SUMMARY OF ORTHOGONAL QMFs

THM: Let $H_0(\omega)$ be a lowpass filter satisfying $|H_0(\omega)|^2 + |H_0(\omega+\pi)|^2 = 2$.
Then a perfect-reconstruction QMF is given by
$G_0(\omega) = H_0(-\omega); \quad G_1(\omega) = H_1(-\omega)$,
where $H_1(\omega) = -H_0(\pi-\omega)e^{-j\omega(2N-1)}$, which is equivalent to $h_1(n) = (-1)^n h_0(2N-1-n)$ using $DTFT[h_0(2N-1-n)] = e^{-j\omega(2N-1)}H_0(-\omega)$.
Similarly $G_1(\omega) = -G_0(\pi-\omega)e^{-j\omega(2N-1)}$.

PROOF:

1. Must satisfy no-aliasing condition $G_0(\omega)H_0(\omega+\pi) + G_1(\omega)H_1(\omega+\pi) = 0$.
2. Substituting for $H_0(\omega)$ and $H_1(\omega)$ gives $G_0(\omega)G_0(\pi-\omega) + G_1(\omega)G_1(\pi-\omega) = 0$.
3. Replacing ω with $\pi-\omega$ in $G_1(\omega)$ above gives $G_0(\omega)G_0(\pi-\omega) + (-G_0(\pi-\omega)e^{-j\omega(2N-1)})$ $\times(-G_0(\pi-(\pi-\omega))e^{-j(\pi-\omega)(2N-1)})$ $= G_0(\omega)G_0(\pi-\omega) - G_0(\pi-\omega)G_0(\omega) = 0$ QED using $-e^{-j\pi(2N-1)} = 1$.

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4. Must satisfy perfect-reconstruction $G_0(\omega)H_0(\omega) + G_1(\omega)H_1(\omega) = 2$.
5. Substituting for $G_0(\omega)$ and $G_1(\omega)$ gives $|H_0(\omega)|^2 + |H_0(\omega+\pi)|^2 = 2$ QED.

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6. Need time reversal between g_0, h_0 AND h_0, h_1;
7. odd-valued time shift between g_0, g_1 and h_0, h_1