SPREAD SPECTRUM SEQUENCE ESTIMATION AND BIT SYNCHRONIZATION USING AN EM-TYPE ALGORITHM

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ABSTRACT

Maximum Likelihood (ML) estimation method for simultaneous amplitude, time delay, and data demodulation in direct sequence spread spectrum communication is proposed. The likelihood function is analytically intractable, so we consider a recursive estimation algorithm. The Expectation-Maximization (EM) algorithm has found increasing use in similar problems, however, for this case it is analytically intractable. Recently, a variant of the EM algorithm, called Space Alternating Generalized EM (SAGE), has been derived. In this work we apply the SAGE algorithm to the sequence estimation problem in a way which allows for simple sequential updates of the parameters. The resulting algorithm maximizes the penalized likelihood function, where the penalty is chosen to ensure good synchronization performance. Simulation results show that the algorithm has fast convergence, and essentially achieves optimal performance.

1. INTRODUCTION

There has been increasing activity recently in applying advanced signal processing methods to digital communication. Several authors [1, 2] pointed to the potential benefits of optimal detection schemes in multi-user wireless communication systems such as mobile radio and personal communication networks. These types of communication channels are generally classified as being non coherent and asynchronous. In addition, they are time varying and suffer from adverse effects such as multipath and fading. In order to make use of optimal multi-user detection schemes it is necessary to have good estimates of signal phase, magnitude, and time delay. This problem becomes critical in direct sequence spread spectrum (DS-SS) communication, where data bits modulate a wide-band pseudo noise (PN) sequence, therefore an accurate estimate of the signal parameters is a pre-requisite for acceptable detection performance. Traditionally, the approach was to estimate each parameter by a specialized circuit or algorithm, such as a phase locked loop for phase estimation, and delay locked loop for time delay estimation [7]. However, this approach is costly in terms of transmission capacity, because it requires transmission of a training sequence in order to give the receiver sufficient time to synchronize. In this work we pose the simplified problem of simultaneous amplitude, time delay, and data demodulation in a single user DS-SS communication system. To solve this problem, we propose an ML estimation technique based on the EM algorithm, which iterates on the parameter updates in a manner which guarantees an increase of the likelihood function. The classical EM algorithm [4] requires simultaneous parameter updates and does not yield a computationally efficient procedure. However, by formulating the problem in terms of a penalized version of the parameters it is possible to decouple the parameters and update them sequentially using the recently developed SAGE algorithm [6]. Application of the EM algorithm to a similar problem has also been considered in [3], however this method cannot be easily extended to the DS-SS problem, because it requires initial coarse synchronization so that the received signal can be match filtered by a locally generated replica of the spreading sequence. The SAGE algorithm is shown to produce a rapidly convergent penalized ML estimation of the amplitude, time delay, and data bits.

This paper is organized as follows. In Section 2 a mathematical model of the sequence estimation problem is presented. In Section 3 the SAGE algorithm is briefly reviewed, and the modified system model is defined. The resulting parameter update equations are then derived. In Section 4 simulation results are reported. Finally, in Section 5, some conclusions of this work are presented.

2. SYSTEM MODEL

We now introduce a single user model of the DS-SS communication system. This problem cannot be efficiently solved by an EM algorithm such as the one proposed in [5], due to the inherent coupling between the parameters of the model. Later, this model will be modified so that the EM methodology can be applied using the SAGE algorithm. The system model defined below is very common in spread spectrum radio communication systems. These systems typically use direct sequence modulation. In multi-user systems, each user is assigned a separate sequence (code). These codes are designed to be mutually orthogonal, however this condition is hard to achieve in an asynchronous system. The following system parameters are defined.

- a - unknown complex baseband amplitude
- τ - unknown time delay in $[0, T)$ with respect to the receiver's clock
- b_n - unknown data bit in $\{+1, -1\}$ corresponding to the n-th bit interval
- T - bit interval period
- $p(t)$ - direct sequence (PN) assigned to the user with support on $[0, T)$ and normalized to unit energy
- N - number of data bits

We assume that the received signal has been quadrature down-converted to baseband. The unknown RF carrier phase can be absorbed into a complex amplitude parameter a, so the following complex baseband model results:

$$y(t) = a \sum_{n=0}^{N} p(t - \tau - (n-1)T) b_n + n(t) \ ; \quad 0 \le t \le NT$$

The noise $n(t)$ is assumed to be a complex, zero mean AWGN process with power spectral density $N_0/2$. The ML estimator is the parameter value which maximizes the log-likelihood function

$$\hat{\theta}_{ML} = \text{argmin}_{\theta} \left\{ \frac{2}{N_0} \int_0^{NT} |y(t) - a \sum_{n=0}^{N} p(t - \tau - (n-1)T) b_n|^2 dt \right\}$$

where $\theta \equiv [a, \tau, b_0, \ldots, b_N]^T$. Direct solution of the ML equation above is analytically intractable except in very special cases. This has sparked interest in finding recursive approximations to the ML estimator. Recently an EM algorithm for the recursive solution of a similar base-band signaling problem has been proposed [3]. This algorithm is based on sampling the outputs of a matched filter at the receiver clock timing. Generally these samples are not a sufficient statistic, therefore this method is sub-optimal. Additionally in [3], the signal amplitude is assumed known, which may not be a realistic assumption, e.g., in a fast-fading environment. In principle, the algorithm of [3] can be adapted to the present problem, however its use requires that the receiver establish initial coarse synchronization, i.e., timing error smaller than one chip time.

3. THE SAGE ALGORITHM

The Space Alternating Generalized EM (SAGE) algorithm developed in [6], is a variant of the EM algorithm which aims to avoid the trade-off between rate of convergence and complexity of optimization. A brief review of the SAGE algorithm follows.

Let θ be a parameter taking values in a parameter space Θ, which is a subset of the p dimensional Euclidean space R^p (C^p). Our goal is to find the penalized maximum likelihood estimate of θ given the observation $Y = y$. At stage i we define an index set S^i to be a non-empty subset of the set of integers $I_p = \{1, \cdots, p\}$, and we denote its complement by $\bar{S}^i$. Corresponding to these index sets we define θ_{S^i} and $\theta_{\bar{S}^i}$ as the elements of θ indexed by S^i and $\bar{S}^i$ respectively. In the SAGE algorithm the maximization of the penalized log likelihood function is replaced by a maximization of a sequence of other functionals $\{\phi^{S^i}(\theta_{S^i}; \theta^{i-1})\}_{i=1,2,\ldots}$. For this purpose a data space X^{S^i}, called *hidden data space*, is defined such that it satisfies the admissibility condition: the conditional pdf of Y given X^{S^i} does not depend on θ_{S^i}. This condition includes the EM complete data space requirement as a special case. The penalized SAGE objective is given by

$$\phi^{S^i}(\theta_S; \theta^{i-1}) = Q^{S^i}(\theta_S; \theta^{i-1}) - P(\theta_S, \theta^{i-1}_{\bar{S}^{i-1}}) \quad (1)$$

$$Q^{S^i}(\theta_S; \theta^{i-1}) \equiv E\{\log p(X^{S^i}; \theta_S, \theta^{i-1}_{\bar{S}^{i-1}} | Y = y; \theta^{i-1})\}$$

where $P(\theta)$ is a suitably defined penalty function. The SAGE algorithm generates a sequence of estimates $\{\theta^i : i = 0, 1, \ldots\}$ starting from an initial parameter guess θ^0, by alternating between different hidden data spaces X^{S^i}, computing $\phi^{S^i}(\theta_S; \theta^{i-1})$ using (1), and maximizing it over θ_S.

Before applying the SAGE algorithm, it is beneficial to slightly modify the system model. We observe that the time delay and data bit parameters are coupled, so it is helpful to assign a separate time delay parameter to each bit interval, and to include a penalty function which forces these parameters to be close together. Therefore, we define the time delay parameter $\underline{\tau}$ as $\underline{\tau} = [\tau_0, \tau_1, \cdots, \tau_N]^T$ where τ_n is the delay of the n-th bit with respect to the receiver's clock for $n = 0, \ldots, N$. In addition we penalize the time delay estimate with a quadratic penalty function $P(\underline{\tau}) = \underline{\tau}^T R \underline{\tau}$, where R is a non-negative definite symmetric matrix. It is now clear that the signal model needs to be modified because each bit has variable duration. We consider the following modified definition

$$y(t) = a \sum_{n=0}^{N} p_n(t; \tau_n, \tau_{n+1}) b_n + n(t) \; ; \quad 0 \leq t \leq NT \qquad (2)$$

and

$$p_n(t; \tau_n, \tau_{n+1}) = \begin{cases} p(t - \tau_n - (n-1)T) & \tau_{n+1} \leq \tau_n \; ; \\ p(t - \tau_n - (n-1)T) + & \\ q(t - \tau_n - nT) & \tau_{n+1} > \tau_n \; ; \end{cases}$$
$$\tau_n + (n-1)T \leq t \leq \tau_{n+1} + nT$$

where $q(t)$ is an arbitrary normalized PN signal on $[0, T]$ and τ_{N+1} is defined to be identically zero. According to this definition the direct sequence is either truncated or extended. The calculation of the conditional expectation in (1) is straightforward, and leads directly to the following SAGE algorithm for simultaneous amplitude, time delay, and data sequence estimation:

For $i = 1, 2, \ldots$

1. Choose index set S^i
2. If $\theta_{S^i} = b_n, n = 0, \ldots, N$ then
$$b_n^{i+1} = \text{argmax}_{b_n} \text{Re} \left\{ \int_{\tau_n^i + (n-1)T}^{\tau_{n+1}^i + nT} y(t) a^{*i} b_n p_n(t; \tau_n^i, \tau_{n+1}^i) dt \right\}$$
3. If $\theta_{S^i} = \tau_n, n = 0, \ldots, N$ then
$$\tau_n^{i+1} = \text{argmax}_{\tau_n} \text{Re} \left\{ \frac{2}{N_0} \int_{\tau_{n-1}^i + (n-2)T}^{\tau_{n+1}^i + nT} y(t) a^{*i} [b_{n-1}^i p_{n-1}(t; \tau_{n-1}^i, \tau_n) + b_n^i p_n(t; \tau_n, \tau_{n+1}^i)] dt \right\} - r_{nn}(\tau_n - \textstyle\sum_{j \neq n} \tau_j^i r_{jn} / r_{nn})^2$$
4. If $\theta_{S^i} = a$ then
$$a^{i+1} = \frac{1}{N} \sum_{n=0}^{N} b_n^i \int_0^{NT} y(t) p_n(t; \tau_n^i, \tau_{n+1}^i) dt$$

The parameter θ above has been redefined as $\theta \equiv [a, \tau_0, \ldots, \tau_N, b_0, \ldots, b_N]^T$. We make the following observations

- Maximization with respect to τ_n in step 3 has to be done by a line search, which may be costly in computation time.
- A Rayleigh fading amplitude model can be implemented by subtracting a quadratic penalty from the objective in stage 4 of the form $|a|^2/\sigma_a^2$ where σ_a^2 is the fading variance.
- Single bit estimates are updated using a correlator, with fixed previous estimates τ_n^i and a^i. In [3] multiple bits are updated with the Viterbi algorithm. Time delay τ_n is updated locally using fixed previous estimates of amplitude and adjacent time delay and bit estimates. Multiple bit correlator gives amplitude estimate a^{i+1}.

4. NUMERICAL RESULTS

A single user noncoherent asynchronous receiver based on the model described above was simulated on a computer. The direct sequences were maximal length sequences [7]. The amplitude, time delay, and data bits were kept fixed in each simulation. We observed that a good initial time delay estimate is crucial for finding the global maximum. Therefore, ML time delay estimates were found in each bit interval, and then averaged over all intervals. The convergence of the algorithm was very fast. The amplitude and data bits estimates converged in 2-3 iterations, and the time delay estimates converged in 5-6 iterations. Two types of penalties were tried: the first one penalized only adjacent bit intervals by a quadratic cost, i.e. $(\tau_m - \tau_n)^2$ where $m = n + 1$ or $m = n - 1$; the second type penalized every possible pair of bit intervals. We refer to the first type as a loose penalty, and to the second as a tight penalty. Both penalties were scaled by $1/T_c^2$ where T_c is the chip-time, in order to force the time estimates τ_n to be within one chip-time. The time delay estimate $\hat{\tau}$ was taken as the average of the time delay vector estimator $\underline{\hat{\tau}}$.

Figure 1 shows simulation results of a sequence of 30 data bits, at 7 values of SNR between 0 to 16 dB. Each point consists of 20 independent simulation runs with either one of the aforementioned penalties. The direct sequence is 7 chips long. The figure shows the variance of $\hat{\tau}$ and $\hat{a}$. The CR bound on the variance of an unbiased estimator of a for the case of known time delay and data bits is shown for reference. We observe that the estimator $\hat{a}$ achieves the CR bound at high SNR. Figure 2 shows simulation results of a sequence of 10 bits, with 35 simulation runs at each point. A threshold effect due to small time bandwidth product for both estimators is observed at an SNR of 6 dB. The variance of both estimators increases rapidly below threshold. Figure 3 shows simulation results of a sequence of 10 data bits, with a direct sequence of length 15. We notice a threshold effect at an SNR of about 10 dB. There seems to be a slight difference in performance between the cases of loose and tight penalty. The tight penalty appears to perform better above threshold, and conversely the loose penalty appears to perform better below threshold. However, more simulations are needed to make a conclusive statement.

5. CONCLUSIONS

We have presented an application of the SAGE algorithm for simultaneous estimation of amplitude, time delay, and data bits in a DS-SS communication system. The derivation of the algorithm is simple and it can be extended to other cases of interest. Computer simulations showed that the algorithm has fast convergence, and essentially achieves optimal performance at high SNR. More simulations are needed to evaluate the BER performance of the algorithm. Choice of the penalty function can be studied once its influence on performance and convergence rate is understood. Convergence analysis of the algorithm is very difficult, and is a subject of an ongoing research.

6. REFERENCES

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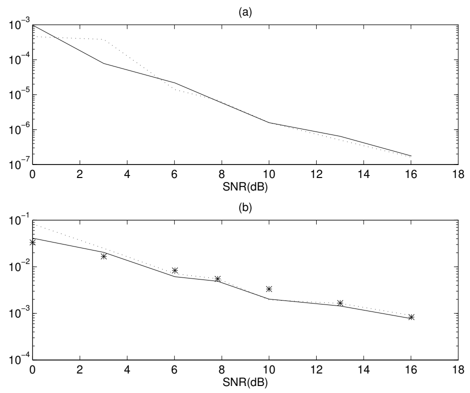

Figure 1: Simulation results, $N = 30$ data bits, 7 chips, solid: loose penalty; dotted: tight penalty; stars: CR bound. Fig. (a) shows $Var\{\hat{\tau}\}$, Fig. (b) shows $Var\{\hat{a}\}$.

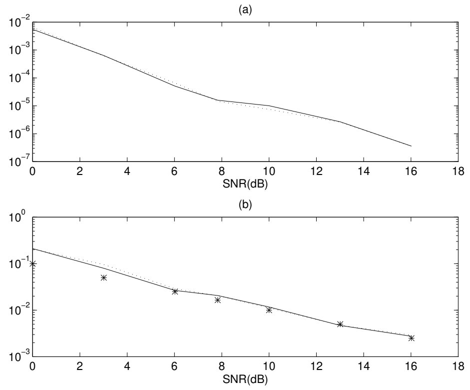

Figure 2: Simulation results, $N = 10$ data bits, 7 chips, solid: loose penalty; dotted: tight penalty; stars: CR bound. Fig. (a) shows $Var\{\hat{\tau}\}$, Fig. (b) shows $Var\{\hat{a}\}$.

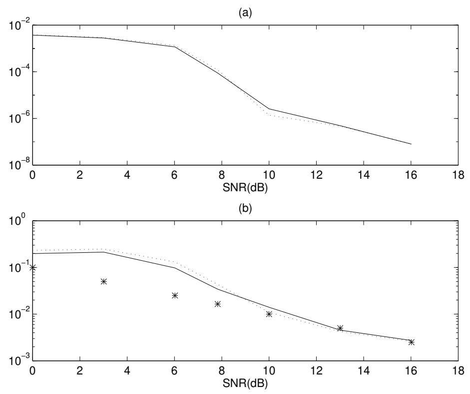

Figure 3: Simulation results, $N = 10$ data bits, 15 chips, solid: loose penalty; dotted: tight penalty; stars: CR bound. Fig. (a) shows $Var\{\hat{\tau}\}$, Fig. (b) shows $Var\{\hat{a}\}$.