GIVEN: Observe $r(t) = \begin{cases} s(t)+n(t) & \text{under } H_1 \\ n(t) & \text{under } H_0 \end{cases}, \quad 0 \le t \le T.$

$n(t)$ NWGN (Non-White Gaussian Noise); 0-mean, $K_n(t,s)$.

GOAL: Decide between H_0 and H_1 using data $\{R(t), 0 \le t \le T\}$.

K-L: Solve integral eqn $\int_0^T K_n(t,s)\phi_i(s)ds = \lambda_i \phi_i(t), 0 \le t \le T$.

$\rightarrow$ $n(t) = \sum n_i \phi_i(t), 0 \le t \le T$. Similarly for $r(t)$ and $s(t)$.

$n_i = \int_0^T n(t)\phi_i(t)dt \sim N(0,\lambda_i)$; $s_i = \int_0^T s(t)\phi_i(t)dt$.

Project onto $\phi_i(t)$: Multiply by $\phi_i(t)$ and $\int_0^T dt$:

$\rightarrow$ $r_i = \begin{cases} s_i + n_i & \text{under } H_1 \\ n_i & \text{under } H_0 \end{cases}, i = 1, 2 \ldots$ Let $\begin{matrix} r = [r_1, r_2 \ldots] \\ s = [s_1, s_2 \ldots] \end{matrix}$.

SOLVE: $\ell = \Delta m^T Q R = \sum \frac{R_i s_i}{\lambda_i} = \int\int R(t) Q(t,u) s(u) dt\, du$ where

(substitute $R_i = \int_0^T R(t)\phi_i(t)dt$ and $s_i = \int_0^T s(u)\phi_i(u)du$)

$Q(t,s)$ $= \sum \frac{\phi_i(t)\phi_i(s)}{\lambda_i} \rightarrow$ solve $\int K(t,u)Q(u,s)du = \delta(t-s)$.

WSS: $n(t)$ WSS, interval $(-\infty,\infty) \rightarrow Q(\tau) = \mathcal{F}^{-1}\{1/S_n(\omega)\}$.

SOLN: LRT is: $\ell = \int\int s(t)Q(t,u)R(u)dt\, du \gtrless \gamma$=threshold where

$\ell \sim N(d^2, d^2)$ under H_1; $\ell \sim N(0, d^2)$ under H_0 where

d^2 $= \int\int s(t)Q(t,u)s(u)dt\, du = \sum \frac{s_i^2}{\lambda_i}$=Fisher discriminant.

See "Vector Gaussian Detection"; notes from Feb. 12.

NOTE: *Matched filter* except match to $\int Q(t,u)s(u)du$, not $s(t)$.

Antipodal signaling in continuous time:

Which $s(t)$ maximizes d^2 subject to $\int_0^T s(t)^2 dt = E$?

$s(t) = E\phi_N(t)$ where λ_N is minimum eigenvalue.

BUT: $\sigma^2_{n(t)} = K_n(t,t) < \infty \rightarrow \sum \lambda_i = \int_0^T K_n(t,t)dt < \infty$

$\rightarrow$ $\lim_{i\to\infty} \lambda_i = 0 \rightarrow \lambda_\infty = 0 \rightarrow d^2 = \infty$! Perfect detection!

Singular detection; so model $n(t)$ with white component.