EECS 551 Fall 1997

PROBLEM SET #5

ASSIGNED: Oct. 2, 1997
DUE DATE: Oct. 9, 1997

Read Sections 4.4 (wavelet bases) and 4.5 (wavelet series) of V&K.
This week's theme: Applications of discrete-time wavelet basis expansions.

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1. *Wavelets for sparsification of linear systems of equations:*
   Consider the discrete-time deconvolution problem in which we observe
   $y(n) = h(n) * u(n), 0 \leq n \leq 7$ and $h(n) = 0.99^{|n|}$ is the impulse response.
   We wish to reconstruct $u(n), 0 \leq n \leq 7$ from $y(n), 0 \leq n \leq 7$ $(u(n) = 0, n \notin [0, 7])$.
   a. Formulate this deconvolution problem as a linear system of equations.
   b. Use the Haar basis to get another linear system of equations which is sparse.
      A SPARSE linear system matrix has most of its elements very small→negligible.
      HINT: $y = Hx \rightarrow Qy = (QHQ^T)(Qu)$ where Q is the matrix A_0 on p.145.
      POINT: Sparse systems of equations are easier and faster to solve.

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2. *Subband Discrete Fourier Transform (DFT)*:
   We want to compute 2^N-point DFT of $x(n)$. We know $X(k) \approx 0$ for $k > 2^{N-1}$.
   We need the higher-order DFT to get good resolution of the frequency components.
   However, we can stand to compute only an *approximation* to $X(k)$.
   a. Show neither decimation-in-time nor decimation-in-frequency FFTs help here.
   b. Preprocess $x(n)$ by taking its Haar transform: $\mathcal{H}\{x(n)\} = x_1(n)$.
      Show $X(k) \approx (1 + e^{-j2\pi k/2^N})X_1(k)$ where $X_1(k) = DFT[x_1(n)/\sqrt{2}]$.
      Explain why this roughly *halves* the computation required for $X(k)$.
   c. Now suppose $X(k) = 0$ except in a few known frequency bands of interest.
      Show that taking the DFT of a subband decomposition (wavelet transform) of $x(n)$ saves a substantial amount of computation.

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3. V&K #3.2. HINT: p.153.
4. *Discrete-time fractals:*
   A *fractal* is self-similar:
   $x(2n)$ and $x(n)$ "look alike."
   The MATLAB program shown at right
   computes a *fractal* signal $x(n)$.
   a. PLOT $x(n), n \in [1, 1024]$ and $[1, 128]$.
   It also computes the Haar transform.
   as rows of the matrices $y0$ and $y1$.
   b. PLOT the $4^{th}, 5^{th}, 6^{th}$ rows
      of $y0$ on a single plot. Repeat for $y1$.
   c. Show any wavelet transform of $x(n)$
      at different scales is almost identical.

Excuse heard in genetic engineering class: "My homework ate the dog."