Delay-optimal hybrid ARQ protocol design for channels and receivers with memory as a stochastic control problem

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***Abstract*—Automatic repeat request (ARQ) protocols are utilized as a flexible way to adapt data transmission to channel variations whenever a feedback channel is available. The transmitter encodes the information into a packet and the receiver attempts to decode it. If decoding is not successful, the receiver signals the transmitter to either resend the same information or send additional information about the data. In this paper we consider ARQ protocols where the transmitter controls the amount of error correction capability introduced in the information sequence to minimize the expected delay. We formulate this problem as a stochastic control problem and study two cases of interest depending on whether or not the receiver feeds back information about the channel state. Some of the benefits of this formulation are an expression for the optimal packet size and delay as a solution of a fixed point equation and a unified treatment for channels with Markov statistics and for receivers with memory.**

I. Introduction

Error control techniques can generally be classified either as forward-error-correction (FEC) or as automatic-repeat-request (ARQ) [1, Ch. 15]. In FEC an information packet is encoded by introducing redundancy in a controlled way in order to cope with channel noise. Although FEC can provide a communication system with a fixed throughput which is equal to the code rate, the disadvantage of a conventional FEC system without feedback is that the transmitter does not adjust the error correction capability according to the channel variations. As a result, the code rate must be low enough for the worst-case channel. In an ARQ system on the other hand, the receiver signals the transmitter to either resend the same information or send additional information about the data if a decoding error occurs. This way the ARQ scheme adapts to channel variations since the transmission will be completed in a short period in a benign channel and in a longer period if the channel is harsher. Therefore, ARQ error control systems are more suitable than FEC systems for error control in data communication systems having time varying channels where a feedback channel is available. The advantages of both FEC and ARQ techniques can be obtained by combining the techniques within a single hybrid-ARQ protocol [2]–[11].

Many adaptive ARQ protocols have been suggested in the literature to improve the system delay (or equivalently throughput). In [5], [9], [12], different block (packet) sizes and multicopy transmission schemes were used as adaptation mechanisms, respectively. In [7], [13], [14], the authors suggested to vary the FEC code rate to compensate for the variations in the channel conditions. Finally, the authors in [15] proposed a methodology for the analysis of the joint statistics of the energy consumed and the delay incurred in a general ARQ protocol. The focus of the above papers is analysis rather than design of an optimal strategy. The latter is usually accomplished by numerical optimization of the system parameters. In [16] the authors consider the design of the downlink of a communication system without forward error correction and ARQ as a stochastic control problem. Recently [17], [18] utilized a stochastic control approach to obtain delay-optimal rate allocation strategies in multi-user environments taking into account the users' arrival statistics. In the above works, coding is not explicitly modeled nor are receivers with memory considered.

In this paper, the design of an ARQ system is formulated as a stochastic control problem whereby the transmitter controls the code rate of each transmitted packet in order to achieve a minimum overall average delay. The proposed approach is flexible enough to incorporate receivers with memory, i.e., receivers that decode the packet at time t based on all past and present information available regarding this packet. Channel memory is taken into account by modelling the channel as a Markov process. Two cases of interest are studied. In the first one, the receiver feeds back to the transmitter channel state information (CSI), while in the second one it does not. The solution of this stochastic control problem, i.e., the optimal strategies and corresponding optimal delays are given in different forms for the two cases of interest. When CSI is fed back, the solutions are given in the form of a dynamic program, which results in a fixed point equation over functions in $\mathbb{R}^M$ (where M is the number of channel states). When CSI is not fed back, the solutions are given in the form of a dynamic program, which results in a fixed point equation over probability mass functions. The proposed methodology is then applied to a simple two-state channel and numerical results are obtained.

The remaining of this paper is structured as follows. Section II presents the system model, while the decoding error performance is discussed in section III. The problem is formulated and solved in section IV for the two cases regarding CSI feedback. The special case of a two-state channel is examined in section V and the conclusions are summarized in section VI.

II. SYSTEM MODEL

Consider an ARQ protocol where the transmitter receives K information bits from the higher protocol layer (K is assumed constant). The transmitter utilizes a very low rate (mother) error correcting code and chooses to transmit n_t symbols from this code at time t. The packet of length n_t is transmitted to the channel using energy E per transmitted symbol (where E is also assumed constant), resulting in a total energy of En_t per packet.

The channel is assumed to be a memoryless symmetric channel during the packet transmission, described by the conditional density $Q_i(y|x)$, where x, y are the input and output symbols respectively. To model slow channel variations, we assume that the channel state changes from packet to packet according to a Markov chain model described by $Pr(h_{t+1}|h_t) = P(h_{t+1}, h_t)$, $Pr(h_0) = P(h_0)$, where $h_t \in [M] \stackrel{\text{def}}{=} \{1, \ldots, M\}$ and the $M \times M$ matrix $P(\cdot, \cdot)$ and the M-dimensional vector $P(\cdot)$ are assumed known. Thus when the Markov process is at state h_t, the channel conditional density is $Q_{h_t}(\cdot|\cdot)$.

At the receiver, all current and past symbols related to the same K-bit information sequence are utilized for decoding. The receiver sends back an acknowledgement (ACK) or a negative acknowledgement (NACK), depending on whether the packet is correctly decoded or not. Upon reception of a NACK, the transmitter at time $t+1$ will select a new set of n_{t+1} symbols to transmit until an ACK is received.

Regarding the decoding process, the receiver either has knowledge of the channel realization h_t (i.e., it has perfect channel state information (CSI) through accurate measurements) and utilizes this information during decoding, or does not have CSI or chooses not to utilize it in the decoding process. In this paper, we assume the former, i.e., perfect receiver CSI.

There are two different scenarios regarding the kind of additional information the receiver feeds back to the transmitter. In the first scenario, the receiver sends back the CSI, i.e., the channel realization h_t (together with the ACK/NACK signal). In the second scenario, no CSI is fed back to the transmitter (so only the ACK/NACK signal is sent). As it will be shown, this difference changes the problem fundamentally. In both cases, the feedback channel is assumed noiseless.

Finally, it is assumed that the higher layer protocol always has packets to transmit, i.e., heavy traffic is assumed, although incorporating a Markov arrival process does not change the problem fundamentally.

III. DECODING PERFORMANCE ANALYSIS

The probability of error at the decoder at time t depends on the number of information bits per packet K, the particular code and modulation used, the transmission energy E, as well as the actual channel model conditioned on the state h_t, and the decoding algorithm. It further depends on the particular realization of the channel $h^t \stackrel{\text{def}}{=} (h_1, \ldots, h_t)$ as well as the selection of the packet lengths n^t up to time t.

In order to provide a unified treatment we choose to abstract all the above design details by approximating the decoding performance through the random coding exponent. In particular, at time t, the probability of decoding error is approximated by

$$Pe_t = \exp\left(-s_t \max_{0\le\rho\le1} E_t(n^t, h^t, \rho)\right) \tag{1}$$

where

$$E_t(n^t, h^t, \rho) = \sum_{i=1}^{t} \frac{n_i}{s_t} E_{r0}(h_i, \rho) - \rho\frac{K}{s_t} \tag{2a}$$

$$= \sum_{j=1}^{M} \frac{s_{j,t}}{s_t} E_{r0}(j, \rho) - \rho\frac{K}{s_t}. \tag{2b}$$

This last expression is a simple generalization of the random coding exponent [19, Ch. 7] where different parts of the coded sequence face different channels (the derivation is omitted due to space limitations). $s_{j,t} = \sum_{i=1}^{t} n_i I(h_i = j)$ is the total number of coded symbols accumulated so far that have been transmitted over the channel with realization $h_i = j$, $s_t = \sum_{j=1}^{M} s_{j,t}$ is the total number of coded symbols accumulated so far and $I(\cdot)$ is the indicator function. It is noted that $E_{r0}(j, \rho)$ are known functions, each depending only on $Q_j(\cdot|\cdot)$. Recall that implicit in the definition of each function $E_{r0}(j, \rho)$ is a choice over the input distribution that maximizes the error exponent. In the following we assume that this maximizing input distribution is the same for all $j \in [M]$. This is a rather mild assumption as practical channels of interest $Q_i(\cdot|\cdot)$ are either symmetric discrete memoryless channels (DMC) or additive white Gaussian noise (AWGN) channels in which case the maximizing input distribution is uniform over the input alphabet, or Gaussian, respectively. This assumption is required in order to be able to generate a single "mother" code at the transmitter without knowledge of the channel realization. It is further assumed that the generated mother code has a maximum size s_{max} coded symbols.

Finally, observe that for a fixed K, the code performance at each time t is fully characterized by the vector $\mathbf{s}_t \stackrel{\text{def}}{=} (s_{1,t}, \ldots, s_{M,t})$. This is also true for probability of error expressions other than the ones derived using the random coding exponent (e.g., union bound based error calculations for linear block or convolutional codes can also be put in a similar form) and thus the rest of the development is also valid for these cases.

IV. PROBLEM FORMULATION

It is clear from the system description that by controlling the transmission parameter n_t the transmitter can either opt for a lower rate, i.e., more data protection against the channel in the expense of higher delay per packet and lower number of packet retransmissions, or choose a high code rate, i.e., less data protection for each individual packet resulting in shorter delay per packet at the expense of larger number of retransmissions. We are interested in the optimal policy that minimizes the total average delay per information bit. In the following we investigate separately two versions of this stochastic control problem, based on whether the receiver feeds back CSI or not.

A. Perfect CSI Feedback

The problem under consideration can be modelled as a controlled Markov process $\{x_t\}$, where $x_t = 0$ is an absorbing state reached after a successful transmission, and $x_t = (h_{t-1}, \mathbf{s}_{t-1})$ represents a state after an unsuccessful transmission over a channel with state h_{t-1}. The evolution of this process is described by the following transition probabilities where we denote by $\mathbf{e}_i$ the vector having all coordinates equal to zero and the i-th coordinate equal to 1.

$$Pr(x_{t+1} = 0 | x_t = 0) = 1 \tag{3a}$$

$$Pr(x_{t+1} \neq 0 | x_t = 0) = 0, \tag{3b}$$

$$Pr(x_{t+1} = 0 | x_t = (i, \mathbf{s})) = 1 - \sum_{j \in [M]} f(\mathbf{s} + n_t \mathbf{e}_j) Pr(h_t = j | h_{t-1} = i) \tag{3c}$$

$$Pr(x_{t+1} = (j, \mathbf{s}') | x_t = (i, \mathbf{s})) = f(\mathbf{s}') Pr(h_t = j | h_{t-1} = i), \quad \mathbf{s}' = \mathbf{s} + n_t \mathbf{e}_j \tag{3d}$$

$$Pr(x_{t+1} = (j, \mathbf{s}') | x_t = (i, \mathbf{s})) = 0, \qquad \mathbf{s}' \neq \mathbf{s} + n_t \mathbf{e}_j, \tag{3e}$$

where $f(\mathbf{s}')$ is the probability of packet error at time t as described in (1), (2).

Note that the state x_t is perfectly observed by the controller (transmitter) through the ACK/NACK mechanism of the ARQ together with the CSI feedback.

The instantaneous cost incurred at time t is

$$c(x_t, n_t) = \begin{cases} n_t/K & , x_t \neq 0 \\ 0 & , x_t = 0 \end{cases}, \tag{4}$$

since cost is incurred only when we transmit, and we are interested in minimizing the average normalized delay per information bit. An implicit assumption in (4) is that the delay is dominated by the "transmission" delay, i.e., the propagation delays (both in the forward and feedback channels) are small compared to the former. This assumption is valid for local area networks but it can be invalid for long distance links, such as satellite links etc.

The optimization problem of interest can be stated as

$$J^* = \min_{\mathbf{n} \in \mathcal{N}_M} J_{\mathbf{n}} = \min_{\mathbf{n} \in \mathcal{N}_M} E\{\sum_{t=1}^{\infty} c(x_t, n_t)\}, \tag{5}$$

where $E\{\cdot\}$ denotes expectation[1], and minimization is over all sequences of control actions $\mathbf{n} \overset{\text{def}}{=} (n_1, n_2, \dots) \in \mathcal{N}_M$. The minimization is restricted to the set $\mathcal{N}_M$ of all Markov policies of the form $n_t = n_t(x_t)$ without loss of optimality [20, Ch. 6].

The solution of this problem is given in the form of a backward dynamic program [20, Ch. 6]. Indeed, since there is a maximum number of code symbols s_{max}, both the state and action spaces are finite. Thus, if we define the average cost-to-go as

$$J_t(x_t) \overset{\text{def}}{=} \min_{n_t^\infty} E\{\sum_{i=t}^{\infty} c(x_i, n_i) | x_t\}, \tag{6}$$

then the dynamic program is expressed as

$$\begin{aligned} J_t(x_t) &= \min_{n_t} E\left\{c(x_t, n_t) + J_{t+1}(x_{t+1}) | x_t\right\} \\ &= \min_{n_t} \left\{c(x_t, n_t) + E\{J_{t+1}(x_{t+1}) | x_t\}\right\}, \end{aligned} \tag{7}$$

with $J^* = E_{x_1}\{J_1(x_1)\}$. Restricting ourselves to time-invariant policies, i.e., $n_t(x_t) = n(x_t)$ without loss of optimality (since both the cost and the Markov process are time invariant), and dropping the time index from all the above equations, it is straightforward to derive that $J(0) = 0$ and

$$J(i, \mathbf{s}) = \min_n \{n/K + \sum_{j=1}^{M} J(j, \mathbf{s} + n\mathbf{e}_j) f(\mathbf{s} + n\mathbf{e}_j) P(j, i)\}. \tag{8}$$

The solution of this fixed-point equation is the vector function $\mathbf{J}^*(\mathbf{s}) = (J^*(1, \mathbf{s}), \dots, J^*(M, \mathbf{s}))$ defined from $[s_{max}]_0^M \to \mathbb{R}_+^M$ and the corresponding minimum normalized average delay is $J^* = \sum_{i=1}^{M} J^*(i, \mathbf{0}) P(i)$, where we used the notation $[s_{max}]_0 \overset{\text{def}}{=} \{0, 1, \dots, s_{max}\}$. The minimizing rate function $\mathbf{n}^*(\mathbf{s}) = (n^*(1, \mathbf{s}), \dots, n^*(M, \mathbf{s}))$ is the optimal control law: at time t, upon observation of a NACK and the previous channel state $h_{t-1} = j$ the transmitter will update its locally stored state $\mathbf{s}$ to $\mathbf{s}' = \mathbf{s} + n^*(i, \mathbf{s})\mathbf{e}_j$ (where $n^*(i, \mathbf{s})$ was the optimal control law at time $t - 1$[2]) and transmit the next $n^*(j, \mathbf{s}')$ code symbols from the mother code. If an ACK is received instead, then the state is reset to $\mathbf{s} = \mathbf{0}$ and a new packet is transmitted with length $n^*(j, \mathbf{0})$ coded symbols.

Several observations are in order. Based on the procedure described above, we can evaluate the minimal delay as a function of E. If one wants to minimize over the energy consumed per information bit, then it should be clear that since this cost is proportional to $c(x, n)$, the optimal policy will be the same as the one derived earlier. In this case it is intuitive that minimizing delay and energy are equivalent goals.

If the transmitter can control both the packet size and the transmitted symbol energy E_t, then the problem of minimizing the average delay trivializes, since we can always send infinite amount of energy (or at least operate always with the peak available energy) to minimize the delay without incurring any cost. However, the problem discussed above, i.e., minimization of the average energy will be meaningful. In general, one can formulate a stochastic control problem where the objective is to minimize the average delay with a constraint on the average energy (see [21] for a treatment of constrained Markov decision processes). A similar problem can also be formulated if we define a maximum number of allowable retransmissions and then try to minimize average delay, with an average

[1]It can be shown that as long as there is a control action n_t resulting in $f(\mathbf{s} + n_t \mathbf{e}_j) < 1$ for all $j \in [M]$ and $\mathbf{s}$, then the Markov process will get to the absorbing state w.p. 1, and thus $J_{\mathbf{n}}$ will be finite. This is always true in any communication system.

[2]Strictly speaking, $h_{t-2} = i$ should be included in the state description x_t for this method to work. This addition is not changing the key features of the problem and it further complicates notation, so it is not adopted here.

energy and error probability constraint, the latter defined as the probability that the packet will not be successfully transmitted during the last allowable retransmission.

In order to implement the optimal policy, the fixed point equation (8) needs to be solved first. However, this processing is done at the design stage, i.e., off-line. This is usually done by appropriately quantizing the space $[s_{max}]_0^M$ and solving the algebraic fixed-point equation implied by (8). Once this is done, the control laws $n^*(i,\mathbf{s})$ (for $i \in [M]$, $\mathbf{s} \in [s_{max}]_0^M$) need to be stored at the transmitter. Again, sampled versions of these functions are stored and some sort of interpolation is performed on-line. Alternatively, if the optimal solution implies that transmission beyond a maximum of T steps results only in diminishing returns, then starting at $\mathbf{s} = \mathbf{0}$ all possible trajectories of $\mathbf{s}$ can be traced for all possible channel realizations up to time T and the corresponding optimal values of $n^*(i,\mathbf{s})$ can be evaluated off-line and stored at the transmitter. This requires exactly M^T such values to be generated and stored.

It is finally mentioned that (8) can be solved analytically in the asymptotic case $K >> 1$. The solution is (the proof is omitted due to space limitations)

$$J(i,\mathbf{s}) = J(\mathbf{s}) = \max\{\frac{K - \sum_{j\in[M]} C_j s_j}{K \max_{j\in[M]} \overline{C}_j}, 0\}, \tag{9a}$$

where C_j is the channel capacity when at state i, and $\overline{C}_i = \sum_{j\in[M]} C_j P(j,i)$. The corresponding optimal control is

$$n^*(i,\mathbf{s}) = \begin{cases} 0, & \overline{C}_i < \max_{i\in[M]} \overline{C}_i \\ \frac{K-\sum_{j\in[M]} C_j s_j}{\max_{j\in[M]} C_j}, & \overline{C}_i = \max_{i\in[M]} \overline{C}_i. \end{cases} \tag{9b}$$

This characterization is useful as it can serve as an accurate initial guess when a numerical solution of (8) is sought. In addition, it gives intuition on the optimal strategy[3] for finite K: the optimal strategy is not to transmit anything if the expected channel capacity conditioned on the current state is not the highest possible. On the other hand, if the expected channel capacity conditioned on the current state is the highest possible then the optimal strategy is to transmit a number of code symbols so that the overall resulting rate (anticipating the best channel) is exactly equal to the capacity of the realized channel so far.

B. No CSI Feedback

When the receiver does not feedback any CSI, the problem changes fundamentally. The reason is that the state of the controlled Markov process described in the previous subsection is not observed at the controller. In essence the controller's task in this case is that of dual estimation and control, i.e., the transmitter implicitly tries to infer the channel state and simultaneously choose the optimal control action. Following the standard methodology [20, Ch. 6] in solving these partially observable stochastic control problems, we seek an "information state" in order to transform the stochastic control problem into a deterministic optimal control problem. We consider the Markov process $\{x_t\}$ as defined previously. The transmitter at time t is only partially observing the state x_t through the signal $y_t \in \{\text{ACK}, \text{NACK}\}$, which is equivalent to whether x_t is zero or not. The belief of the transmitter regarding the state x_t at time t conditioned on the received signal y_t is an information state. In particular we have $Pr(x_t = 0|\text{ACK}) = 1$, $Pr(x_t \neq 0|\text{ACK}) = 0$, and $Pr(x_t = 0|\text{NACK}) = 0$. For $x_t = (i,\mathbf{s})$, the mass function $\pi_t(i,\mathbf{s}) \stackrel{\text{def}}{=} Pr(x_t = (\mathbf{s},i)|\text{NACK})$ has the following recursive form

$$\pi_{t+1}(j,\mathbf{s}) = \frac{f(\mathbf{s}) \sum_{i=1}^M P(j,i)\pi_t(i,\mathbf{s}-n_t\mathbf{e}_j)}{\sum_{j\in[M]}\sum_{\mathbf{s}} f(\mathbf{s}) \sum_{i=1}^M P(j,i)\pi_t(i,\mathbf{s}-n_t\mathbf{e}_j)}, \tag{10}$$

or in shorthand notation

$$\pi_{t+1} = T(n_t, \pi_t). \tag{11}$$

Defining a cost-to go function $J_t(\pi_t)$ with π_t being the current state observed by the controller, the solution to the cost minimization problem can be expressed in the following dynamic program

$$J_t(\pi_t) = \min_{n_t}\{n_t + J(T(n_t,\pi_t))Pr(y_{t+1} = \text{NACK}|\pi_t, n_t)\}. \tag{12}$$

Again, we can restrict ourselves to time-invariant policies $n_t(\pi) = n(\pi)$ without loss of optimality and after dropping the time indices the solution is expressed in the following fixed point equation

$$J(\pi) = \min_n\{n + J(T(n,\pi))g(n,\pi)\}, \tag{13a}$$

where

$$g(n,\pi) = \sum_{j\in[M]}\sum_{\mathbf{s}} f(\mathbf{s}) \sum_i P(j,i)\pi(i,\mathbf{s}-n\mathbf{e}_j). \tag{13b}$$

Observe that π is a probability mass function over $[s_{max}]_0^M \times [M]$.

In order to implement the optimal policy first the fixed-point equation has to be solved off-line thus obtaining $J^*(\pi)$ and the control law $n^*(\pi)$ for all π. This is usually accomplished by appropriately quantizing the random variable $\mathbf{s}$ (e.g., partitioning $[s_{max}]_0^M$ into Q sets), and then sampling the space of probability mass functions over MQ-ary variables (e.g., using L samples) thus transforming (13) into an approximate fixed point equation in $\mathbb{R}_+^L$. For general partially observed problems, the transmitter initializes the state π_1 according to the a-priori knowledge regarding the Markov process, and at each time t performs the control law $n^*(\pi_t)$ and updates the state π_t according to the recursion in (11) and the current input y_t. In this particular problem, however, the only input of interest is $y_t = \text{NACK}$, since when an ACK is received, the transmission terminates successfully. As a result, additional off-line processing is possible. In particular, the deterministic

[3]For this problem, any choice of code symbols smaller than the one stated in (9b) is indeed optimal. This is a result of the fact that no transmission (i.e., $n = 0$) is not penalized in this model.

system described in (11) can be run *off-line* with initial state $\pi_1 = p$ producing the sequence $\pi_1, \pi_2, \ldots$ and the corresponding optimal values $n^*(\pi_1), n^*(\pi_2), \ldots$. It is these values that have to be stored in the controller and used for the 1st, 2nd, etc retransmission. The minimal average delay incurred is then given by $J^* = J(\pi_1)$.

V. ARQ over the Gilbert-Elliot Channel

We now focus on a specific example to illustrate the above described methodology.

Regarding the channel, we assume a fading channel with two states ($M = 2$), a good ($h_t = 1$) and a bad ($h_t = 2$) state with $P(2,1) = a$ and $P(1,2) = b$. In the good (bad) state the channel behaves as an additive white Gaussian noise (AWGN) channel with a signal-to-noise ratio equal to SNR_1 (SNR_2). This is a commonly used channel model that captures the salient features of a time-varying fading channel with memory and is known as the Gilbert-Elliot channel model [22], [23].

A more convenient parameterization of this class of channels is through the average SNR, $\overline{SNR} = P(1)SNR_1 + P(2)SNR_2$ (where $P = (b/(a+b), a/(a+b))$ is the steady state distribution), the ratio of the good to bad SNR, $\gamma = SNR_1/SNR_2$, and the average time spend in the good and bad state, $T_1 = 1/a$, and $T_2 = 1/b$. Further, it is mentioned that the fastest, in the sense of most unpredictable, channel is the one for which $Pr(h_{t+1}|h_t) = Pr(h_{t+1})$, which implies $a + b = 1$; for smaller values of $a + b$ the channel dynamics become slower.

We first present results for the degenerate case where $SNR_1 = SNR_2$. In this case knowledge of channel state does not convey any information, and thus the two formulations in section IV should give identical results. Indeed, in this case the state $\mathbf{s}$ collapses to the scalar variable s and (8) degenerates to the fixed point equation

$$J(s) = \min_n \{n + J(s+n) f(s+n)\}. \tag{14}$$

Similarly, the fixed point equation (13) admits a solution of the form $J(\pi) = J(\delta_{s-s_0}) \stackrel{\text{def}}{=} J(s_0)$ satisfying the same equation as above, where δ_k denotes the Kronecker delta function. Fig. 1 depicts the minimal average delay as a function of the average SNR for different values of the packet size K (s_{max} was considered large enough so that it is not an active constraint). As can be observed, larger data packets result in better performance due to the exponential decrease of the error probability with K. The performance of the optimized hybrid ARQ scheme is very close to capacity even for very small K. Regarding the sequence of optimal packet lengths, it can be seen that the optimal strategy involves sending the longest part of the codeword in the first transmission and essentially ceasing transmission in subsequent retransmissions. This is consistent with the information theoretic analysis where the optimal transmission in a memoryless channel is obtained using FEC.

Next a non-degenerate two-state channel is considered. We consider a channel with $\gamma = 2$. The steady-state probability of the good channel, is fixed to $P(1) = 0.5$ and the channel

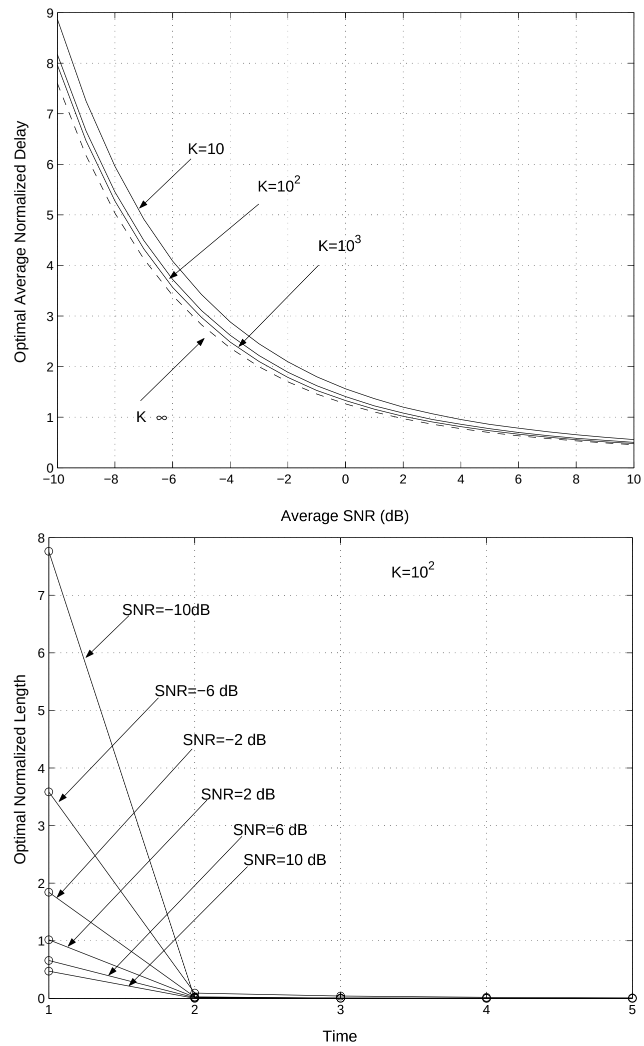

Fig. 1. Optimal average delay (a) and normalized codeword length (b) for the degenerate channel with $SNR_1 = SNR_2$.

parameters are selected such that $a + b = 1$, resulting in a completely unpredictable next state, or $a + b = 0.01$, resulting in a slowly varying channel. Results are shown in Fig. 2. Observe that for the unpredictable channel, the numerically evaluated result coincides with the theoretical derived for $K \to \infty$. The slow channel cost $J(2, \mathbf{0})$ coincides with the average cost for the fast channel, while the slow channel cost $J(1, \mathbf{0})$ is slightly better than the average cost for the fast channel. The result is a somewhat better average cost for the slow channel.

It is noted that for the case of $K \to \infty$, no-CSI feedback is equivalent to perfect CSI feedback for this two-state channel. The reason is that a NACK essentially reveals to the transmitter that the channel was in the bad state, since the optimal transmission scheme is to send enough coded symbols just to exceed capacity anticipating the good channel. This is obviously not true for channels with $M \geq 3$.

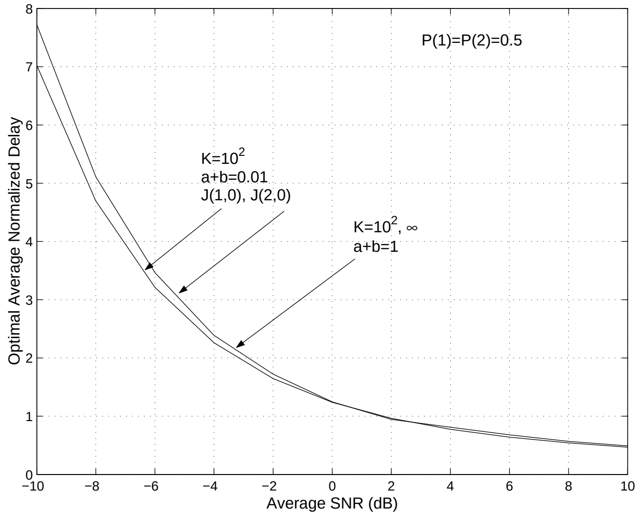

Fig. 2. Optimal average delay for the channel with $SNR_1 = 2SNR_2$. The message length is set to $K = 100$.

VI. Conclusions

The design of optimal ARQ protocols for channels and receivers with memory is considered in this paper. The transmitter controls the error correcting code rate in order to optimize the average delay. The problem is formulated as a stochastic control optimization problem and is solved for the two scenarios where the receiver feeds back or does not feed back CSI to the transmitter. In the former case the resulting system is a controlled Markov process with perfect observation and the solution is given in the form of a fixed point equation. In the latter, the Markov process is partially observed and the solution is given in the form of a fixed point equation over functions. The applicability of the proposed methodology is demonstrated for the special case of the Gilbert-Elliot two-state channel. It is shown that for slowly varying channels CSI feedback results in improved performance, while in both scenarios the complexity of implementing the optimal policy is minimal. An interesting future direction for research is the problem of minimizing the average delay with an average energy constraint, as well as the light-traffic case whereby delay is incurred both due to queueing and transmission.

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