Estimation and prediction of Respiratory Motion with Real-time Phase Adjustment

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Abstract

We propose a hierarchical infrastructure for estimation, tracking, and prediction of respiratory motion. The hierarchical formulation allows incorporating information on different knowledge levels. To study breathing-related motion, in particular, it is desirable to account for semi-periodicity globally, and slow frequency/displacement variation locally. With the absence of well-justified model and high uncertainty, we use a data-based approach on the finer level to estimate the local variation of both frequency and displacement, utilizing classic control and chaos theory. A non-parametric warping map is estimated and used to "counteract" local variation, resulting in a much more regular post-warping signal. This corresponds to phase adjustment to bring each semi-period in the global level into synchronization. Globally, we model the phase-adjusted signal as a noisy observation of an intrinsic periodic system, and the best periodic pattern is obtained with an optimization approach.

Compared with conventional methods, the proposed framework relaxes the perfect periodic assumption required by most global models (e.g., Lujan '03, Neicu '03, Ruan '06), and thus offers more flexibility and robustness towards phase asynchronization. It also overcomes the myopia present in local state space models, and makes incorporation of semi-periodicity both natural and easy. It offers the freedom to balance the roles of physical knowledge and data fidelity differently at each level. A model-based method on the global level incorporates the well-recognized semi-periodicity pattern of respiratory motion, and data-driven phase estimation for warping and un-warping fully utilizes the observation and enjoys the freedom of nonparametric setup.

We have also derived a recursive estimation algorithm, which makes real-time processing efficient. Introduction of forgetting factor takes into account temporal sequentiality and the mild Markovianity in time series, and offers further flexibility in capturing long-term pattern drifting.

An initial experiment shows that when we attempt to predict one period ahead (which is about 5 seconds in most cases), the proposed method reduces tracking error (in RMSE) by about 50% compared with conventional perfect periodic extension. Traditional linear state models simply fail in such long-term prediction tasks.

1. Phase Estimation

Even though it is well-recognized that breathing-related motion is semi-periodic, there is no readily available way to extract the end points of periods from an observed trajectory. It is necessary, therefore, to obtain an instantaneous phase estimation in order to identify the correspondence across periods. Notice that the instantaneous frequency follows naturally as the derivative of the instantaneous phase.

Several methods exist for defining an instantaneous phase, but each of them bears certain disadvantages that makes the direct application problematic in studying respiratory motion. We briefly list them blow: The terminology "phase" technically means "a particular appearance or state in a regularly recurring cycle of changes", with two key features: periodicity and distinguishability. Therefore, phase can be generally defined as follows.[1]

Instantaneous Phase	Notation	Pros	Cons
natural phase	ϕ^N	motion of self-sustained oscillator	constant frequency assumption problematic
linear phase	ϕ^L	general, robust	requires characteristic marker events, unavailable!
Hilbert phase	ϕ^H	holomorphic signal analysis	sensitive to short-lived small fluctuations

Table 1: Existing definitions for instantaneous phase

Definition 1 (C^1 Restricted Definition of Phase) *Given a continuous temporal function f, its phase ϕ is defined as a continuous differentiable function of time t satisfying that for all t: (1) $\frac{\partial}{\partial t}\phi(t) > 0$ and*

[1] For convenience, we assume the phase function to be first order differentiable, which is a quite mild restriction in general.

(2)there exists a unique continous function F on $[0, 2\pi)$ such that $f(t) = F(\phi(t) mod 2\pi)$. Without loss of generality, we assume $\phi(0) = 0$.

Notice that the above definition does not uniquely determine the phase function. Indeed, it determines the phase map up to a diffeomorphism. Intuitively, any differentiable order-preserving warping of a given legitimate phase map would yield another phase map.

To incorporate the desirable properties of the existing instantaneous phases (i.e.,ϕ^N, ϕ^L, ϕ^H) and alleviate their disadvantage, we propose the following procedure:

1. Augment the observed signal $s(t)$ onto a 2-dimensional phase plane by its own proper delay $s(t - \tau)$. This is a numerically robust way that is analogous to the differential operation.

2. Take a proper Poincaré section. Indeed, iterative ellipse fitting (Ruan '07) followed by identifying Poincaré section with its minor axis allows an efficient and robust way to find isochrones. Such isochrones fall in high velocity region are much more robust to amplitude noise than extrema for phase purposes.

3. linearly interpolate between neighboring isochrone points. This guarantees monotonicity, and makes the phase strictly correspond to the zero Lyapunov exponent.

2. Phase Unwarping for State Estimation, Tracking and Prediction

With the phase map ϕ, the observed trajectory can be modeled as an intrinsic periodic system composed with warping and subsequently corrupted by additive noise.

2.1. Static Unwarping for Estimation

In the static scheme, a globally consistent basic pattern is assumed to be present. Therefore, it is natural to consider the underlying ideal signal as a perfect periodic function manifested with local stretching/shrinking. The observation can be modeled as:

$$s(t) = f_T(\phi(t)) + n(t). \tag{1}$$

Here $n(t)$ represents additive noise which we assume to be white, and f_T indicates the underlying periodic function with period T. Notice that since we conform to convention, and define the identified ϕ modulus 2π as isochrone time instances, f_T is a 2π periodic function. The particular choice of T is of incidental interest in itself.

As ϕ is monotone, and thus invertible, we can right-compose equation 1 with ϕ^{-1} and get

$$s(\phi^{-1}(\theta)) = f_T(\theta) + \eta. \tag{2}$$

Since f_T is perfectly $2\pi-$periodic, we represent a basic pattern as $\tilde{f} = f_T|_{[0,2\pi)}$, then the minimum mean squared error (MMSE) estimator is simply the average:

$$\tilde{f}(\theta) = \text{mean}\{s(\phi^{-1}(\psi)) : \psi = 2k\pi + \theta \quad \text{for some } k \in \text{N}\} \tag{3}$$

After obtaining $\tilde{f}(\theta)$ (and equivalently $f_T(\theta)$), we can obtain a "denoised" $s(t)$ by taking $f_T(\phi(t))$.

2.1.1. Adaptive Unwarping

In the adaptive unwarping setting, we relax the assumption of a global static $\tilde{f}(t)$. As new data becomes available, we incorporate the new information, and update our estimation for $\tilde{f}(t)$ correspondingly:

$$\tilde{f_t}^*(\theta) = \text{argmin}_{\tilde{f}(\theta)} \int_{\theta=0}^{2\pi} \sum_{i=1}^{K(\theta)} \lambda^{K(\theta)-i} ||s(\phi^{-1}(2\pi i + \theta)) - \tilde{f}(\theta)||^2, \tag{4}$$

where $K(\theta)$ is the biggest integer such that $K(\theta)2\pi + \theta \leq \phi(t)$. The adaptive weight $0 < \lambda \leq 1$ indicates how the past is discounted and the current samples are emphasized. It reflects the trade-off between system responsiveness and noise sensitivity and can be pictured as a "forgetting factor", where $\lambda = 0$ indicates forgetting everything in the past, and $\lambda = 1$ weight all the samples equally, ignoring the relative closeness of its position to the current time.

The solution turns out to be:

$$\tilde{f_t}^*(\theta) = \frac{1}{C_t(\theta)} \sum_{i=1}^{K(\theta)} \lambda^{K(\theta)-i} s(\phi^{-1}(2\pi i + \theta)), \tag{5}$$

where $C_t(\theta) = \sum_{i=1}^{K(\theta)} \lambda^{K(\theta)-i}$ is the normalization factor. This solution has the intuitive interpretation as the weighted averaging of historical data corresponding to the same phase.

The above expression can be computed recursively, so there is no growing memory issue, unlike the batch-processing case. Observe that for a given θ, $\tilde{f_t}^*(\theta)$ indeed changes in a discrete fashion with the growth of t, as only a growth in the upper summation bound $K(\theta)$ can affect the right hand side of equation 5. The same analysis applies to C_t. Therefore, it suffices for us to derive a recursion on K. For simplicity, we denote $\tilde{f_K}(\theta) \equiv \tilde{f_t}^*(\theta)$ and $C_K(\theta) \equiv C_t(\theta)$. The recursion is obtained as follows:

$$\begin{aligned} \tilde{f}_{K+1}(\theta) &= \frac{1}{C_{K+1}} \sum_{i=1}^{K+1} \lambda^{K+1-i} s(\phi^{-1}(2\pi i + \theta)) \\ &= \frac{C_K}{C_{K+1}} \tilde{f}_K(\theta) + \frac{1}{C_{K+1}} s(\phi^{-1}(2(K+1)\pi + \theta)). \end{aligned} \tag{6}$$

Extension from estimation to prediction is straight forward. Upon extrapolation of the warping map ϕ, the fundamental pattern $\tilde{f}_t^*(\theta)$ can be extended into the future.

3. Verification

We verified this study with the trajectories of an external fiducial placed on the patients' chest wall using the RPM system. Under IRB approved protocol, we have obtained breathing trace data recorded at 30Hz from 12 patients. The recorded RPM data have relative units. To better illustrate the major idea of this paper, we scale the breathing trace data to have variation similar to that of the lung tumor motion in superior-inferior direction. The post-scaling characteristic parameters of this population are listed in Table 2.

Patient ID	Mean motion amplitude (mm)	Standard deviation (mm)	Patient ID	Mean motion amplitude (mm)	STD (mm)
1	5.61	2.2	7	13.42	1.38
2	17.24	2.16	8	37.37	4.79
3	9.14	0.60	9	26.09	0.18
4	7.75	0.56	10	7.83	0.22
5	21.48	7.31	11	10.76	0.69
6	20.49	6.38	12	15.81	2.77

Table 2: Dataset information and Experiment Results

We show one typical case of phase map estimation based on the proposed Poincaré return map in Figure 1. Due to the noticeable noise present in the observed trajectory, the computed Hilbert phase exhibits large variations, instability and violates the bijection condition. In contrast, the proposed phase that is based on a simple Poincare sectioning followed by local linear interpolation, guarantees the local smoothness of the warping function.

With the static method, we first warp the observed signal according to phase estimation and average it to obtain the globally consistent basic pattern. The basic pattern is then unwarped back to the time unit from phase space and this offers estimation and tracking for the current state. We illustrate the obtained estimation in Figure 2. As a base-line reference, we compare it with the best-fit periodic signal found with a projection method. Even though both methods provide reasonable estimation results, a noticeable sacrifice in matching performance is present in the conventional method, due to local frequency drift around 10*sec*.

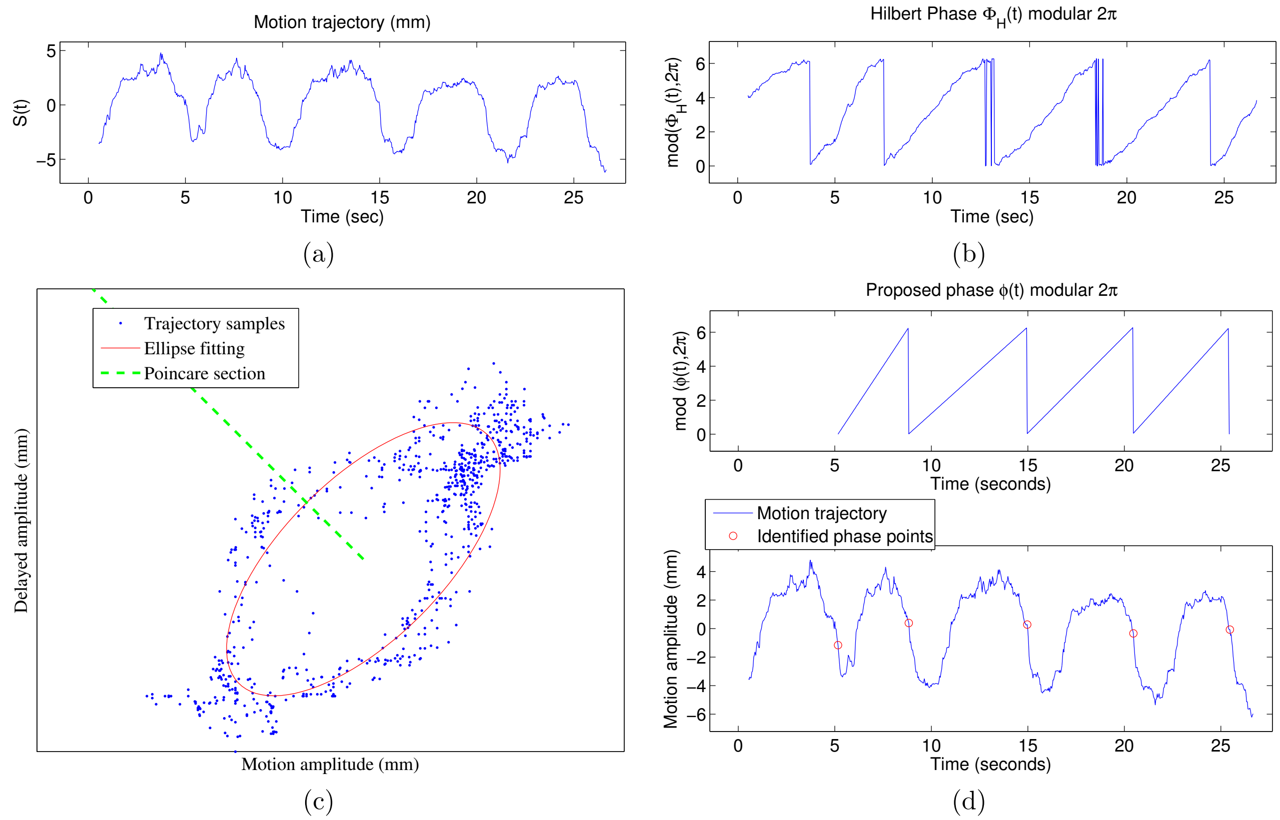
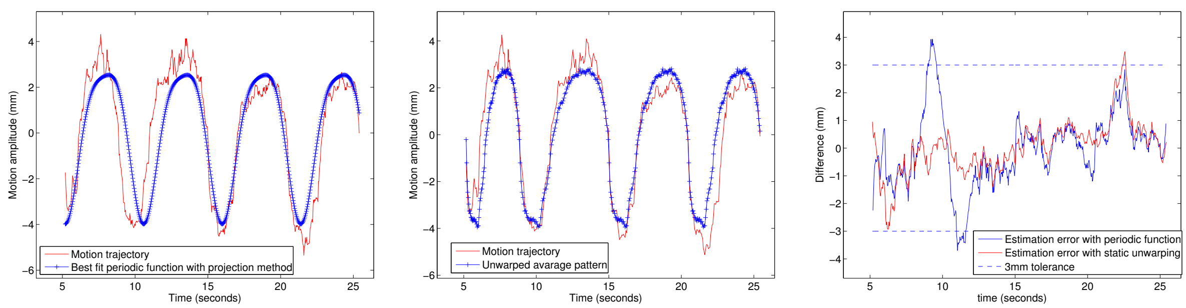


Figure 1: Extraction of isochrone instants using Poincaré map: (a) original signal $s(t)$; (c) isochrone extraction (d) phase extracted according to the isochrones in (c).

This is also illustrated clearly as the locally systematic positive(negative) estimation error before(after) the $10sec$ time instant.

The adaptive recursive algorithm further relaxes the consistent basic pattern assumption and allows the "shape" to change slowly over time too. Moreover, the recursion enables taking in new data as they become available in real-time, unlike the static method which is a retrospective batch-process. Therefore, the adaptive algorithm is a practical tool for real-time application, as well as for analysis.

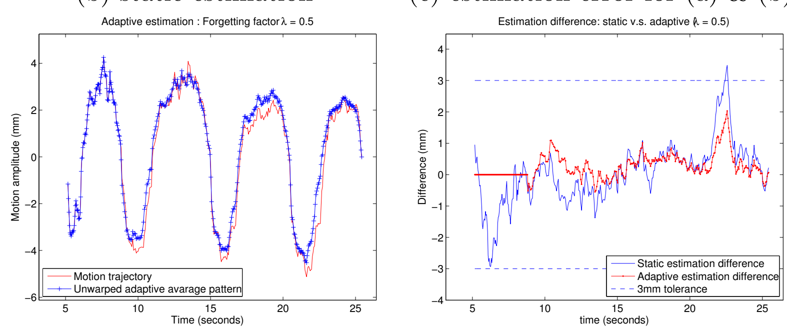


(a) periodic fitting

(b) static estimation

(c) estimation error for (a) & (b)

perfect periodic fitting is based on subspace projection method (Ruan et al '06)
static estimation assumes a consistent pattern
adaptive estimation is carried out with recursion



(d) adaptive estimation

(e) estimation error for (b) & (d)

Figure 2: Comparison of different estimation methods. top row: retrospective estimation assuming constant pattern; bottom row: adaptive estimation with recursion.