A unique resonance mode observed in a prism-coupled micro-tube resonator sensor with superior index sensitivity

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Abstract: Silica micro-tube resonators were studied by using a prism-coupled method and act as refractive index sensing elements. Two types of refractive index sensing modes were observed experimentally by tuning the incident angles in the prism-coupled configuration. The resonance mode with sensitivity below 100nm/RIU is a result of the typical evanescent wave interaction with the fluid, while the mode with record high sensitivity of ~ 600nm/RIU is a form of non-evanescent wave with high optical field in the low index fluid region. An analysis of the field distribution of the resonance modes in the micro-tube also revealed the existence of high order modes with strong optical field inside the low index liquid core, which leads to the high index sensitivity. Theoretical calculation of the sensitivity for this specific mode obtained by the Mie scattering method is consistent with the experimental result. A ray optics picture is presented to elucidate the physical nature of this special resonance mode observed in the micro-tube resonator.

OCIS codes: (140.4780) Optical resonators; (230.5750) Resonators; (170.4520) Optical confinement and manipulation; (130.6010) Sensors;

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1. Introduction

Optical sensors using evanescent waves to interrogate the presence of analytes on the sensor surface or in the surrounding environment typically rely on detecting effective refractive index change. In order to detect very low concentration or minute amount of analytes using optical sensors, long waveguide lengths (exceeding cm) are typically required in order to accumulate a detectable phase shift [1]. This would also require significant amount of samples that may not be readily obtainable in many sensing applications. To address this problem, sensors based on optical micro-cavities can be used. Such sensors offer a unique advantage by reducing the size of the device by orders of magnitude, without sacrificing the interaction length by virtue of the high quality (Q) factor resonances, thereby significantly reducing the amount of sample needed for the detection. The resonance effect provides an effective long interaction length for the sensor to achieve sufficient sensitivity. Also typical bio-sensing experiment requires that the devices can handle aqueous analytes. Therefore fluidic handling capability is an indispensable part of the sensor platform. High quality-factor microsphere cavities using Whispering gallery mode (WGM) resonances have been demonstrated to respond to a monolayer of protein adsorption [2], however integration with fluidic system is very challenging and typically requires fluidic chambers much larger than the active device element. Though micro-ring [3,4] and micro-disk [5] sensors can be mass fabricated using batch processing techniques, they suffer from limited Q factors due to the surface roughness induced scattering loss.

A silica micro-tube based resonator sensor is very attractive for biochemical sensing applications due to its ability to handle aqueous analytes and its high Q factor [6]. The high-Q (~10^6 as demonstrated in Ref. 6) is a result of the low scattering loss due to very smooth surfaces. However there are also limitations to the micro-tube resonators. Since only the inner surface can be used as the sensor interface, so the total evanescence optical field interacting with the solutions is less than half that of micro-ring [7] resonators if the two devices have similar field distribution for the guided and evanescent waves, thereby resulting in a lower sensitivity. Moreover, the previously demonstrated micro-tube resonator sensors required a tube thickness of less than 5 microns [8] in order to obtain a refractive index sensitivity of 2.6nm/RIU. Significantly higher index sensitivity can be obtained by further reducing the wall thickness to below sub-micron [9]. But such strategies cause the micro-tube to be very brittle and difficult to handle in practical sensing applications.

In this letter we propose a prism coupled micro-tube platform, which not only allows probing the resonance modes in a micro-tube with much thicker tube walls for easy sample handling, but more importantly it allows the incident angle be adjusted to excite different high order modes in the micro-tube resonator and thereby realize different sensitivities. The index sensing experiment was demonstrated by using a prism coupled silica micro-tube with a wall thickness of 32 µm. Two different sensitivities (expressed as resonance shift per Refractive Index Unit change) were observed by varying the incident angles, one around 100 nm/RIU and the other around 600 nm/RIU. To the best of our knowledge, this index sensitivity of 600nm/RIU is the highest reported in the micro-cavity based sensors. We attribute this high sensitivity to a new type resonance mode, which has the highest optical field present in the low index fluid region, which maximizes the interaction of light with the analyte solution flowing though the micro-tube.

2. Experimental Setup and Results

Figure 1 shows the schematic of the experimental setup. Light incident onto a SF11-glass prism was evanescently coupled into a fused silica micro-tube through a controlled air gap.

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The fused-silica micro-tube was purchased from Polymicro Inc. and has an inner radius of $R_1 = 134$ µm and an outer radius of $R_2 = 166$ µm. In our experiment, the silica micro-tube was first connected to two pieces of plastic tube at each end. The analyte solution flowing through the silica tube was controlled by a peristaltic pump. Next, the silica tube was positioned perpendicular to the incident light plane and mounted at the center of the prism. The coupling gap between the micro-tube and the prism was controlled by a deposited Al film with 400-500 nm thickness on one side of the silica micro-tube. The silica micro-tube was then bonded to the prism and mounted on a rotation stage. The output from a wavelength tunable diode laser around 1550 nm wavelength (Santec TSL-220) was collimated by a GRIN lens collimator and incident onto the prism with a spot size of ~ 500 µm. The incident angle was tuned by rotating the rotation stage so as to satisfy the phase matching condition to the different order resonance modes in the micro-tube resonator. The light reflected from the prism, which was coupled out of the resonator, passed through an aperture and then a polarizer, and finally was focused onto a photo-detector. The polarizer was used to select the TE and TM polarization in the output from the resonator. In our experiment we focused on the measurement of the TE polarization, because the TE mode was observed to posses higher Q factors than did the TM modes.

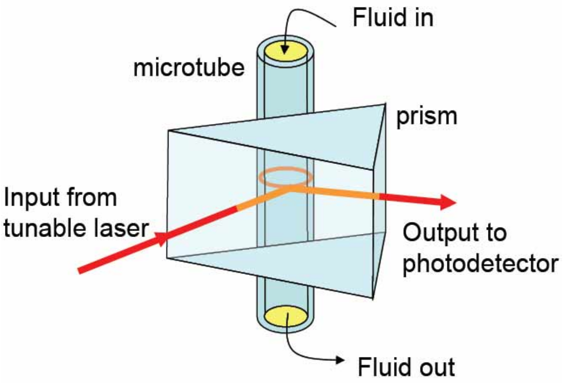


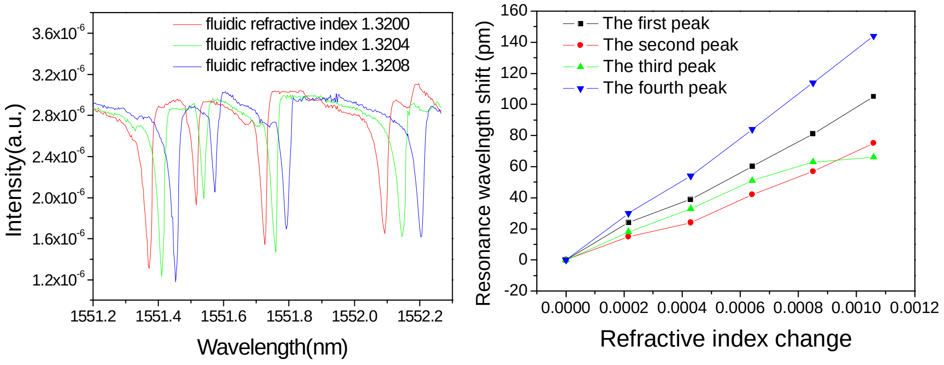
Fig.1 The schematic of the experimental setup

The index sensing experiment was performed by changing the refractive index of the liquid that flows through the micro-tube, and simultaneously monitoring the corresponding resonance wavelength shifts. The refractive indices of the liquid were controlled by premixing a very small amount of isopropyl alcohol (IPA) with deionized (DI) water. The refractive index of the solution can be estimated based on the dielectric constants of both liquids and the molar ratio of each component [10]:

$$n = \sqrt{\kappa \cdot n_{IPA}^2 + (1-\kappa) \cdot n_{water}^2} \tag{1}$$

where κ is the molar ratio of IPA in the solution. The refractive indices of water and IPA are taken to be 1.320 and 1.378 around the 1550 nm wavelength range

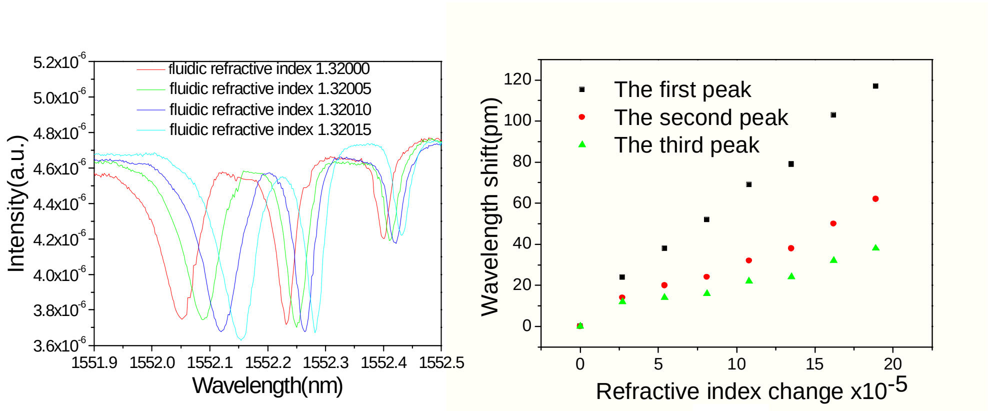
The beam illuminated on the prism is a Gaussian beam that covers a range of spatial wave vector components. Therefore, the light can be coupled into several resonance modes within a certain wavelength range that satisfies the phase matching condition. Fig. 2(a) shows three resonance spectra measured from the reflected beam for liquids with three different refractive indices filled in the micro-tube. The refractive index difference between each spectrum was ~ 4×10^{-4}, and the incident angle to the prism was ~ 37.5^o. Four resonant modes are shown in each spectrum. These four resonant modes shift differently for the same fluidic index changes of 4×10^{-4}: the resonance wavelength shift is ~ 39pm for the first peak, ~ 28pm for the second peak, ~ 30pm for the third and ~ 54pm for the fourth. Fig. 2(b) shows the resonant wavelength shifts as a function of the refractive index change. By linearly fitting the curve in Fig. 2(b), the device sensitivity, defined as the resonance shift per unit refractive index unit (RIU) for the four resonance modes are ca. 96.7 nm/RIU, 70 nm/RIU, 74.7 nm/RIU and 135 nm/RIU, respectively. The sensitivity of these resonance peaks is 40 times higher than the sensitivity reported by White et al. [8], which was about 2.6 nm/RIU.



(a) (b)

Fig. 2. (a) Resonance curve shift due to the change of liquid retractive index in the microtube. (b) Resonance wavelength as a function of the change in liquid refractive index in the micro-tube at an incident angle of 37.5°

Further decreasing the incident angle to ~ 35 ° allows the excitation of much higher order resonance modes in the micro-tube resonator. Figure 3(a) shows four resonance spectra measured for different liquid indices at an incident angle of ~ 35°. Again three resonance modes show different shifts. For the same index change of 5×10^{-5}, the resonance shifts are 30pm for the first mode, 18pm for the second and 10pm for the third. Figure 3(b) shows the resonant wavelength shift as a function of the refractive index change. The sensitivity for the three resonance modes are 600 nm/RIU, 360 nm/RIU, and 180 nm/RIU, respectively. The sensitivity corresponding to the first and second mode is much higher than what can be achieved with the typical evanescence wave sensors.



(a) (b)

Fig. 3. (a) Resonance curve shift due to the change of liquid refractive index in the micro-tube, and (b) resonance wavelength shift as a function of the change in liquid refractive index at an incident angle of ~35 degree.

3. Theory of Micro-tube Resonators

We anticipate that the very high sensitivity to the refractive index change in the silica micro-tube is due to the large optical field penetrating into the fluidic core region. In order to

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understand the physics behind the observed high sensitivity, we investigated the optical field distribution in the micro-tube using cylindrical coordinates z, r and ϕ. The time independent field distribution for the resonance mode can be separated into a radial-dependent mode component and an azimuthal-dependent phase term, $\Psi_Z(r)\exp(im\phi)$, where Ψ_Z is the amplitude of the axial magnetic (TE) or electrical (TM) modal field, m is the azimuthal quantization number. For the TE mode in the micro-tube we only need to consider three components: H_z, E_r and E_ϕ. The radial dependent field H_z can be expressed by Bessel function in the following form [11]:

$$H_Z = \begin{cases} AJ_m(n_1 k_0 r) & 0 < r \le R_1 \\ [BJ_m(n_2 k_0 r) + CN_m(n_2 k_0 r)] & R_1 < r \le R_2 \\ DH_m^{(1)}(n_3 k_0 r) & R_2 \le r < \infty \end{cases} \tag{2}$$

where n_1, n_2 and n_3 are the refractive indices of the liquid core region, the silica micro-tube region and the outside air region, k_0 is the wave-vector in the vacuum; $J_m(nk_0 r)$, $N_m(nk_0 r)$ and $H_m^{(1)}(nk_0 r)$ are respectively the m^{th} order cylindrical Bessel function, Neumann function and Hankel function of the first kind. By matching the boundary conditions at the liquid/silica and silica/air interfaces, we obtain the eigen function equation, which can be used to calculate the resonance wavelength. The TE mode resonance with azimuthal number m is determined by:

$$\frac{n_1 J_m(k_0 n_1 R_1)}{n_2 J_m'(k_0 n_1 R_1)} = \frac{(B/C) J_m(k_0 n_2 R_1) + N_m(k_0 n_2 R_1)}{(B/C) J_m'(k_0 n_2 R_1) + N_m'(k_0 n_2 R_1)} \tag{3}$$

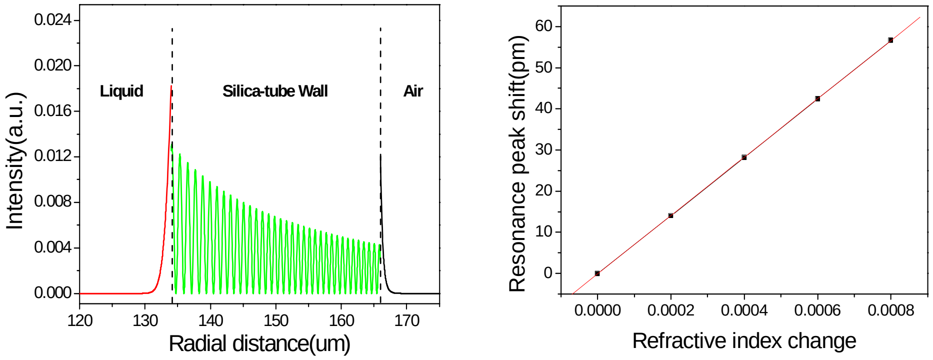
where
$$\frac{B}{C} = \frac{n_3 H_m^{(1)}(k_0 n_3 R_2) N_m'(k_0 n_2 R_2) - n_2 H_m^{(1)\prime}(k_0 n_3 R_2) N_m(k_0 n_2 R_2)}{n_2 H_m^{(1)\prime}(k_0 n_3 R_2) J_m(k_0 n_2 R_2) - n_3 H_m^{(1)}(k_0 n_3 R_2) J_m'(k_0 n_2 R_2)}$$

For a given m number, there are a series of k_0 satisfying equations (3), which are referred to as the ν^{th} ($\nu = 1, 2, 3, ...$) order resonance mode. The radial and tangential components of the electrical field in the TE mode can be expressed in terms of H_z as:

$$E_r = -\frac{m}{\omega \varepsilon_o \varepsilon_r r} H_z \tag{4}$$

and
$$E_\phi = \frac{1}{i\omega \varepsilon_o \varepsilon_r} \frac{\partial H_z}{\partial r} \tag{5}$$

The magnetic field distribution in the micro-tube resonator at the resonance wavelength can be obtained from Eq. (2) and the electrical field distribution can be obtained from equations (4) and (5). In the simulation we chose the following parameters for the micro-tube and its environment: inner radius $R_1 = 134$ µm, outer radius $R_2 = 166$ µm, and the refractive indices of the silica microtube $n_2 = 1.450$, the liquid (water) in the silica tube $n_1 = 1.320$, and the air outside the micro-tube $n_3 = 1.000$. The electrical field is responsible for the resonance wavelength shift due to the refractive index change in the liquid. Therefore, our study will mainly focus on the electrical field distribution in the micro-tube. In the micro-tube resonator the radial electrical field of TE mode is much larger than the angular electrical field, thus it will suffice to plot the radial electrical field (E_r) distribution in the silica micro-tube. Figure 4 shows the electrical field distribution of the TE_{714}^{35} mode in the silica micro-tube resonator with a resonance wavelength around 1.55 µm, where the subscript, 714, stands for azimuthal mode number and the superscript, 35, stands for radial mode number. As shown in Fig. 4(a), the optical field is predominantly guided in the high index region (silica tube wall) and decay exponentially in the low refractive index regions (liquid core and air regions). Such a mode is confined in the tube wall by the total internal reflections from the silica/air and silica/water interfaces. Thus, refractive index change transduced by the TE_{714}^{35} mode is via a typical evanescence wave sensing mechanism.



(a) (b)

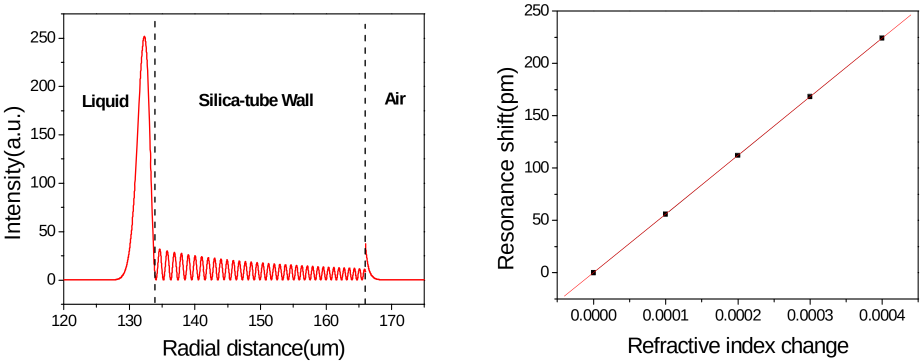
Fig. 4. (a) Radial electrical field intensity distribution for the resonance mode E_{r714}^{35}. (b) Resonance wavelength shift related to liquid's refractive index change for the resonance mode E_{r714}^{35}

We simulated the resonance wavelength shift related to the refractive index change of liquid by using the Mie theory. In Mie theory calculation [12], the optical field outside the tube wall is replaced by:

$$H_z = H_z^{inc.} + H_z^{scat.} \tag{6}$$

where $H_z^{inc.} = J_m(k_0 n_3 r)$ represents the incident wave, and $H_z^{scat.} = D_1 H_m^{(1)}(k_0 n_3 r)$ represents the scattered wave. The coefficient of the scattered wave D_1 can be calculated by matching H_z and E_θ at the boundaries r = R1 and r = R2. The resonance curve can be obtained by plotting the coefficient $\left|D_1^2\right|$ as a function of the wavelength by choosing a specific azimuthal mode number *m*. When changing the refractive index of the liquid by $2\text{x}10^{-4}$, the resonance peak gradually moves to the longer wavelength. The resonance peak shift as a function of the refractive index change is plotted in Fig. 4(b). A linear fit of the calculated data points gives the sensitivity of this mode, which is ~ 70nm/RIU. This number is very close to the experimental sensitivity obtained at the incident angle of ~ 37.5 degree. Generally, for such resonance mode having an evanescence wave in the liquid core region, the sensitivity normally is lower than 100nm/RIU.

Using a similar approach, we investigated resonance modes having higher radial mode numbers. The radial electrical field distribution of the TE_{700}^{37} mode in the silica micro-tube resonator is plotted in Fig. 5(a). The radial electrical field still decays exponentially in the air region, however the electrical field distribution in the liquid region is drastically different from that of the typical evanescence wave resonance mode (e.g. TE_{714}^{35} discussed above). Not only does the radial electrical field for the first peak completely move into the liquid region, but also it reaches a magnitude that is significantly higher than that in the silica tube walls. The optical field in the liquid region is no longer an evanescent field, but rather account for about 60% of the total field. Such a resonance mode with strong electrical field in the liquid region is very sensitive to the liquid's refractive index change, making it an excellent choice for index sensing. We further simulated the resonance peak shift resulting from the index change of liquid by using the Mie scattering method. The results are shown in Fig. 5(b). As can be seen in the figure, the index sensitivity of this mode is around 560nm/RIU, which is very close to the 600nm/RIU that we observed in the experiment.



(a) (b)

Fig. 5.(a) Radial electrical field intensity distribution for resonance mode E_{r700}^{37}, (b) Resonance wavelength shift related to liquid’s refractive index change for resonance mode E_{r700}^{37}

Next we present a ray optics [13] picture in order to shed light on the origin of resonance modes like TE_{700}^{37}, and to understand the significant field penetration into the liquid region. This kind of resonance modes exist as if the rays are bounced by the liquid/silica interface and confined in the liquid region. From the ray optics point of view, when the light propagating in the liquid region is incident onto the liquid/silica interface ($r = R_1$), the light will be partially reflected (termed ray 1) and partially transmitted at this inner boundary; and the transmitted rays continue to propagate within the silica tube wall until they hit the silica/air interface ($r = R_2$), where they undergo total internal reflection at this outer boundary (termed ray 2). When these two rays meet again at the liquid/silica interface at the same location and are in phase, they will interfere constructively. Furthermore, if the two rays form closed loop within the tube circumference, a resonance mode is created. The schematic ray picture of this mode is shown in Fig. 6.

This type of modes can exist only under certain conditions related to the geometry of the micro-tube and the refractive indices of the three regions. Let us assume that the refractive index of the liquid is n_1, micro-tube n_2 and outer media n_3. We first define the incident and reflection angles at the inner boundary ($r = R_1$) as θ_1 and that at the outer boundary ($r = R_2$) as θ_2. In order to have the light transmitted through the inner boundary still well confined at the outer boundary by the total internal reflection, the angles θ_1 and θ_2 should satisfy the following three conditions:

$$\frac{\sin(\pi-\theta_1)}{R_2}=\frac{\sin\theta_2}{R_1},\ \sin\theta_1<\frac{n_1}{n_2} \text{ and } \sin\theta_2>\frac{n_3}{n_2} \qquad (7)$$

These conditions will lead to another condition on the dimensions of the micro-tube: $\frac{R_2}{R_1}<\frac{n_1}{n_3}$. Therefore if the liquid inside the tube is water and outside media is air that corresponds to our experiment, the requirement on the ratio of the outer and inner radius of the micro-tube is $\frac{R_2}{R_1} < 1.32$. This condition is satisfied for the silica micro-tube used in our experiment ($R_2/R_1 = 166/134 = 1.24 < 1.32$).

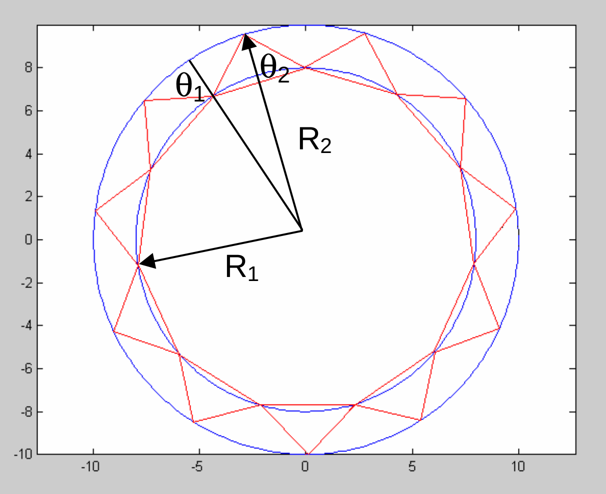


Fig. 6. The schematic ray picture of resonance mode with light transmitted into the inner boundary.

The detection limit of the micro-resonator-based refractive index sensing device is directly related to the Q factor of the resonator and the sensitivity of the resonance mode discussed above. In our experiment we can resolve the one twenty-fifth of the resonance line width change, given that resonance mode with the high field inside the liquid region has a Q factor of ~ $2\text{x}10^4$. The limit of the smallest detectable liquid index change is ~ 5 x 10^{-6} (refractive index units). The Q factor of the micro-tube is determined by the total loss of resonance mode in the resonator, which includes the radiation loss, the absorption loss and the scattering loss. Thus, the resonator's overall Q factor can be expressed as:

$$\frac{1}{Q} = \frac{1}{Q_{radiation}} + \frac{1}{Q_{absorption}} + \frac{1}{Q_{scattering}} \tag{8}$$

Because the micro-tube forms a curved waveguide, even for the special mode described above that has high field intensity in the liquid region, the optical wave transmitted into the liquid region still can return to the same loop and well confined by the total internal reflection at outer boundary; thus, the radiation loss in this kind of resonance mode is still very low. The scattering loss also can be reduced by choosing the mode with low field intensity at the inner boundary interface. The Mie scattering method revealed that the intrinsic Q value of this kind of mode is around 10^7 without considering the water absorptions loss. When the loss due to high water absorption around 1.55 μm range is taken into account in the Mie Scattering calculation, the Q factor is reduced to about 5 x 10^4, which is consistent with experimental result of Q ~ 2 x 10^4. Therefore we believe that the water absorption loss limited Q is the dominant term in determining the overall Q factor of the resonance mode. Based on these considerations, we can infer that the refractive index detection limit of the micro-tube resonator sensor can be greatly increased by moving the working wavelength from near the IR to the visible range. Using the Mie theory calculation we predict the intrinsic Q of the resonance mode with high field inside the liquid region is above $5\text{x}10^6$ in the visible range. If in experiment we can achieve a Q factor in visible range around 10^6, the smallest detection limit can reach ~ 5 x 10^{-8} (refractive index units), resulting in the micro-tube resonator being one of the best index sensing devices.

4. Summary

A prism coupled method was used to study the silica micro-tube resonator with varied incident angles. At 37.5^{o} incident angle, typical evanescent sensing modes were observed with sensitivity around 100 nm/RIU or lower. At 35^{o} incident angle, a non-evanescent sensing mode was observed with sensitivity around 600 nm/RIU. The very strong optical field penetration in the fluid leads to significant interaction with the fluid and results in the very high sensitivity. Sensors based on such resonance modes are ideally suited for applications where minute changes in the bulk refractive index needs to be determined.

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