pr[0 or 1 arrivals in 5 minutes]$=\frac{(3\cdot5)^0e^{-3\cdot5}}{0!} + \frac{(3\cdot5)^1e^{-3\cdot5}}{1!} = 16e^{-15}.$

1e. $Pr = \int_5^\infty 3^2Te^{-3T}dT = 16e^{-15} = 0.0000049$ (as in (d)).

1f. $x(10) = 0 \Leftrightarrow \begin{matrix}\text{5 successes}\\ \text{\&5 failures}\end{matrix} \rightarrow Pr = \binom{10}{5}(\frac{1}{2})^{10} = 0.246$

2a. $H(\omega) = \mathcal{F}\{3e^{-2t}1(t)\} = \frac{3}{j\omega+2}$. $S_x(\omega) = 4|\frac{3}{j\omega+2}|^2 = \frac{36}{\omega^2+4}$.

2b. $K_x(t-s) = \mathcal{F}^{-1}\{\frac{36}{\omega^2+4}\} = 9e^{-2|t-s|}$. $\sigma^2_{x(t)} = K_x(0) = 9.$

2c. $x(7) \sim \mathcal{N}(0,9) \rightarrow Pr[x(7) < 6] = \Phi(6/\sqrt{9}) = \Phi(2) = 0.9776$

2d. $x \sim \mathcal{N}(0,K)$ where $x = \begin{bmatrix} x(3)\\ x(7)\\ x(9)\end{bmatrix}; 0 = \begin{bmatrix}0\\0\\0\end{bmatrix}; K = 9\begin{bmatrix} 1 & e^{-8} & e^{-12}\\ e^{-8} & 1 & e^{-4}\\ e^{-12} & e^{-4} & 1\end{bmatrix}$

2e. $\hat{x}(7) = E[x(7)] + \frac{\lambda_{x(7),x(5)}}{\sigma^2_{x(5)}}(x(5) - E[x(5)]) = \frac{9e^{-4}}{9}6 = 6e^{-4} = 0.110.$

2f. $x(t)$ Markov$\rightarrow \hat{x}(7)[x(5), x(3)] = \hat{x}(7)[x(5)] = 0$ (independent of #2e).

3a. $\hat{x}(i)[x(j), x(k)] = [\lambda_{x(i),x(j)} \quad \lambda_{x(i),x(k)}]\begin{bmatrix} \sigma^2_{x(j)} & \lambda_{x(j),x(k)}\\ \lambda_{x(j),x(k)} & \sigma^2_{x(k)}\end{bmatrix}^{-1}\begin{bmatrix}x(j)\\x(k)\end{bmatrix}$

$= [j \quad k]\begin{bmatrix} j & k\\ k & k\end{bmatrix}^{-1}\begin{bmatrix}x(j)\\x(k)\end{bmatrix} = [j \quad k]\frac{\begin{bmatrix} k & -k\\ -k & j\end{bmatrix}}{jk-k^2}\begin{bmatrix}x(j)\\x(k)\end{bmatrix} = x(j).$

3b. Makes sense: II$\rightarrow$Markov$\rightarrow$disregard $x(k)$ given $x(j)$ to estimate $x(i)$.

3c. $f_{x(i),x(j)}(X_i, X_j) = f_{x(i)|x(j)}(X_i|X_j)f_{x(j)}(X_j)$

$= f_{x(i)-x(j)|x(j)}(X_i - X_j|X_j)f_{x(j)}(X_j) = f_{x(i)-x(j)}(X_i - X_j)f_{x(j)}(X_j)$

$= f_{x(i-j)}(X_i - X_j)f_{x(j)}(X_j) = \frac{2^ie^{-2X_i}(X_i-X_j)^{i-j-1}X_j^{j-1}}{(i-j-1)!(j-1)!}, X_i > X_j > 0.$

3d-f. $y(n) = 1$ with prob. p^n; $y(n) = 0$ with prob. $1-p^n$.

3d. $\lim_{n\to\infty} Pr[|y(n) - 0| > \epsilon] = \lim_{n\to\infty} Pr[y(n) = 1] = \lim_{n\to\infty} p^n = 0.$

3e. $\lim_{n\to\infty} E[(y(n) - 0)^2] = \lim_{n\to\infty} 1^2 \cdot Pr[y(n) = 1] = \lim_{n\to\infty} 1^2p^n = 0.$

3f. $\lim_{n\to\infty} \sum_{i=1}^n Pr[|y(i) - 0| > \epsilon] = \lim_{n\to\infty} \sum_{i=1}^n Pr[y(n) = 1] = \lim_{n\to\infty} \sum_{i=1}^n p^i$
$= \sum_{i=1}^\infty p^i = \frac{p}{1-p} < \infty \rightarrow$converge with prob. 1.