Homework #4
(Do not hand in, for practice only)

1. Consider a volume coil and a surface coil. Let the volume coil have sensitivity,
   $S_v(x) = 1$, and the surface coil have the following sensitivity pattern (as a function of
   distance from the coil):

   $$S_s(x) = \frac{1}{\left(1 + \left(\frac{x}{a}\right)^2\right)^{3/2}}, \text{ where } a \text{ is the coil radius.}$$

   Let the noise variance of the volume coil be $\sigma_v^2 = 1$ and the noise variance of the
   surface coil be $\sigma_s^2 = 0.001 \cdot a^3$, where a is assumed to be in units of cm.
   a. For $a = 5$ cm, determine for which distance from the object surface it is
      advantageous (from a signal to noise ratio standpoint) to use the surface coil over
      the volume coil (and vice versa). SNR = (signal intensity)/σ, where σ is the noise
      standard deviation.
   b. For $a = 10$ cm, determine for which distance from the object surface it is
      advantageous to use the surface coil over the volume coil (and vice versa).

2. Consider 1 gram of gray matter brain tissue. Assume that the physiological
   parameters for this tissue at rest are:
   f = perfusion rate = 0.55 ml/min/g
   Oxygen extraction fraction (OEF) = 0.5
   Cerebral metabolic rate of oxygen (CMRO2) = a OEF f, where a is a constant
   V = Fractional blood volume = 0.05
   Q = Concentration of deoxyhemoglobin = b V OEF, where b is a constant

   $R2' = \dfrac{2Q}{3b}$ (in ms^{-1}), the relation component due to magnetic field perturbations

   $R2 = 1/60$ (in ms^{-1})

   a. What is the resting state $T2^*$?
   b. For TE = 30 ms, what is the image intensity (assume TR $\gg T1$)?

   Now assume that the brain tissue becomes active resulting in an increase in the
   oxygen metabolism (CMRO2) of 5%. In order the satisfy the metabolic needs of the
   tissue, the perfusion rate (f) increases by 40%, which also results in a blood volume
   (V) increase of 20%.

   c. What is the new OEF? Has this gone up or down?
   d. What is the new Q? Has this gone up or down?
   e. What is the new $R2'$? Has this gone up or down?
   f. What is the new $T2^*$? Has this gone up or down?
   g. For TE = 30 ms, what is the image intensity (assume TR $\gg T1$)? Has this
      gone up or down?