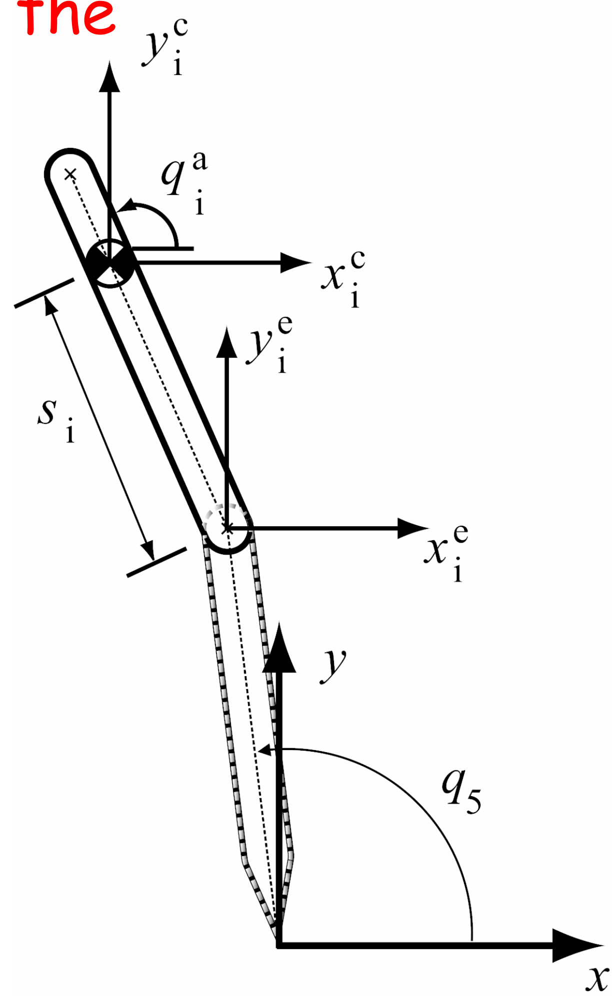
Computing the Lagrangian

What if the data is given in terms of the inertia of the link about the joint?



Kinetic Energy of i-th Link

$$
\begin{aligned}
K_i &= \frac{1}{2} m_i \left((\dot{x}_i^e)^2 + (\dot{y}_i^e)^2 \right) + \\
&\quad m_i \begin{bmatrix} \dot{x}_i^e \\ \dot{y}_i^e \end{bmatrix}^T \begin{bmatrix} \cos(q_i^a) & \sin(q_i^a) \\ -\sin(q_i^a) & \cos(q_i^a) \end{bmatrix} \begin{bmatrix} s_i \\ 0 \end{bmatrix} \dot{q}_i^a \\
&\quad + \frac{1}{2} J_i (\dot{q}_i^a)^2
\end{aligned}
$$

$J_i =$ inertia about joint axis

$$K_i = \tfrac{1}{2} \dot{q}^T D_i(q) \dot{q}$$

Computing the Lagrangian

What if the center of mass is off-axis?

Kinetic Energy of i-th Link

$$
\begin{aligned}
K_i \;=\;& \frac{1}{2} m_i \left((\dot{x}_i^e)^2 + (\dot{y}_i^e)^2 \right) + \\
& m_i \begin{bmatrix} \dot{x}_i^e \\ \dot{y}_i^e \end{bmatrix}^T \begin{bmatrix} \cos(q_i^a) & \sin(q_i^a) \\ -\sin(q_i^a) & \cos(q_i^a) \end{bmatrix} \begin{bmatrix} s_i \\ t_i \end{bmatrix} \dot{q}_i^a \\
& + \frac{1}{2} J_i (\dot{q}_i^a)^2
\end{aligned}
$$

$J_i =$ inertia about joint axis

$$K_i = \frac{1}{2} \dot{q}^T D_i(q) \dot{q}$$