

EX #1: To generate sample functions $x(n)$ of a Bernoulli process with $p=0.6$:

1. Spin a wheel of fortune *once*, resulting in a number $\omega_o \in [0, 1)$.
2. Let ω_o have decimal expansion $0.w_1w_2w_3w_4w_5 \dots$ where $w_i = 0, 1 \dots 9$.
3. Set $x(n)=1$ if $w_n=0,1,2,3,4$ or 5 and $x(n)=0$ if $w_n=6,7,8$ or 9 .

$$\omega_o = 0.141592653589793\dots \rightarrow \{x(n)\} = \{1,1,1,1,0,1,0,1,1,1,0,0,0,1,\dots\}.$$

$$\omega_o = 0.718281828459045\dots \rightarrow \{x(n)\} = \{0,1,0,1,0,1,0,1,0,1,1,0,1,1,\dots\}.$$

1. Different sequence of 0's and 1's associated with each $\omega_o \in \Omega=[0,1)$.
Each sequence is a *sample function* of the Bernoulli random process.
2. Digits independent of each other (if ω_o irrational); $\Pr[x(n)=1]=0.6$.
3. $\Pr[\omega_o \text{ irrational}]=1$ so this *almost surely* works (need ∞ precision).

EX #2: Monte Carlo simulation of compounded rate of return on investments:

1. Let $x(n)$ be the *annual* rate of return on investments in year $2000+n$.
2. Let $x(n) \sim \mathcal{N}(0.1, 0.0004) \Leftrightarrow E[x(n)] = 10\%$ (historical average).
3. Confidence limits or intervals (see pp. 275-280 and recitation) are:

$$\Pr[8\% < x(n) < 12\%] = 67\%; \quad \Pr[6\% < x(n) < 14\%] = 95\%.$$

4. Let $y(n) = \prod_{i=1}^n (1+x(i))$. Assume $x(n)$ iidrvs. Then $E[y(n)] = (1.1)^n$.
5. Generate 10 *sample functions* of $x(n)$ (Matlab's **randn**) and $y(n)$.
Results plotted below. Curves are different *sample functions* of $y(n)$.

- Note that the actual return can vary **greatly** from the expected return!
So don't plan on retiring too quickly! "Your performance may vary..."

