

Nonrigid Registration Using Regularization that Accommodates Local Tissue Rigidity

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Outline

- Problem Statement
- Regularization Setup
 - 1. Typical geometric regularizations
 - 2. Local tissue rigidity issue review
 - 3. Penalty design for non-rigidity
 - 4. Spatial varying regularization weights
 - 5. Parameterization & Optimization
- Experimental Results
- Discussions & Future Work

Parameterized Registration with Regularization

- Registration: to find a transformation T that minimizes a weighted sum of a dissimilarity energy and a regularization energy - two antagonist goals.

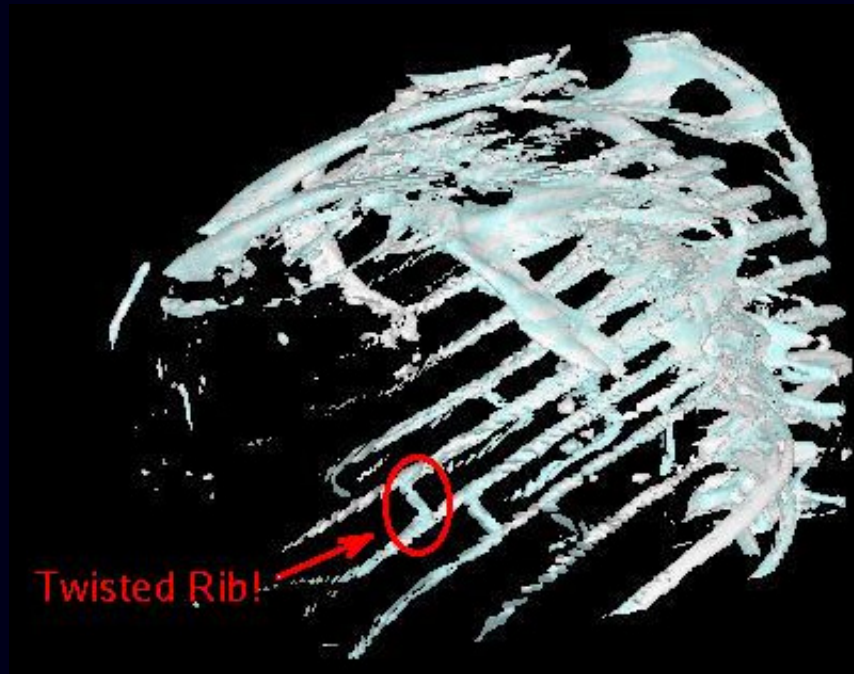
$$T^* = \arg \min_E(T), \quad E(T) = E_{sim}(T, X, Y) + E_{reg}(T).$$

where

- $X, Y : \mathbb{R}^3 \supset \Omega \rightarrow \mathbb{R}$: intensity maps for reference and homologous image;
 - $T : \Omega \rightarrow \Omega$ transformation for the region of interest;
 - $\Gamma : \{T(\theta) : \theta \in \Theta\}$, feasible set of parametric transformations;
 - $\underline{x} \in \Omega$, the coordinate of a spatial location.
- Conventional Regularization Methods
 - global smoothness \Rightarrow e.g., $E_{reg}(T) = \int \|dT\|^2$
 - topology preserving $\Rightarrow |J(T(\underline{x}))| > 0 \forall \underline{x}$,
where $|J(\cdot)|$ denotes the determinant of (local) Jacobian.
 - volume preserving \Rightarrow e.g., $E_{reg}(T) = \int (|J(T)| - 1)^2$

Problem with Conventional Methods

- Intensity based registration with global smoothness regularization
- Superposition of extracted bone structure from reference (white) and deformed homologous (light blue) images.



Drawback of Conventional Regularization Methods

- homogeneous regularization tends to ignore tissue type difference.
⇒ bone warping is a common and non-physiological result.
- Observation: bone structures should deform more rigidly than soft tissues.
- Existing Work
 - treat different regions independently [little 97, Huesman 03].
 - spatial-varying filter [Staring 05].

Proposed Regularization Design

- Geometric regularization is composed of a homogeneous roughness penalty and tissue type dependent non-rigidity penalty.

$$E_{reg}(\mathbf{T}) = \gamma_s E_{rough}(\mathbf{T}) + \gamma_r E_{nonrigid}(\mathbf{T})$$

- Smoothness regularization

$$E_{rough}(\mathbf{T}) = \int_{\Omega} \|d\mathbf{T}\|_{Frob}^2$$

- Tissue type dependent rigidity regularization

$$E_{nonrigid}(\mathbf{T}) = \int_{\underline{x} \in \Omega} \gamma(\underline{x}) r(\mathbf{T}_{\underline{x}}), \quad (1)$$

$\gamma(\underline{x})$: spatial varying relative regularization weight;

$\mathbf{T}_{\underline{x}}$: local transformation;

$r(\cdot)$ penalizes the deviation of transform from being rigid.

Design of non-Rigidity Index r

- Possible designs for functional $r : (\mathbb{R}^3 \rightarrow \mathbb{R}^3) \rightarrow \mathbb{R}_{\geq 0}$
 - “test of affineness”
 - topology preserving, volume preserving
 - condition number

- We propose

$$r(\mathbf{T}_{\underline{x}}) = \|\mathbf{J}_{\mathbf{T}}(\underline{x})\mathbf{J}'_{\mathbf{T}}(\underline{x}) - \mathbf{I}\|_{Frob}^2$$

- Justifications

- A local map map $\mathbf{T}_{\underline{x}}$ being rigid \Leftrightarrow its Jacobian $\mathbf{J}_{\mathbf{T}}(\underline{x})$ be an orthogonal matrix.
- A matrix $\mathbf{M} \in \mathbb{R}^{3 \times 3}$ is orthogonal $\Leftrightarrow \|\mathbf{M}\mathbf{M}' - \mathbf{I}\| = 0$, where $\|\cdot\|$ denotes any matrix norm.
- Frobenius norm is a feasible and computationally efficient choice.

Spatial Varying Regularization Weight $\gamma(\underline{x})$

- Spatial varying regularization weight $\gamma(\underline{x})$ reflects tissue type difference.

$$\gamma(\underline{x}) \begin{cases} \text{large} & \text{in rigid regions, e.g., bone structure} \\ \text{small} & \text{in elastic tissue regions, e.g., muscle, fat...} \end{cases}$$

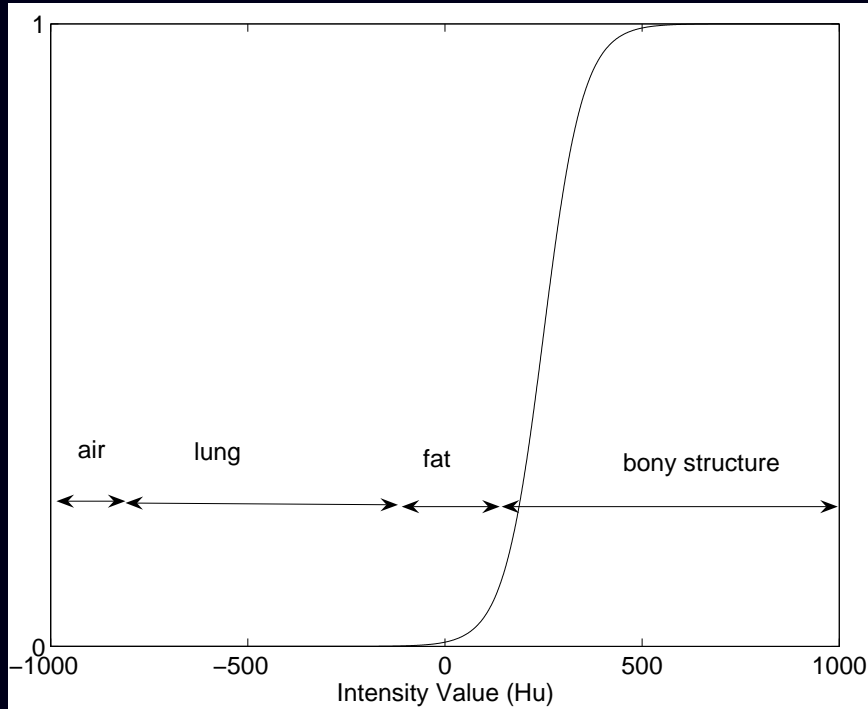
also refer to as “local stiffness factor” for physical interpretation.

- Observation:
In X-ray CT, voxel intensity (CT number) is highly correlated with tissue type, hence a good index for local rigidity.
- Instead of a direct map $\gamma: \Omega \rightarrow \mathbb{R}_+$, define it implicitly as a function of image intensity.

$$\gamma = h \circ X$$

where $h: \mathbb{R} \rightarrow \mathbb{R}_+$ is a (continuous) monotone increasing map from the domain of CT numbers to “rigidity level” of an arbitrary unit.

Illustration for Proposed Weight Function



Construct the weighting function that *softly* distinguishes tissue types.

- Sharp rising edge → inter tissue type sensitivity
- Saturation → intra tissue type robustness
- Simple form (scale and location parameter for tanh)

Experiment Setup

We register X-ray CT images acquired during different breathing phases:

- Description of Registration Technique

- Intensity based, SSD data dissimilarity metric

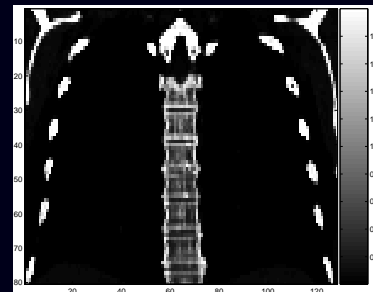
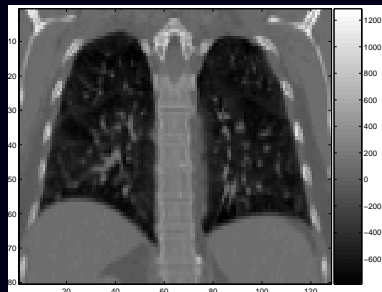
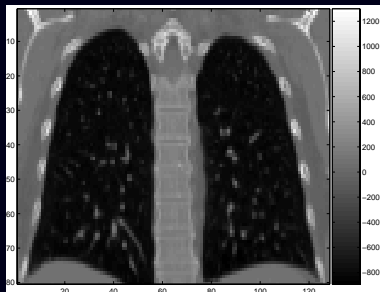
$$E_{sim}(T, X, Y) = \int_{\Omega} (X - Y \circ T)^2$$

- Tensor B-Spline parameterization
- Multi-resolution, gradient based optimization

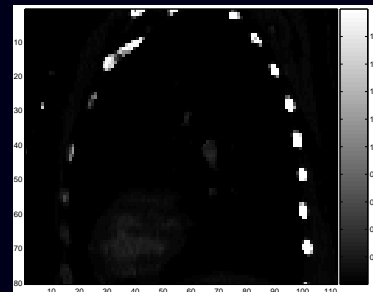
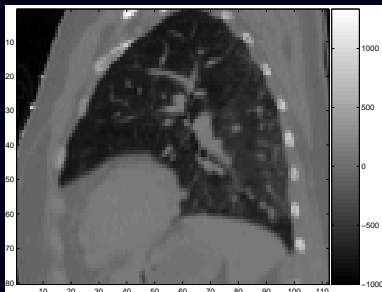
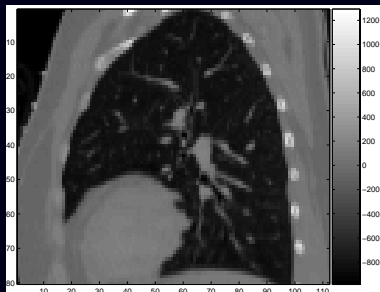
- Description of Experiment Data

- Reference: deep inhale breath-hold (80% tidal breath) thorax CT
- Homologous: exhale
- $512 \times 512 \times 148$ with voxel size $0.2 \times 0.2 \times 0.5 \text{cm}^3$.
- pre-crop the reference image to size $259 \times 175 \times 107$.

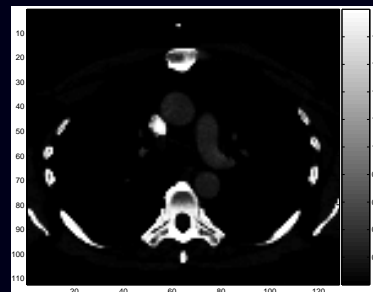
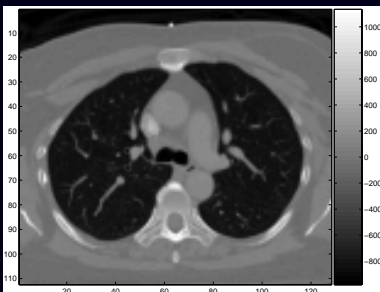
Slice Views of Data and Derived Weighting Map



Coronal View



Sagittal View



Axial View

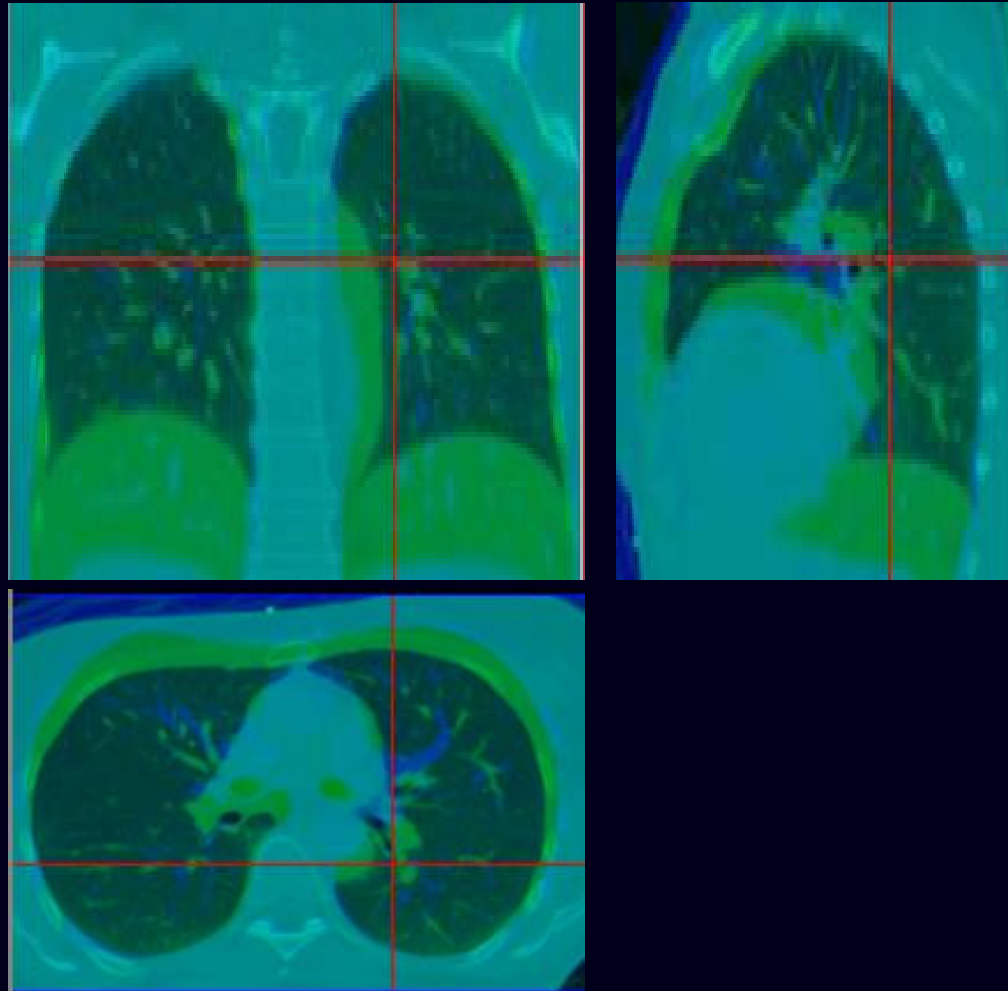
Reference X

Homologous Y

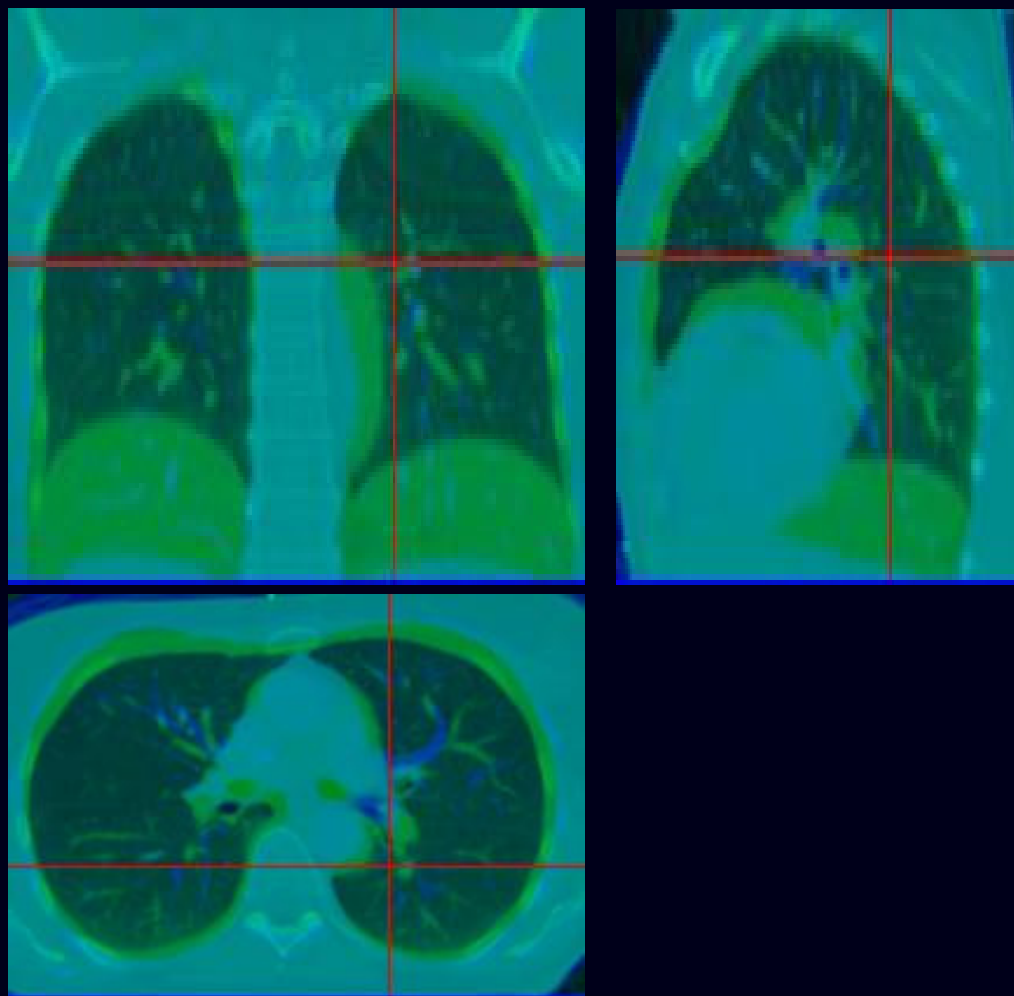
Stiffness Map γ

Registration Results

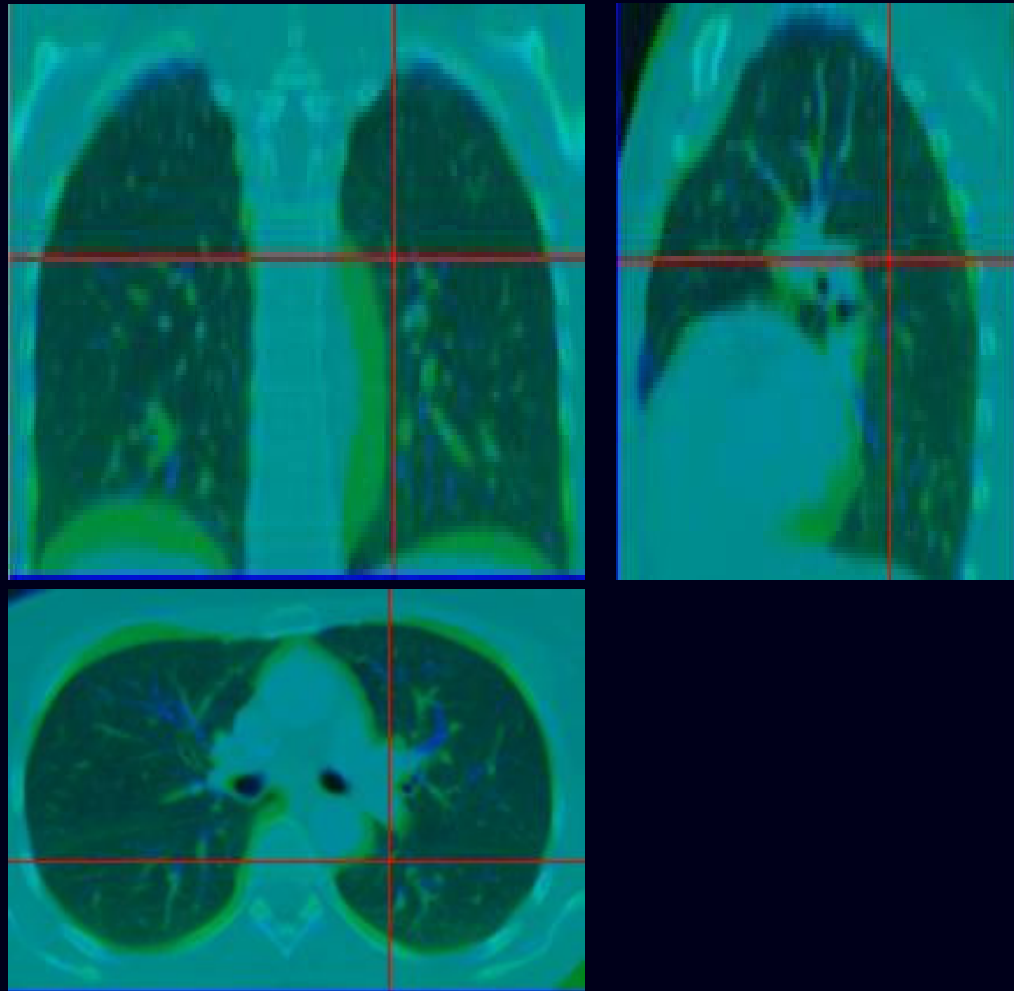
Color scheme: reference image (blue) v.s. deformed homologous (green).



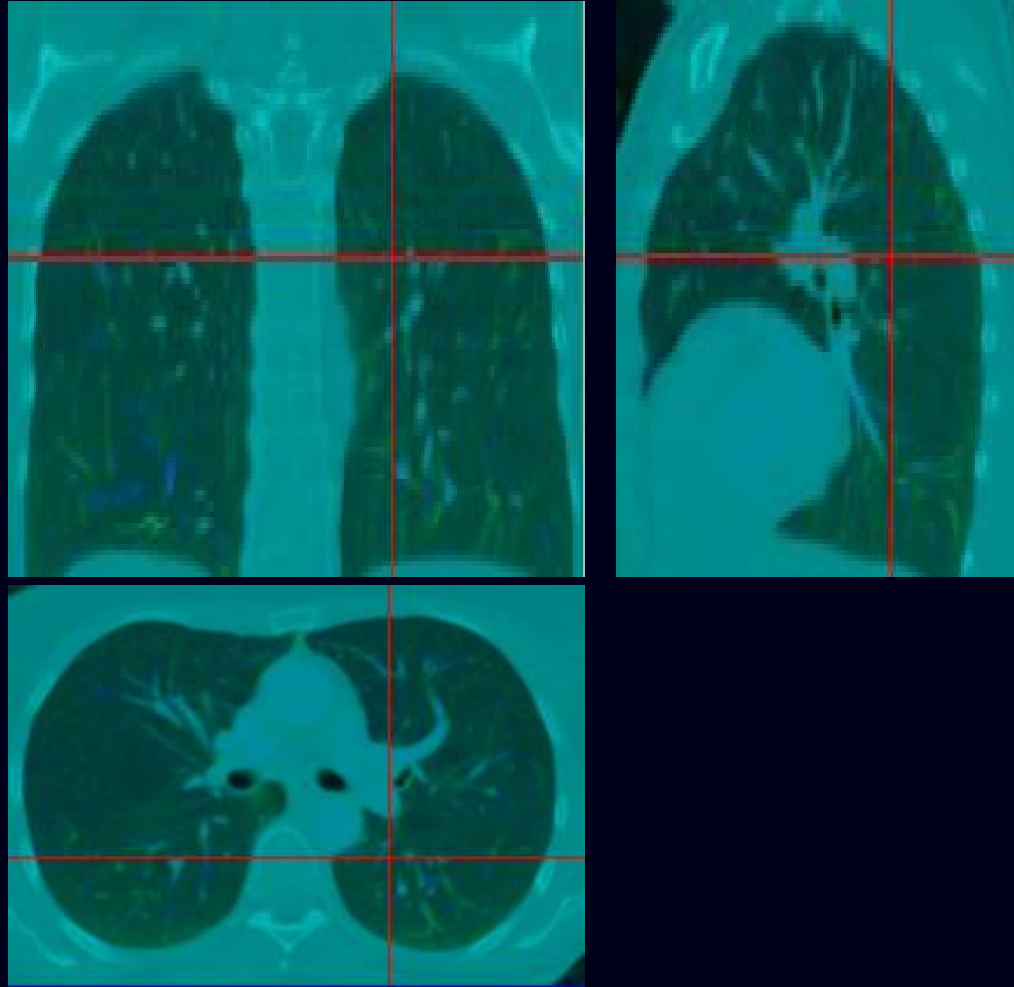
Rigid Transform



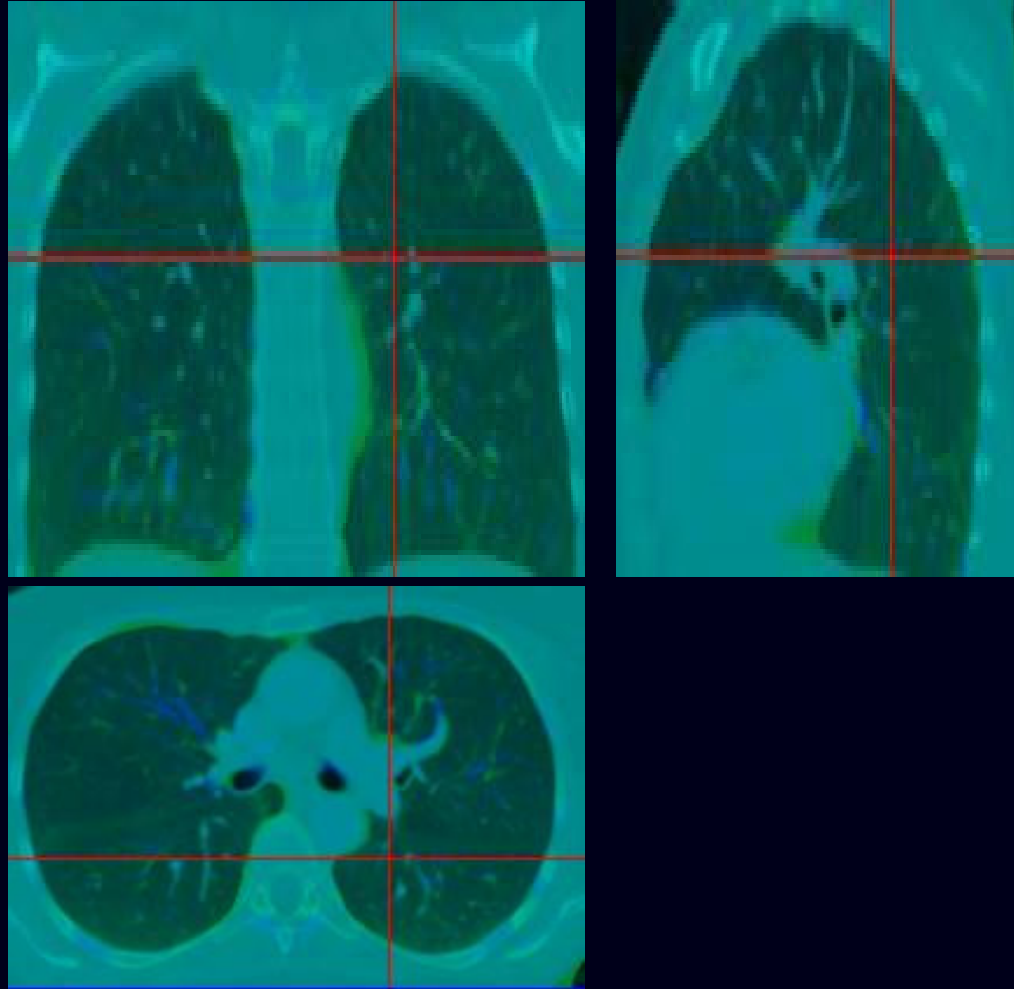
Affine Transform



B-Spline Parametrized Transform - Conventional

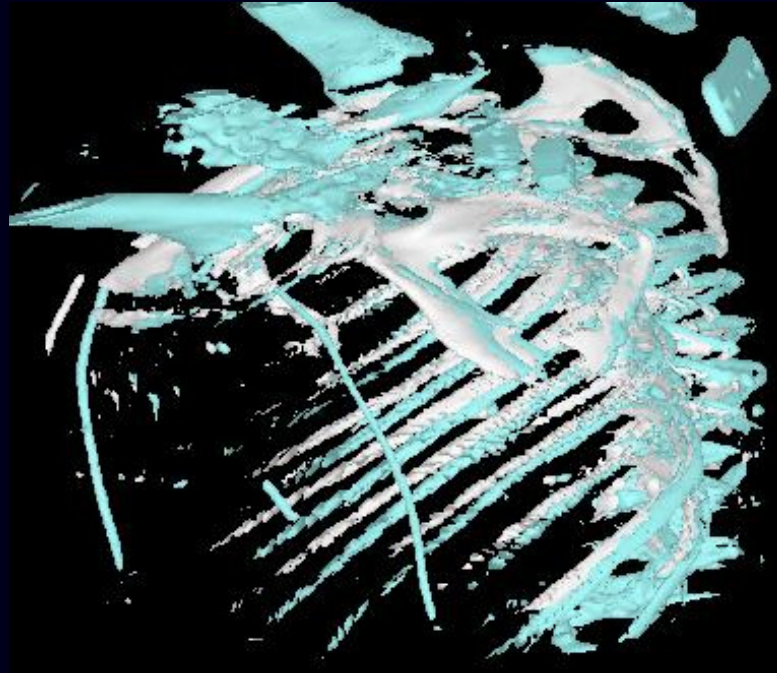


B-Spline Parametrized Transform - Proposed



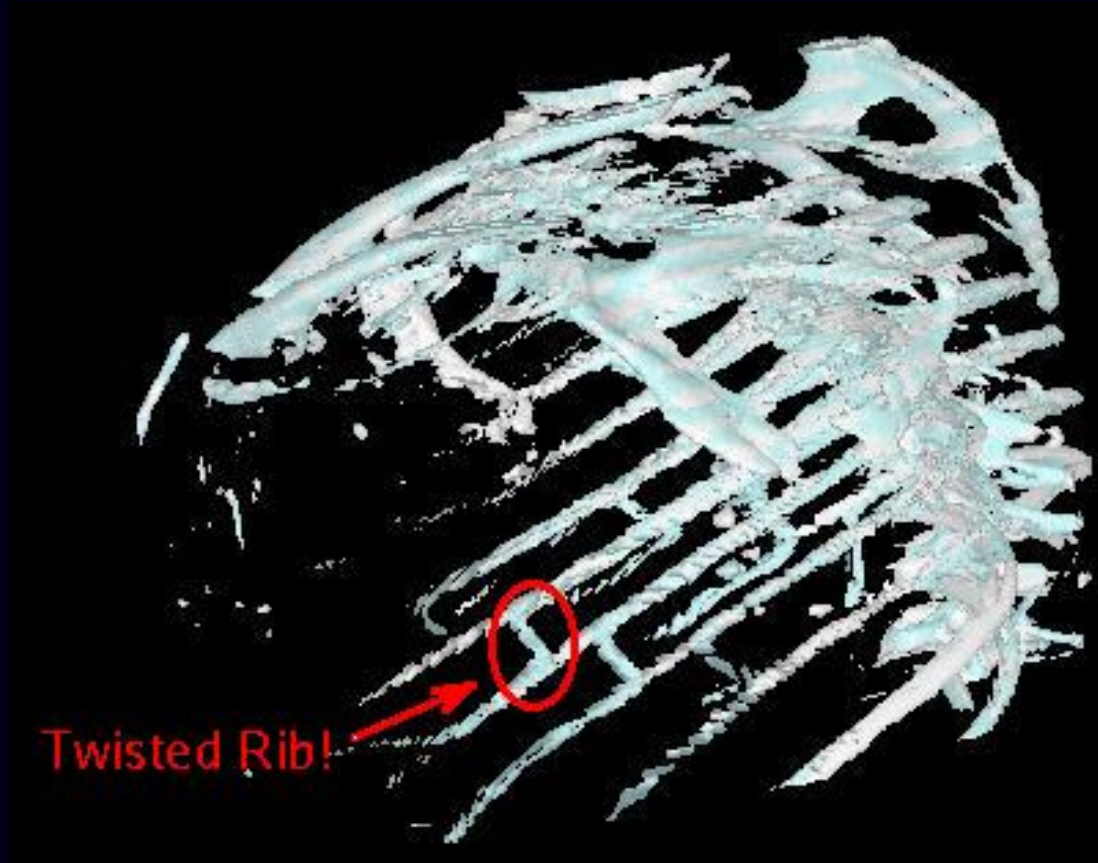
Comparison of Extract Geometry

- Extract geometry by thresholding CT number to reveal bone structure.
- Reference (white) v.s. deformed homologous geometry (light blue).



Without registration \Rightarrow Global misalignment

Geometry from Conventional B-Spline Registration



Ignoring Tissue Difference \Rightarrow Twisted Ribs! (local Minima in Intensity Match)

Geometry from Proposed B-Spline Registration



⇒ General good alignment; more physiological deformation

Summary

- Conclusions
 - Tissue-type dependent rigidity regularization design.
 - Additive regularization acts as a *soft* correcting force in rigid structure and relaxes in elastic regions.
 - No explicit segmentation, robust to partial volume effects.
 - Frobenius norm based design is computationally friendly.
 - Experiment with clinical data shows physiologically promising results.
- Challenges and Future Work
 - Quantitative validation/evaluation methods other than visual inspection.
 - Extension to incorporate direction dependent (anisotropic) properties.
 - Generalization to other modalities.