Eng. 100: Music Signal Processing

DSP lecture

Exam review

Curiosity:

- https://www.youtube.com/watch?v=bAdqazixuRY (robots vs music - Nigel Stanford)

Announcements:

- PDR in discussion/labs this week
- Think about how you will test / validate
  (measurable for PDR, process for CDR)
- REMINDER: THURSDAY DISCUSSION in LAB ROOM!

Midterm course feedback

- lecture notes
  strategy: download in advance, annotate in class
- iClicker angst
  $\approx$ 74 questions over semester
  syllabus points: 37 clicker / 1150 total = $\approx$ 3%
  each question: 3/74 $\approx$ 0.04% of total
  2/3 for trying: 2% vs 3%
  cf midterm 100pts
  Canvas category issue

Midterm exam

- Wed. Mar. 13, 9:00AM sharp to 10:20AM
  - Rooms will be announced on Canvas soon
  - Past exam(s) on HW solution server (Canvas has URL)
    Cover page with formulas!
  - Policies: no paper, no electronics: just pencils and eraser

    We will not answer questions during the exam. If you think a question is ambiguous, write down what you think we meant to ask, and answer that version. *(equity)*

    You are welcome to report possible errors during the exam.
- DSP topics: Labs 1-3, HW 1-4, Projects 1-2, autocorrelation

Not on exam

- "Missing frequencies" in scatter plots
- Calculus, complex numbers, Newtonian physics
- Harmonic product spectrum (HPS)
- Phase vocoder, FM synthesis, additive synthesis
- Gtk.jl code *writing* (but basic Gtk code *understanding* is in scope)

Resources for review

- Lecture slides / recordings
- Homework / solutions (see Canvas announcement)
  Strategy: revisit HW, clicker questions, RQs
  using *only* the formulas on front page of exam.
- Practice exam(s) (on HW solution server)
- Labs / project descriptions (your textbook)

  For each `Julia` line that you "cut and paste" (or wrote) can you:
  - describe what the purpose of that command was?
  - determine the size of the output array?
  - determine number of points that are plotted?
  - determine what would be appropriate labels for the plot?
  - determine by hand what value would be computed?
  - determine the duration or pitch heard with `sound`?

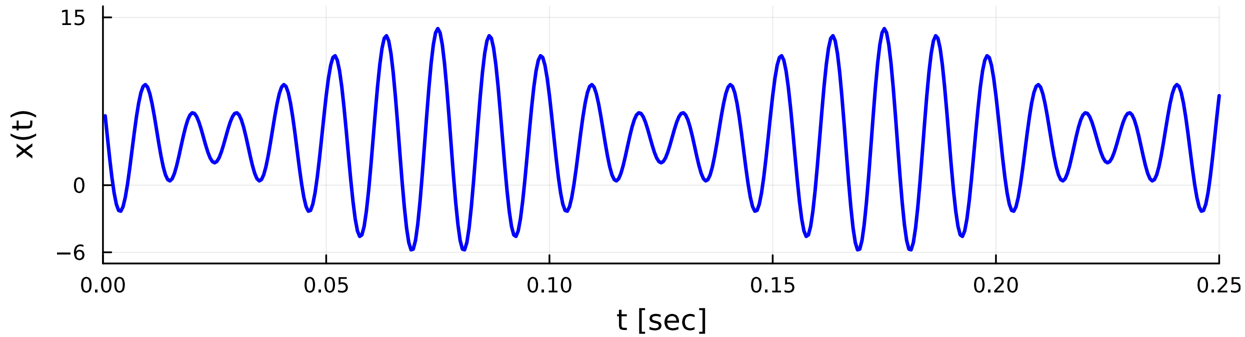
Review thoughts

- chronological vs integrative
- procedural vs conceptual

Signal representations review

10 different ways to represent signals! (Note axes and units for plots.)

- A plot of an analog signal as a function of time: $x(t)$ versus t, such as



Q0.1 What are some questions you could answer from this plot?

??

- A "complicated" formula, such as

$$x(t) = 8\cos^2(2\pi 40t) - 6\sin(2\pi 90t).$$

- A simple formula as a sum of sinusoidal signals (cosines), with positive amplitudes (using trigonometric identities):

$$x(t) = 4 + 4\cos(2\pi 80t) + 6\cos(2\pi 90t + \pi/2).$$

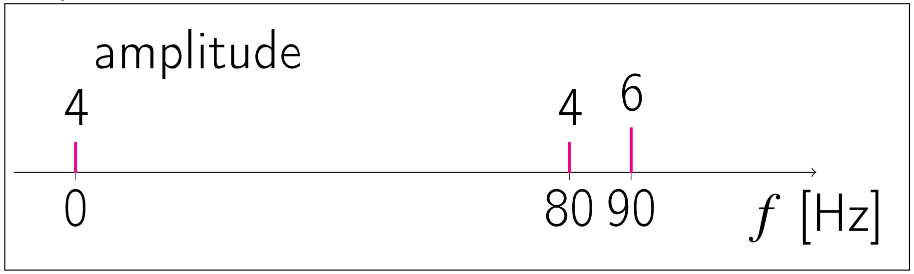
- A list enumerating the frequency (*e.g.*, in Hz), (positive!) amplitude, and phase (f_k, c_k, θ_k) of each component, such as

$$(0, 4, \text{N/A});\ (80, 4, 0);\ (90, 6, \pi/2).$$

The "N/A" means "not applicable" because the DC component has no phase.

MIDI numbers are an alternative way to represent frequencies (for music using equal temperment pitches).

- A line spectrum that graphs the frequencies and amplitudes of each sinusoidal component, such as:



(Technically this is called a magnitude spectrum because we ignore the phase in the figure.)

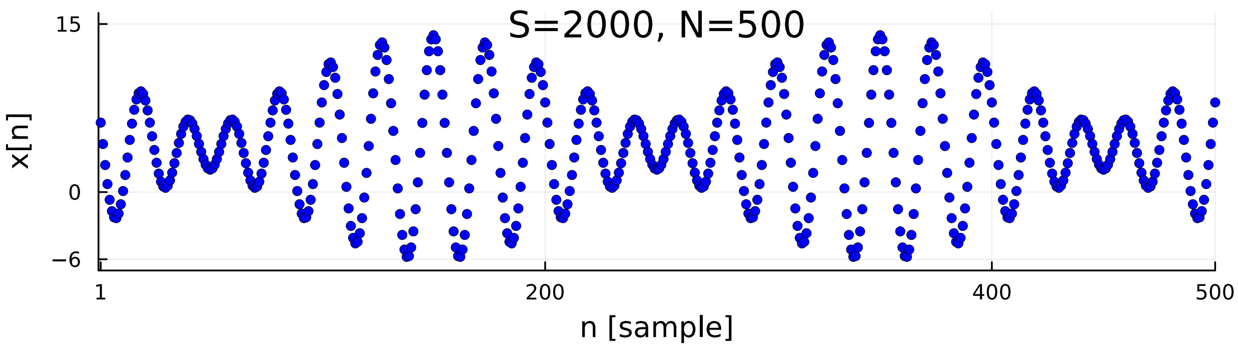
$$x(t) = 4 + 4\cos(2\pi 80 t) + 6\cos(2\pi 90 t + \pi/2).$$

- Julia code, such as: (transition to digital...)

```
S = 2000; N = Int(0.25*S); t = (1:N)/S # "analog" perspective
x = 4 .+ 4 * cos.(2π*80*t) + 6 * cos.(2π*90*t .+ π/2) # plot x(t) vs t
```

```
S = 2000; N = Int(0.25*S); n = 1:N # "digital" perspective
x = 4 .+ 4 * cos.(2π*80*n/S) + 6 * cos.(2π*90*n/S .+ π/2) # plot x[n] vs n
```

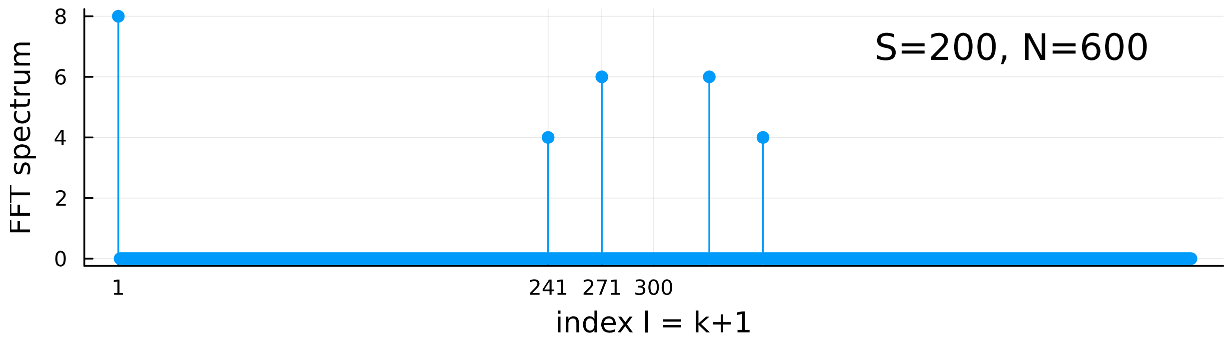
- A plot of samples $x[n] = x(n\Delta) = x(n/S)$, a digital signal:



- Corresponding formula for sampled signal:

$$x[n] = x(n/S) = 4 + 4\cos(2\pi 80 n/S) + 6\cos(2\pi 90 n/S + \pi/2).$$

- The FFT output, *i.e.*, the output of the Julia code `2/N*abs.(fft(x))`



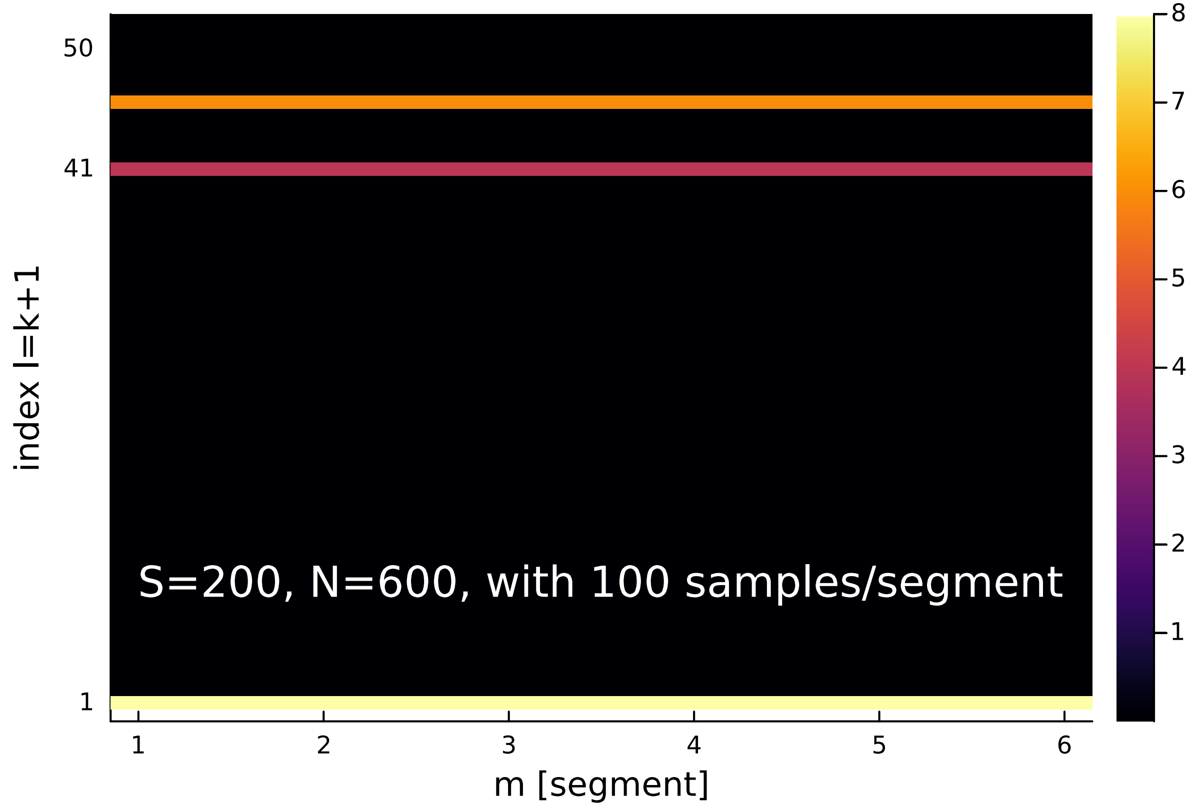
Note that this Julia plot differs somewhat from the line spectrum that we draw by hand.

- The first array value is $2c_0$, so it is "twice as big as it should be."
- The horizontal axis is not frequency f in Hz. (We could fix that by using another argument to `plot`.)
- Each of the nonzero lines appears twice (mirror image). How could we fix that? ??

Review how to convert between each of these representations!

$$x(t) = 4 + 4\cos(2\pi 80t) + 6\cos(2\pi 90t + \pi/2).$$

- The spectrogram of the signal, formed by FFT of signal segments.



Why just horizontal lines?
Note the yellow line across the bottom!

Q0.2 What is the index `l` of the "orange" frequency component?

??

Review problems

Q0.3 What is the fundamental frequency of the signal shown previously?

$x(t) = 4 + 4\cos(2\pi 80t) + 6\cos(2\pi 90t + \pi/2)$

??

Q0.4 What is the bandwidth of that signal?

??

Q0.5 Sketch the spectrum of $x(t) = 5\cos(2\pi 50t) + 12\sin(2\pi 50t) + 6\sin^2(2\pi 50t)$.

??

Q0.6 An array `x` contains 10 samples of a signal $x(t)$ sampled at 200 $\frac{\text{Sample}}{\text{Second}}$. Determine a formula for $x(t)$ when the output of `2/length(x) * abs.(fft(x))` is `[6 0 0 0 7 0 7 0 0 0]`.

??

Q0.7 A signal $x(t)$ has the following spectrum.

Determine a formula for a signal $x(t)$ that has that spectrum.

??

Q0.8 Give a formula for a *different* signal $y(t)$ that has the same spectrum.
Hint: we are showing magnitude spectra.

??

Q0.9 The signal $x(t)$ is sampled with $S = 1000 \, \frac{\text{Sample}}{\text{Second}}$ for 4 seconds. The samples are stored in a vector `x`. Suppose you do `plot(2/N*abs.(fft(x)))`.
Sketch (by hand) what the stem plot will look like. Hint: first determine N.

??

Q0.10 How fast would the signal $x(t) = 8000 \cos^4(2\pi 50 t)$ need to be sampled to avoid aliasing?

??

Determining the frequency of a sinusoid

- numerous applications, not just music
- Analog methods
  - From continuous-time formula: by inspection $x(t) = 200\cos(2\pi 100t + 30)$
  - From analog signal plot: find (fundamental) period T, then use $f = 1/T$
  - From sound: "tuning" like musicians do using beat frequencies
  - From line spectrum: by inspection
  - From a text description:
    "the 3rd harmonic of a periodic signal having period of 10 msec"

    ??
- Digital methods "by hand"
  - From formula: $x[n] = 7\cos\left(\frac{\pi}{8}n\right)$ where $S = 8000$ Hz

    ??
  - From digital signal plot:
    find (fundamental) period m (samples), then use $f = m\Delta = S/m$
  - From `Julia` code: `S = 100; x = cos.(2pi*(1:50)*40/S)`
  - ...

Digital methods

- arccos method

Q0.11 Non-redundant conditions for arccos to work correctly?

??

- FFT: peak in spectrum (at "$k+1$") then $f = \frac{k}{N} S$
- correlation:
  - correlate with (numerous?) candidate sinusoids
  - use both $\cos$ and $\sin$ and combine `c.^2 + s.^2`
  - choose frequency associated with maximum correlation via `argmax`
- autocorrelation:
  - correlate signal with shifted versions of itself.
  - ignore peak at 0; find 2nd peak (at "$m+1$") then $f = S/m$
- Harmonic product spectrum?
- Spectrogram (many FFTs, again using $f = \frac{k}{N} S$)

Gtk

```
using Gtk
using Sound: sound

S = 8000

function playit(f)
    sound(cos.(2π*(1:2S)*f/S), S)
    return nothing
end

g = GtkGrid()
b = GtkButton("go")
signal_connect((w) -> playit(S/8), b, "clicked") # callback
g[1,1] = b # put the button in the grid

win = GtkWindow("gtk3", 300, 400) # window for all the buttons
push!(win, g) # put button grid into the window
showall(win); # display the window full of buttons
```

What are some questions that could be asked here?

Q0.12

??

Sampling

- B (for "bandwidth") denotes highest frequency of a signal

Q0.13 What is the largest value B can have?

??

Q0.14 Example of a signal that "achieves" that upper limit?

??

- To avoid aliasing, use $S > 2B$
- All of the spectrum-related methods require $S > 2B$ to work.

Q0.15 Suppose the signal $x(t) = 7\cos(2\pi 250 t)$ is sampled with $S = 400 \frac{\text{Sample}}{\text{Second}}$ for $N = 800$ samples.
Sketch the figure produced by the `Julia` code

```
plot(2/N*abs.(fft(x)), line=:stem)
```

??

??

Q0.16 What frequency would be found by the FFT method?

??

This example involves sampling, aliasing, spectra, and FFT.

Aliasing explanation for the example

$x(t) = 7\cos(2\pi 250 t)$ when sampled at $S = 400\text{Hz}$ yields

$$x[n] = x(t)\Big|_{t=n/S} = 7\cos\left(2\pi 250 \frac{n}{400}\right) = 7\cos\left(2\pi \frac{250}{400} n\right), \quad n \in \mathbb{Z}$$

$y(t) = 7\cos(2\pi 150 t)$ when sampled at $S = 400\text{Hz}$ yields

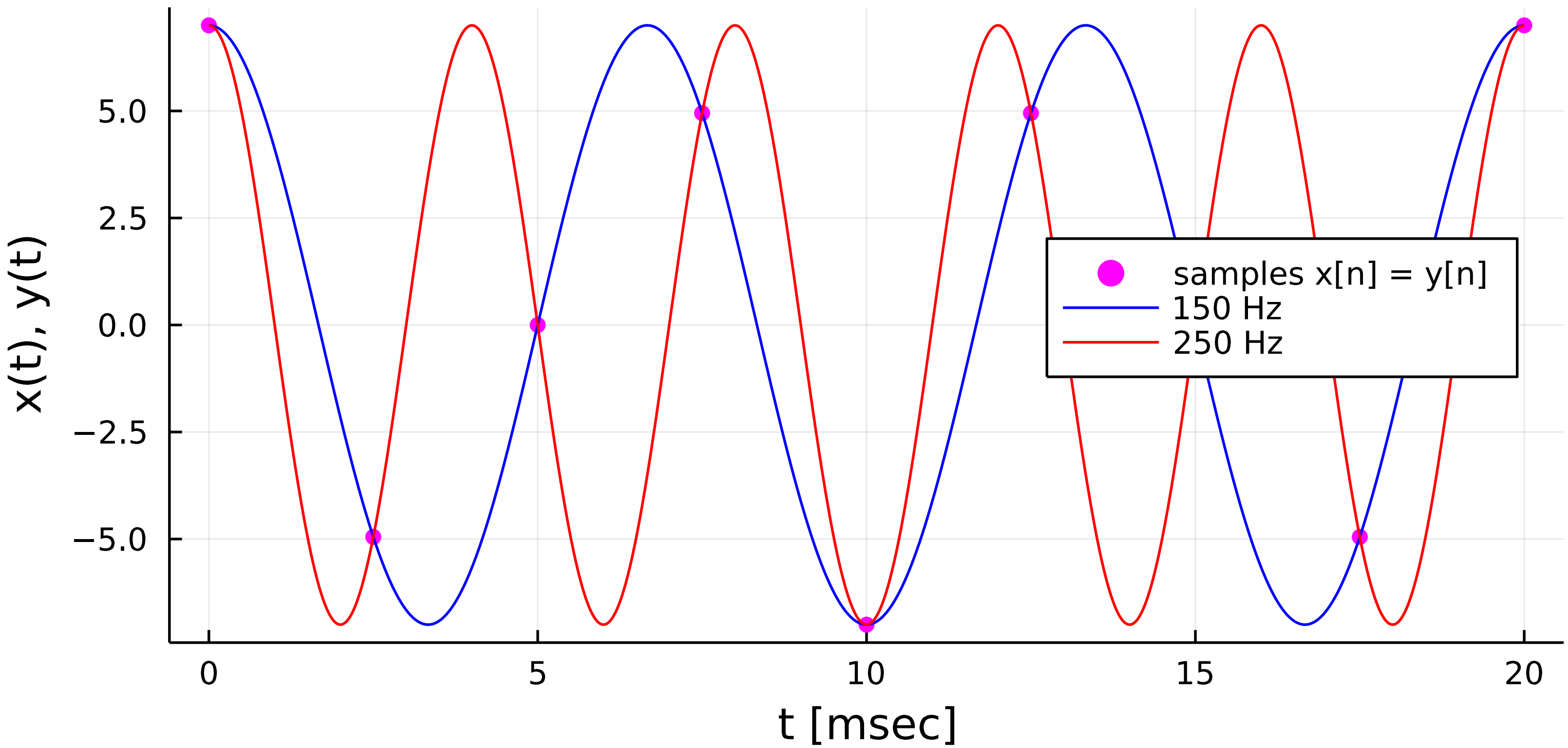
$$y[n] = y(t)\Big|_{t=n/S} = 7\cos\left(2\pi 150 \frac{n}{400}\right) = 7\cos\left(2\pi \frac{150}{400} n\right), \quad n \in \mathbb{Z}$$

But for all $n \in \mathbb{Z}$ (integers):

$$\cos\left(2\pi \frac{250}{400} n\right) = \cos\left(-2\pi \frac{250}{400} n\right) = \cos\left(-2\pi \frac{250}{400} n + 2\pi n\right) = \cos\left(2\pi \frac{150}{400} n\right)$$

Where did we use the fact that n is an integer? ??

So even though $x(t)$ and $y(t)$ are completely different signals, after sampling $x[n]$ and $y[n]$ are the same. This is aliasing.



5.0
2.5
0.0
−2.5
−5.0
x(t), y(t)
0
5
10
15
20
t [msec]
samples x[n] = y[n]
150 Hz
250 Hz

Sampling rates and maximum frequency

Q0.17 What is the highest frequency we could find by arccos method? (HW1 “challenge” problem.)

??

Q0.18 What is the highest frequency we can find by the FFT method?

`plot((2/N)*abs.(fft(x)))` gives:

$$[2c_0 \; \underbrace{c_1 \; c_2 \; \ldots \; c_{N/2-2} \; c_{N/2-1}} \; c_{N/2} \; \underbrace{c_{N/2-1} \; c_{N/2-2} \; \ldots \; c_2 \; c_1}]$$

??

Q0.19 What is the maximum frequency we can find "by eye" from a digital signal $x[n]$, assuming no aliasing has occurred?

??

Q0.20 What is the maximum frequency we can find "by eye" from an analog periodic signal $x(t) = x(t+T)$?

??

Why $S > 2B$ is crucial to avoid aliasing

- Consider $x(t) = \cos(2\pi f t)$ with $f = S/2$
  Plot its samples $x[n]$
  ??

- Consider $y(t) = \sin(2\pi f t) = \cos(2\pi f t - \pi/2)$ with $f = S/2$
  Plot its samples $y[n]$
  ??

Q0.21 Would $S \geq 2B$ suffice to avoid aliasing?

??

- For FFT approach, the highest *reliable* frequency is really for $k = \frac{N}{2} - 1$,
  i.e., $f = \frac{(N/2-1)}{N} S = \left(\frac{1}{2} - \frac{1}{N}\right) S < S/2$

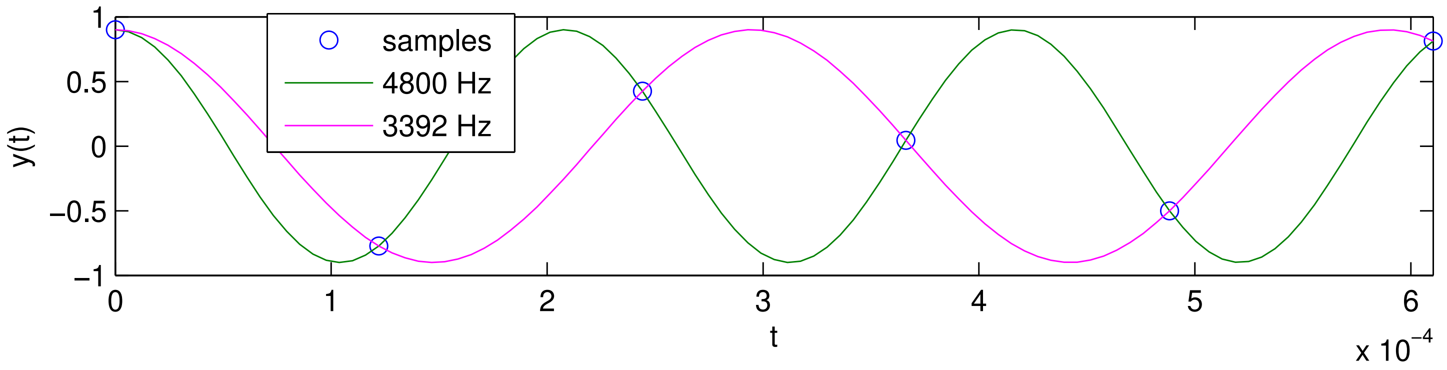
Aliasing: audio example

```
S = 8192; t = 0:1/S:0.3
x = 0.9*[cos.(2pi*2800*t); cos.(2pi*3800*t)]
```

play

```
y = 0.9*[cos.(2pi*3800*t); cos.(2pi*4800*t)]
```

play



arccos method says 3392 Hz, not 4800 Hz for last part of this example

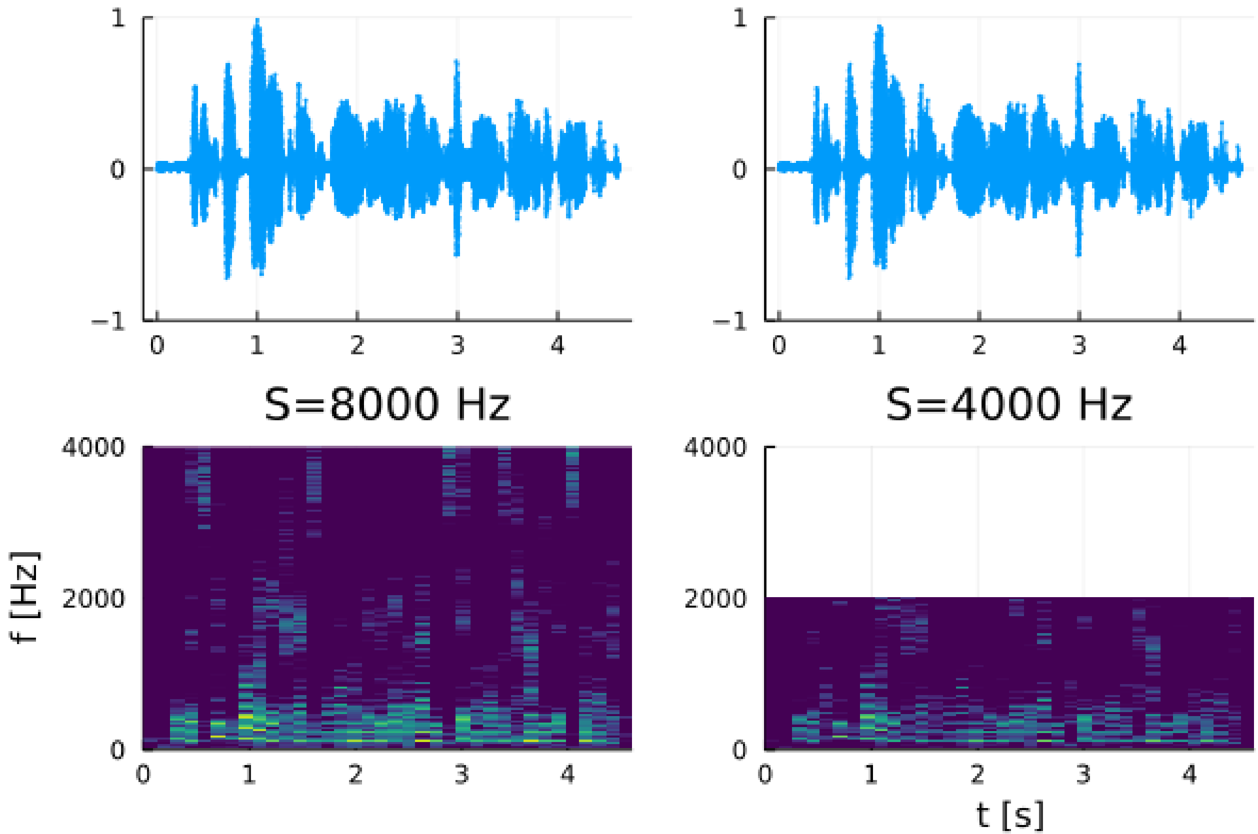
Q0.22 Is $S > 2B$ here?

A: Yes B: No ??

Aliasing: speech example

play Original $x[n]$ with $S_1 = 8000 \frac{\text{Sample}}{\text{Second}}$

play Down-sampled $y[n] = x[2n]$ via `y = x[1:2:end]` with $S_2 = S_1/2 = 4000 \frac{\text{Sample}}{\text{Second}}$



Is $S_2 > 2B$ here? ??

Summary of two important models

- Simple exponential model: $\boxed{y = b\,a^x}$

  Use semi-log plot:

$$\underbrace{\log(y)}_{\tilde{y}} = \underbrace{\log(a)}_{\text{slope}}\,x + \underbrace{\log(b)}_{\text{intercept}}$$

- Simple power model: $\boxed{y = b\,x^p}$

  Use log-log plot:

$$\underbrace{\log(y)}_{\tilde{y}} = \underbrace{p}_{\text{slope}}\,\underbrace{\log(x)}_{\tilde{x}} + \underbrace{\log(b)}_{\text{intercept}}$$

Even though Lab 2 uses these models for musical notes, they are ubiquitous in science and engineering and more.

Other important topics

- Spectrogram: think carefully about "N" in $f = \frac{k}{N} S$
- Removing noise / interference
- ...