EECS 551 Fall 1997

EXAM #2

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Open book, notes, problem sets. Given under Honor Code. SHOW YOUR WORK! You will be graded on your explanations as well as on your answers. DETAILS!

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NOTE: Most problems on this exam are independent of other problems on the exam.

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(40) 1. **Computing continuous-time Haar wavelet transforms:**

For the function $x(t)$ shown at right:

- (10) a. Compute the Haar *scaling* transform $\bar{x}^i_j$ of $x(t)$.
- (10) b. Compute the Haar *wavelet* transform $x^i_j$ of $x(t)$. Be sure to consider all possible integer values of $i$ and $j \geq 0$.
- (5) c. Write Mallat's fast wavelet algorithm for the Haar basis.
- (10) d. Insert answers to (a),(b) into (c); confirm algebra works.
- (5) e. For small $i$ $x^i_j$ looks like a scaled Haar basis. Explain this shape.

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(40) 2. **Computing 1st-order Battle-Lemarie wavelet transforms:**

Here $n^{th}$-order Battle-Lemarie wavelets are based on $n^{th}$-order spline $s_n(t)$. Still using the function $x(t)$ from Problem #1:

- (5) a. Prove, using subspace ideas ONLY (e.g., $V_i, W_i$), that only scale $i = 0$ is needed to represent $x(t)$ using 1st-order Battle-Lemarie scaling functions.
- (10) b. Compute the 1st-order Battle-Lemarie *scaling* transform of $x(t)$ directly using formula $\bar{x}^i_j = \int x(t) 2^{-i/2} \phi(2^{-i}t - j)dt$. HINT: Write $\phi(t) = \sum c_n s_1(t-n)$; express answer in terms of constants $c_n$.
- (10) c. An easier way: Somewhere in the derivation of the 1st-order Battle-Lemarie scaling function there is an equation that gives the answer to (b) directly. Compute $DTFT[\bar{x}^i_j]$ as an explicit function of $\omega$.
- (10) d. Compute the Fourier transform of the 2nd-order Battle-Lemarie scaling function. HINTS: $\int s_2(t)^2 dt = \frac{66}{120}$; $\int s_2(t)s_2(t-1)dt = \frac{26}{120}$; $\int s_2(t)s_2(t-2)dt = \frac{1}{120}$.
- (5) e. Show that your answer to (d) is piecewise quadratic.

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(20) 3. **Computing 2D continuous Haar transforms:**

$z(x,y)$ is defined as $z(x,y) = \begin{cases} 1, x \leq 0, y \geq 0 \\ 3, x \leq 0, y \leq 0 \end{cases}$; $z(x,y) = \begin{cases} 2, x \geq 0, y \geq 0 \\ 4, x \geq 0, y \leq 0 \end{cases}$.

- (10) a. Show that the 2D Haar wavelet transform $z^{i,(m)}_{j,n} = 0$ for all $i, j, n$ and $m = 1, 2, 3$!
- (5) b. So how does the 2D Haar wavelet transform represent $z(x,y)$?
- (5) c. If noise is added to $z(x,y)$, explain how thresholding can remove most of it.

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DON'T FORGET TO WRITE OUT AND SIGN THE HONOR PLEDGE:
"I have neither given nor received aid on this exam,
nor have I concealed any violation of the Honor Code."