EECS 551 Fall 1997

PROBLEM SET #1

ASSIGNED: Sept. 4, 1997
DUE DATE: Sept. 11, 1997

Read Chapters 1 and 2 (review) of Vetterli and Kovacevic (henceforth V&K).
Section 2.5 (multirate) may be new. Skip Section 2.6 (time-frequency) for now.

THIS WEEK: Some APPLICATIONS of DSP ideas to numerical analysis.
I've found many students haven't been exposed to the ideas below.
These should be straightforward, but you will still likely learn something new.

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1. We want to differentiate a signal $x(t)$ bandlimited to 20 kHz.
   a. Sketch a complete digital filtering system. Leave $h(n)$ unspecified.
   Include a sampler (A/D), a digital filter $h(n)$, and interpolator (D/A).
   b. Compute the digital filter $h(n)$ that implements differentiation.
   HINT: Compute $DTFT^{-1}\{j\omega, |\omega| < \pi\}$. Why this?
   c. Show that neglecting $h(n)$ for $|n| > 1$ leads to the central difference rule for differentiation: $\frac{dx}{dt} \approx [x(t+\Delta) - x(t-\Delta)]/2\Delta$
   d. Using MATLAB, PLOT on one plot the frequency responses (DTFT magnitudes) of both $h(n)$ from (b) and the truncated $h(n)$ from (c).
   For what frequencies does the truncation work well?

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2. Now we want to integrate a signal $x(t)$ bandlimited to 20 kHz.
   a. Using MATLAB, PLOT the frequency responses (DTFT magnitudes) of
   i. the running sum rule and the ideal integrator (one plot)
   ii. Simpson's rule and the ideal integrator (one plot)
   This should reproduce the plots on p.6 of the class notes.
   b. Explain why Simpson's rule blows up for ANY critically-sampled $x(t)$
   HINT: Where are the poles of the Simpson's rule transfer function?
   c. Give a rule on sampling $x(t)$ to avoid this problem.

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3. Spectral factorization (we will need this later, for wavelet filters):
   A *minimum phase* signal has all its poles and zeros inside the unit circle.
   a. Show that if $x(n)$ is minimum phase, there exists another minimum phase signal $x^{-1}(n)$ such that $x(n) * x^{-1}(n) = \delta(n)$.
   b. Show that any signal $x(n)$ with no poles or zeros on the unit circle can be factored as $X(z) = X_{min}(z)X_{max}(z)$ where X_{min} and $X_{max}(1/z)$ are minimum phase.
   c. Perform (b) for the signal $x(n) = \{6, -35, 62, -35, 6\}$ where $x(0) = 62$.
   Perform (a) for the minimum phase factor you get.
   d. An even signal $r(n) = r(-n)$ can be *spectrally factored* as $R(z) = H(z)H(1/z)$ if it is *positive definite*. Show that if $r(n)$ is not positive definite then it has a zero or pole on the unit circle.

"Never let your schooling interfere with your education"–Mark Twain