Reduced Signature Adaptive Target Detection in Remote Sensing

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<u>Outline</u>

1. Target Detection Examples in EO/SAR
2. Linear Multivariate Image model
3. Data Reduction via Sufficiency and Invariance
4. Case of Homogeneous Clutter
5. Case of Inhomogeneous Clutter
6. Experimental Results

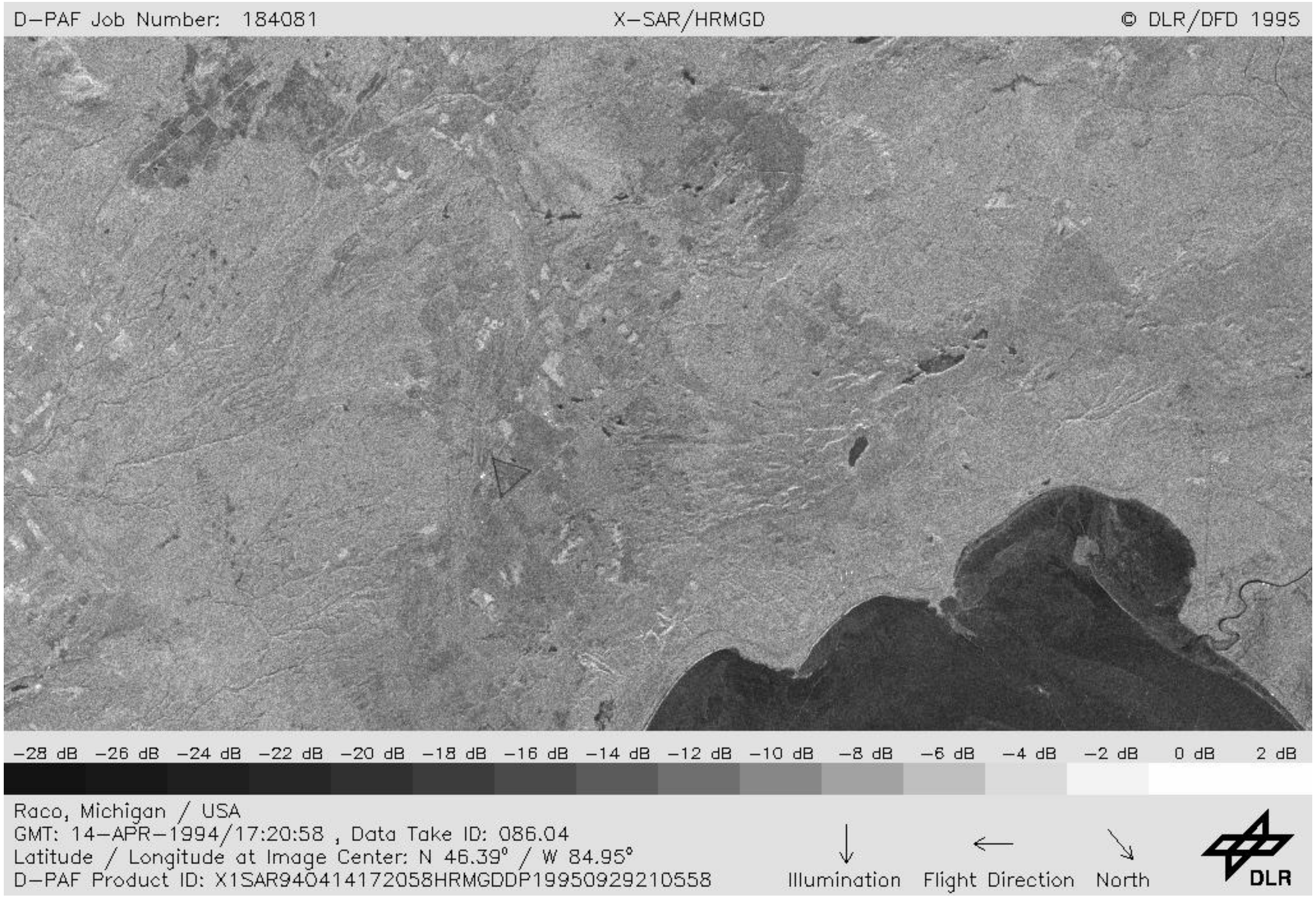


Figure 1. *X-SAR image of Raco, Michigan (DARA/ESA database)*

Figure 2. *EO image of CCGS Des Groseilliers (98m long) (NSIDC - SHEBA database)*

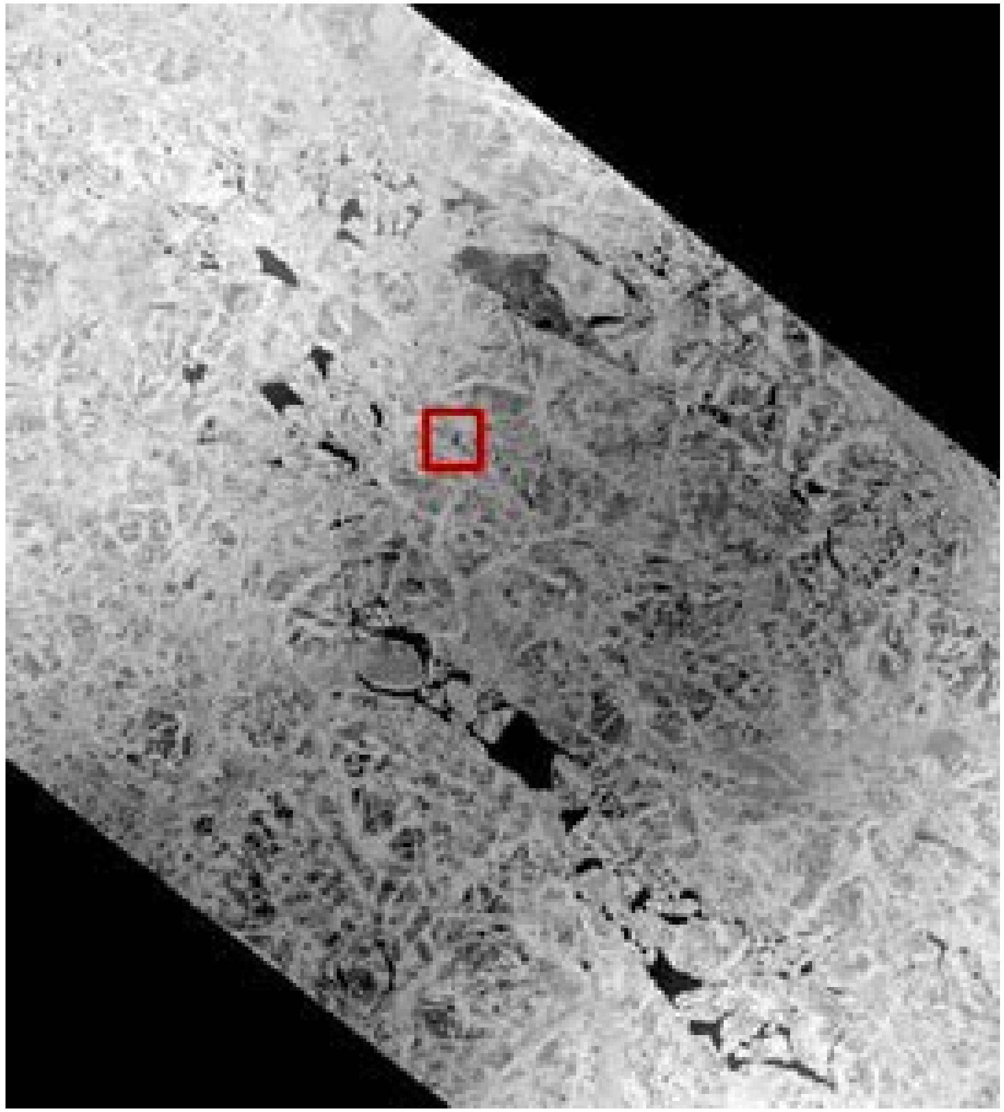


Figure 3. *EO image of arctic swath (aprrox 7km wide) (SHEBA database).*

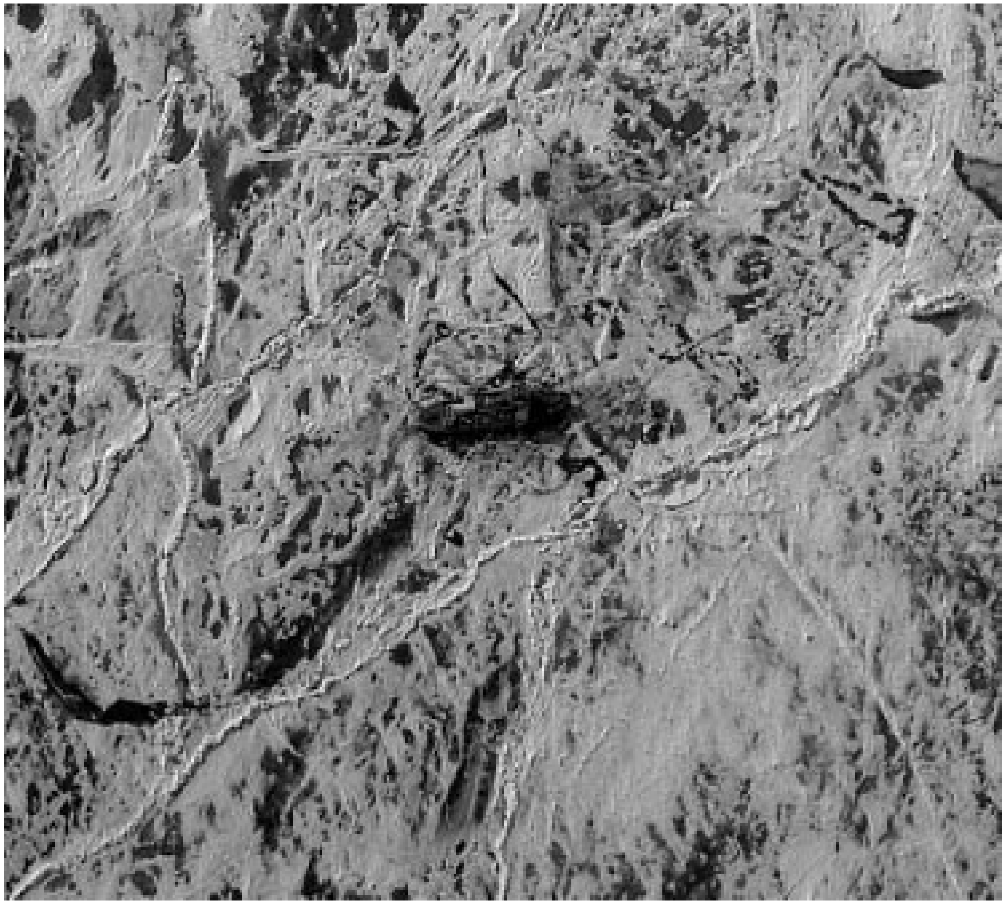
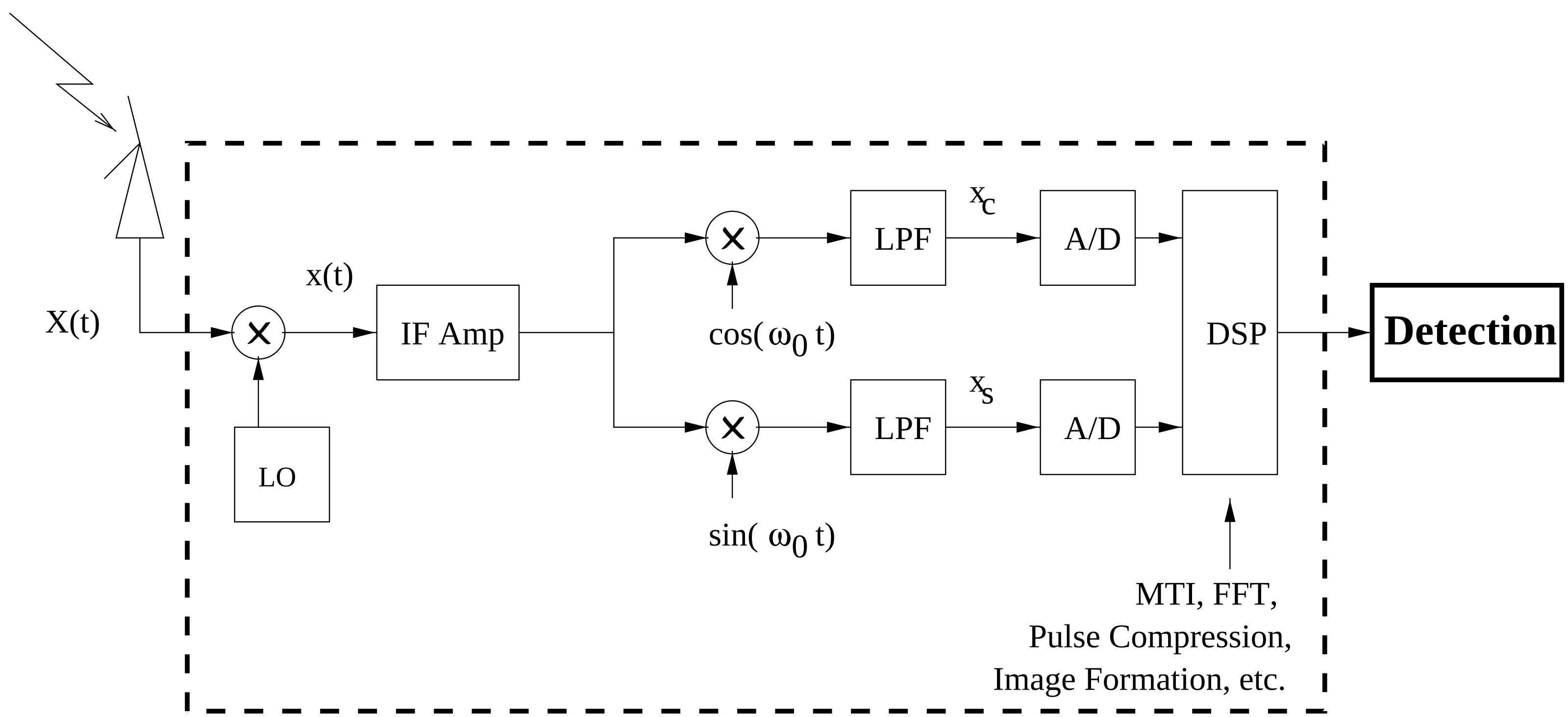


Figure 4. *Blowup of EO image of arctic swath (SHEBA database).*

Radar Preprocessor



LINEAR MULTIVARIATE IMAGE MODEL

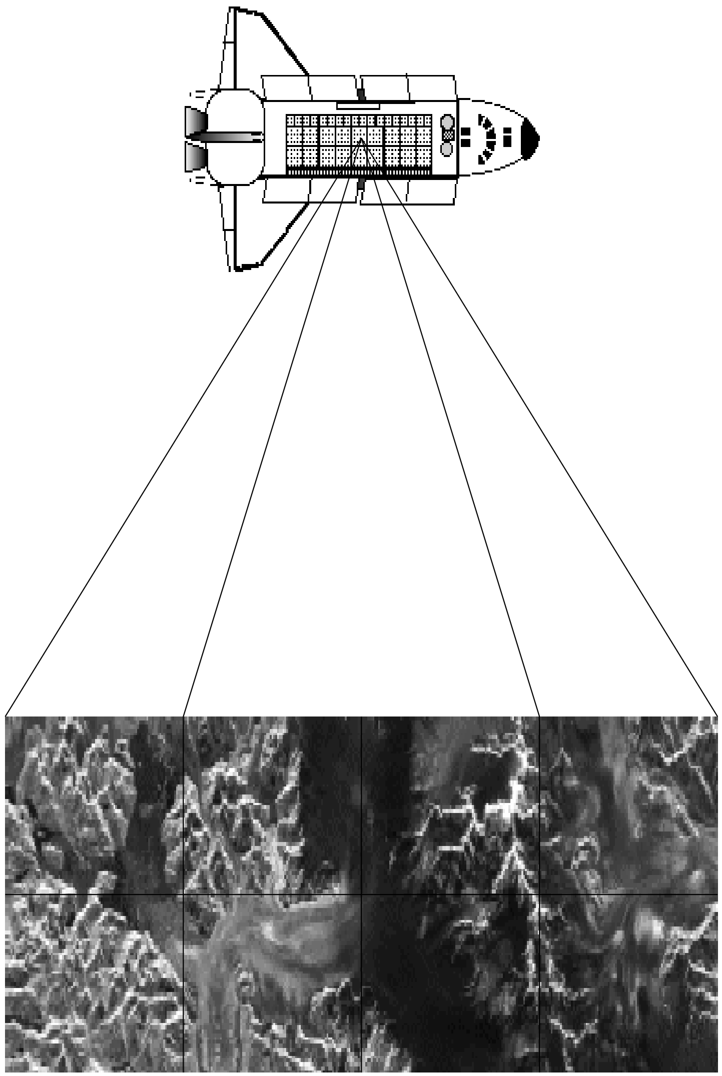
$$\mathbf{X} = [\underline{X}_1, \ldots, \underline{X}_n]$$

$\underline{X}_k$ = $m \times 1$ lexicographic ordered k-th sub-image

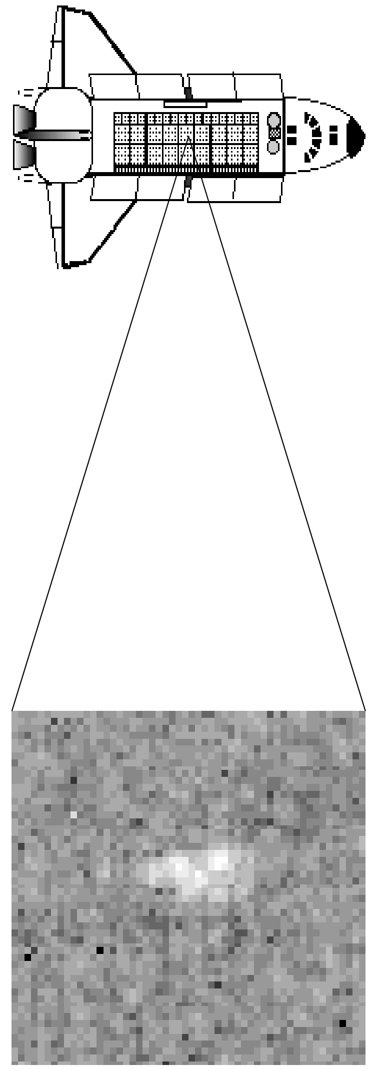
$$\mathbf{X} = \mathbf{S}\,\mathbf{A}\,\mathbf{B} + \mathbf{N}$$

- $\mathbf{S} = [\underline{s}_1, \ldots, \underline{s}_p]$ = an $m \times p$ matrix of target signatures (*known*)
- $\mathbf{A} = \mathrm{diag}(\underline{a})$ = a $p \times p$ diagonal matrix (*unknown*)
- $\mathbf{B}$ a $p \times n$ matrix of target locations

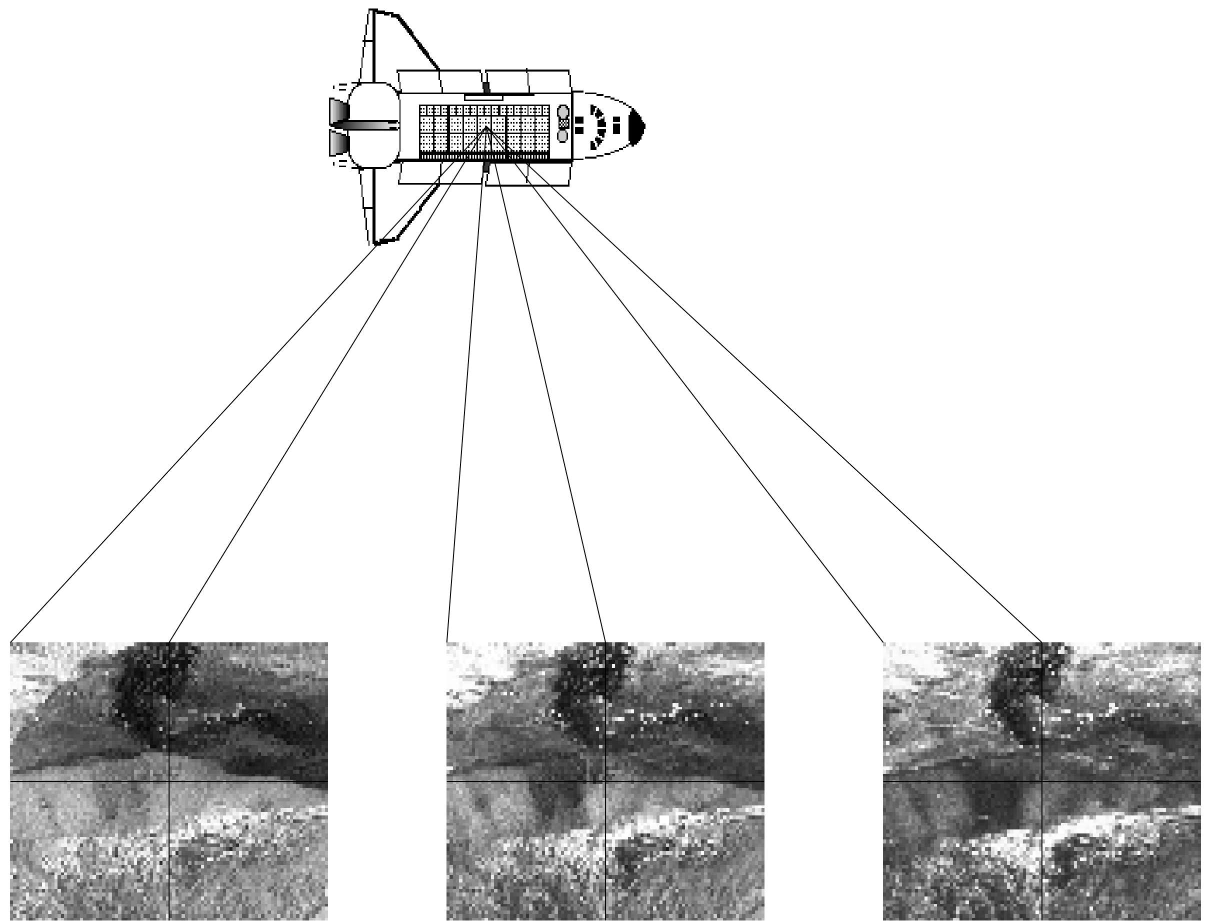
Spatially scanned radar: $\mathbf{B} = [1, 0, \ldots, 0]$ $(p = 1)$



Multiple dwell radar: $\mathbf{B} = [1, \ldots, 1]$ $(p = 1)$



Multispectral radar:



C-band L-band

P-band

Multivariate Gaussian Noise Model

$\mathbf{N}$: $(m \times n)$ multivariate Gaussian matrix
w/ i.i.d. columns $(m \times 1)$ each having covariance matrix $\mathbf{R}(m \times m)$.

$$cov[\text{vec}\{\mathbf{N}\}] = \mathbf{R} \otimes \mathbf{I}_n = \begin{bmatrix} \begin{matrix} \mathbf{R} & O \\ O & \mathbf{R} \end{matrix} & \mathbf{O} \\ \mathbf{O} & \begin{matrix} \mathbf{R} & O \\ O & \mathbf{R} \end{matrix} \end{bmatrix} (mn \times mn)$$

Note :

$$\mathbf{ANB} \sim N_{mn}\left(\underline{0}, \mathbf{ARA}^T \otimes \mathbf{BB}^T\right)$$

(A : $m \times m$, **B** : $n \times n$**)**

III. Detection Criteria

- Statistical model for observations **X**

$$\mathbf{X} \sim f(X;\theta)$$

- Parameter space $\Theta = \Theta_0 \cup \Theta_1$
- two hypotheses:

$$H_0 \quad : \quad \mathbf{X} \sim f(X;\theta_0), \qquad \theta_0 \in \Theta_0$$

$$H_1 \quad : \quad \mathbf{X} \sim f(X;\theta_1), \qquad \theta_1 \in \Theta_1$$

Likelihood Ratio Test (LRT)

:

$$\frac{f_{\theta_1}(\mathbf{X})}{f_{\theta_0}(\mathbf{X})} \underset{H_0}{\overset{H_1}{\gtrless}} \eta$$

η selected such that $P_{FA} = \alpha$, $\alpha \in [0,1]$

Note: LRT is MP test for $\Theta_0 = \{\theta_0\}$ and $\Theta_1 = \{\theta_1\}$

Difficulties:

- LRT is not usually CFAR
- LRT is not usually unbiased
- LRT is not usually UMP

GLR test

GLRT between hypotheses $\theta \in \Theta_0$ vs. $\theta \in \Theta_1$

$$\frac{\max_{\theta\in\Theta_1} f_\theta(\mathbf{X})}{\max_{\theta\in\Theta_0} f_\theta(\mathbf{X})} \underset{H_0}{\overset{H_1}{\gtrless}} \eta$$

1. GLRT is a function of MLEs $\hat{\theta}_0$ and $\hat{\theta}_1$
2. GLRT is not asymptotically CFAR or UMP unless MLEs are consistent
3. GLRT optimization may be intractible
4. GLRT performance can be very poor (not even unbiased) in finite sample regime

$\Rightarrow$ GLRT decision region overinfluenced by ML estimates, eroding H_0 vs. H_1 discrimination ability

Statistical Reduction via Sufficiency

Minimal Sufficient Statistic

$$\Lambda_{\theta_0,\theta_1}(\mathbf{X}) \stackrel{\text{def}}{=} \frac{f(\mathbf{X};\theta_1)}{f(\mathbf{X};\theta_0)} = g(T(\mathbf{X}),\theta_1,\theta_0)$$

$$\{\mathbf{X} : \Lambda_{\theta_0,\theta_1}(\mathbf{X}) = \lambda\}_{\lambda>0} = \{\mathbf{X} : T(\mathbf{X}) = t\}_t$$

- $T(\mathbf{X})$ specifies orbit $\{\mathbf{X} : \Lambda_{\theta_0,\theta_1}(\mathbf{X}) = \lambda\}$ (depends on θ_0 and θ_1).

- $T(\mathbf{X})$ achieves maximal data reduction while preserving information necessary to estimate θ_0 and θ_1 and discriminate between H_0 and H_1.

- Distribution of $T(\mathbf{X})$ depends on particular values of $\theta_0 \in \Theta_0$ and $\theta_1 \in \Theta_1$.

Statistical Reduction via Invariance

Maximal invariant statistic

$$\tilde{\Lambda}(\mathbf{X}) \stackrel{\text{def}}{=} \left\{ \frac{f(\mathbf{X};\theta_1)}{f(\mathbf{X};\theta_0)} : \theta_0 \in \Theta_0, \theta_1 \in \Theta_1 \right\}$$
$$= \left\{ \tilde{h}(\mathbf{X}) : \theta_0 \in \Theta_0, \theta_1 \in \Theta_1 \right\}$$

- $\mathbf{Z} = Z(\mathbf{X})$ specifies orbit of $\{\mathbf{X} : \tilde{\Lambda}(\mathbf{X}) = \tilde{\lambda}\}$ (depends on Θ_0 and Θ_1).

- Maximal invariants can be found when Θ_0 and Θ_1 have simple topological group structure.

Group characterization of Θ_0, Θ_1

Let G be a group of transformations $g : X \to X$ acting on $\mathbf{X}$.

Assume that for each $\theta \in \Theta$ there exists a unique $\bar{\theta} = \bar{g}(\theta)$ such that

$$f_\theta(g(\mathbf{X})) = f_{\bar{\theta}}(\mathbf{X}), \qquad \bar{g}(\Theta_0) = \Theta_0,\ \bar{g}(\Theta_1) = \Theta_1$$

$\bar{g}$ is called the induced group action on Θ.

$\mathbf{Z} = Z(\mathbf{X})$ is a maximal invariant iff

1. (invariant property) $Z[g(\mathbf{X})] = Z(\mathbf{X})$ for all $g \in G$ and
2. (maximal property) $Z(\mathbf{X}) = Z(\mathbf{Y}) \Rightarrow \mathbf{Y} = g(\mathbf{X})$ for some $g \in G$.

Example: detection of scalar signal; a, σ^2 unknown

$$\underline{\mathbf{x}}^T = a[1,0,\ldots,0] + \underline{N}, \qquad \underline{N} \sim N_n(0, \sigma^2 \mathbf{I}).$$

$$\Theta_0 = \{a = 0, \sigma^2 > 0\}, \quad \Theta_1 = \{a \neq 0, \sigma^2 > 0\}$$

$$\log \Lambda_{\theta_0,\theta_1}(\underline{\mathbf{x}}) = \frac{a}{\sigma_1^2}\, \underline{\mathbf{e}}_1^T \underline{\mathbf{x}} + \frac{\sigma_1^2 - \sigma_0^2}{2\sigma_0^2\sigma_1^2}\, \underline{\mathbf{x}}^T \underline{\mathbf{x}} + const.$$

1. **Sufficiency orbits**: $[\underline{\mathbf{e}}_1^T \underline{\mathbf{x}}, \underline{\mathbf{x}}^T \underline{\mathbf{x}}] = [t_1, t_2]$ (circle in $\mathbf{R}^3$)

2. **Invariance orbits**: $|\underline{\mathbf{e}}_1^T \underline{\mathbf{x}}|^2 / \underline{\mathbf{x}}^T \underline{\mathbf{x}} = z$ (cone in $\mathbf{R}^3$)

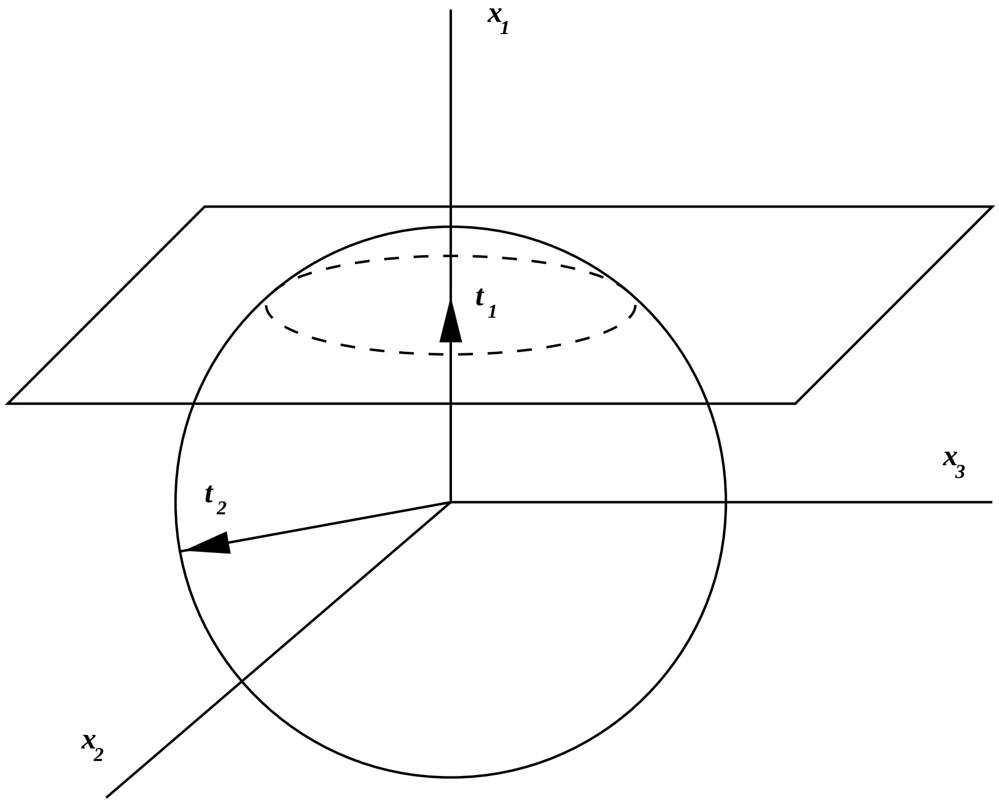


Figure 5. Sufficiency orbit

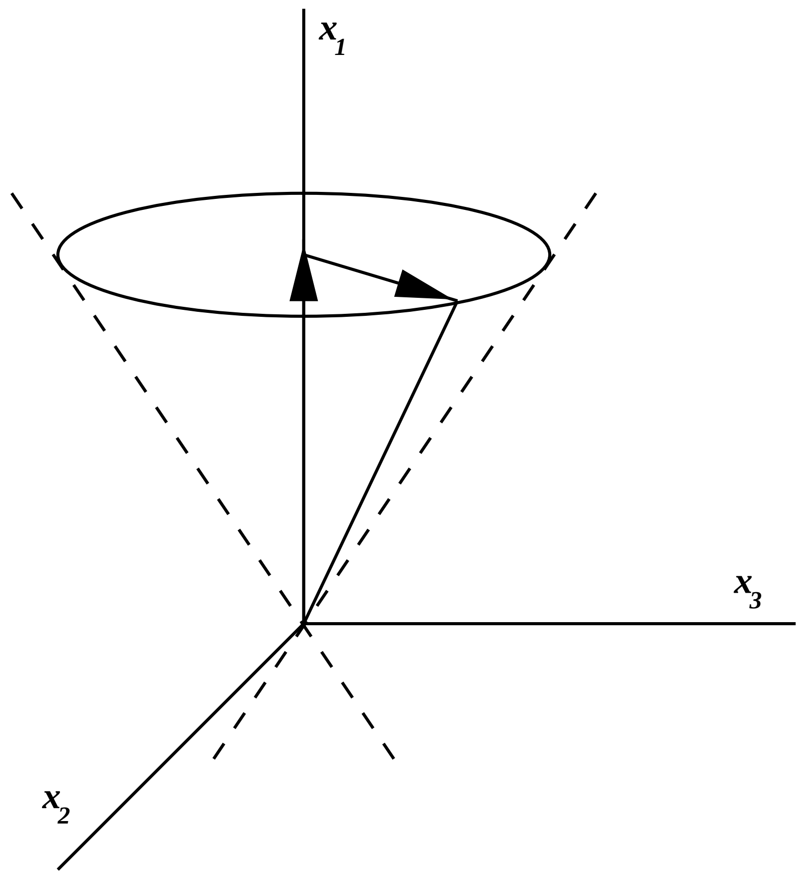


Figure 6. Invariance orbit

Unknown Target in Unknown Clutter

$$\mathbf{X} = \underline{s} \cdot \underline{b}^T + \mathbf{N} \sim N\left(\underline{s} \cdot \underline{b},\ \mathbf{R} \bigotimes \mathbf{I}_n\right)$$

$\underline{s}$ unknown $m \times 1$, $\mathbf{R}$ unknown $m \times m$, $\underline{b}$ known $n \times 1$.

Define: $\mathbf{Q}_b = \left[\frac{1}{||\underline{b}||}\underline{b}, \underline{b}_2^{\perp}, \ldots, \underline{b}_n^{\perp}\right] \Rightarrow \mathbf{X} \rightarrow \mathbf{X}\mathbf{Q}_b$ gives canonical form

$$\begin{aligned} \mathbf{X} &= \begin{bmatrix} s_1 \\ \vdots \\ s_m \end{bmatrix} \cdot [1, 0, \ldots, 0] + \mathbf{N} \\ &= [\underline{x}_1,\ \mathbf{X}_2] \end{aligned}$$

primary secondary

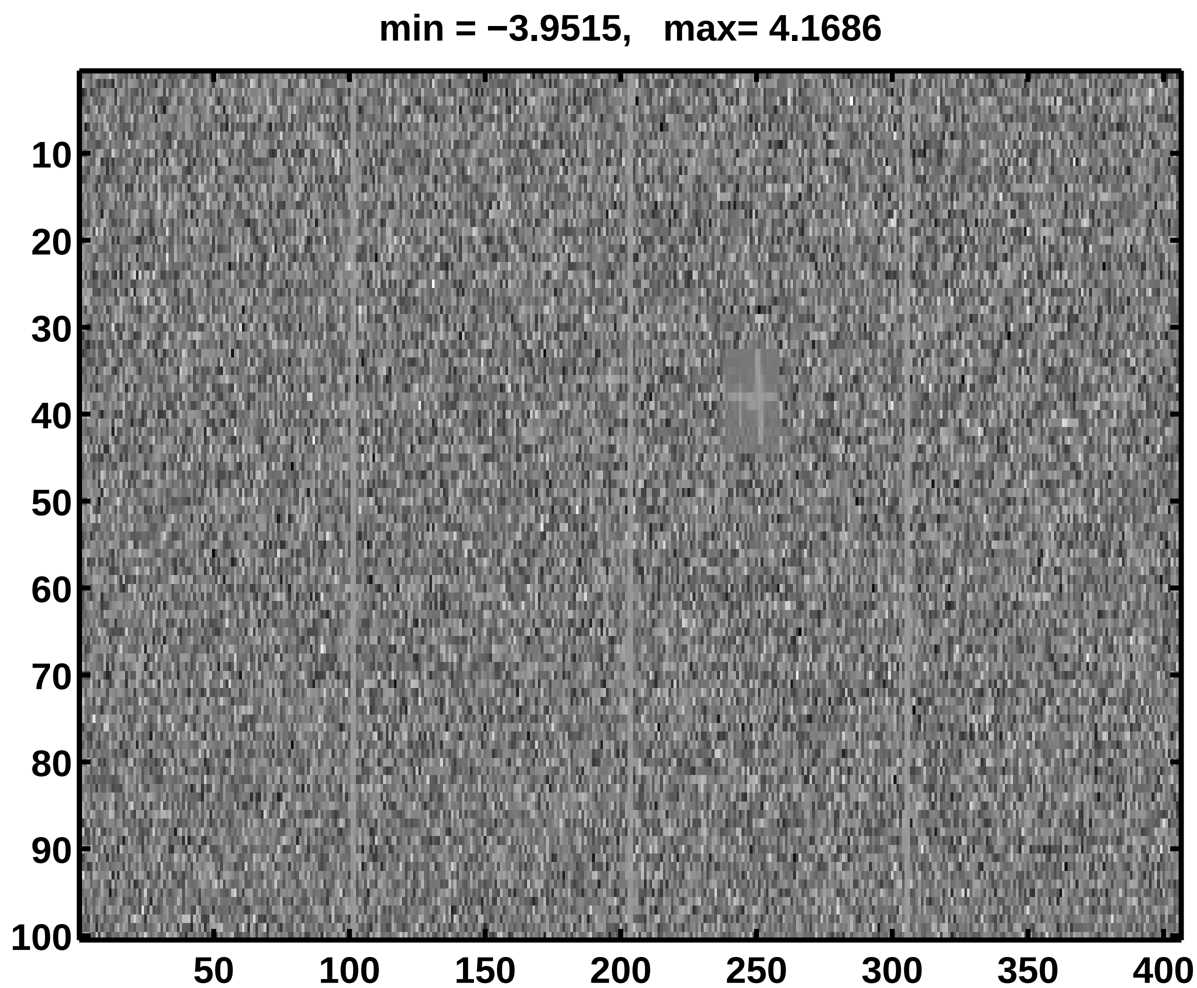


Figure 7. Target in unknown clutter

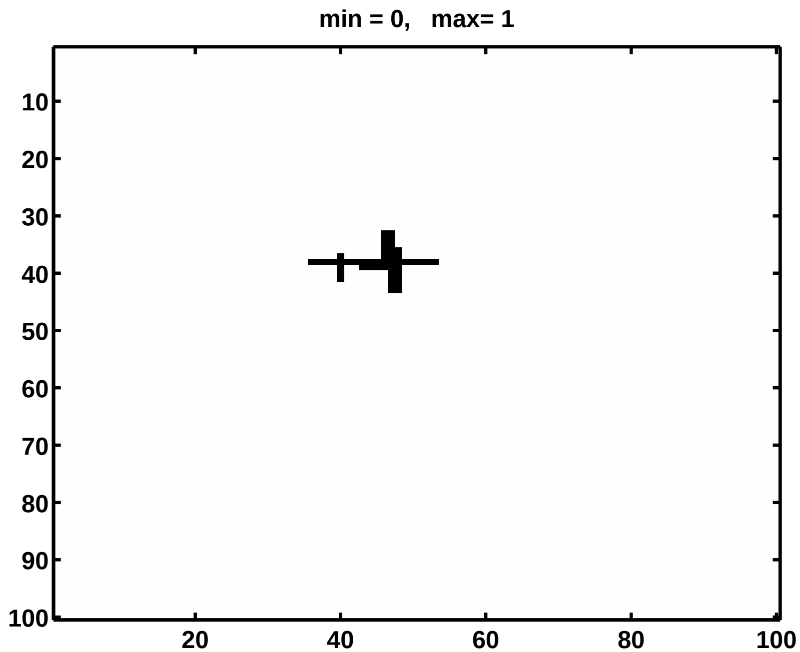


Figure 8. Target alone

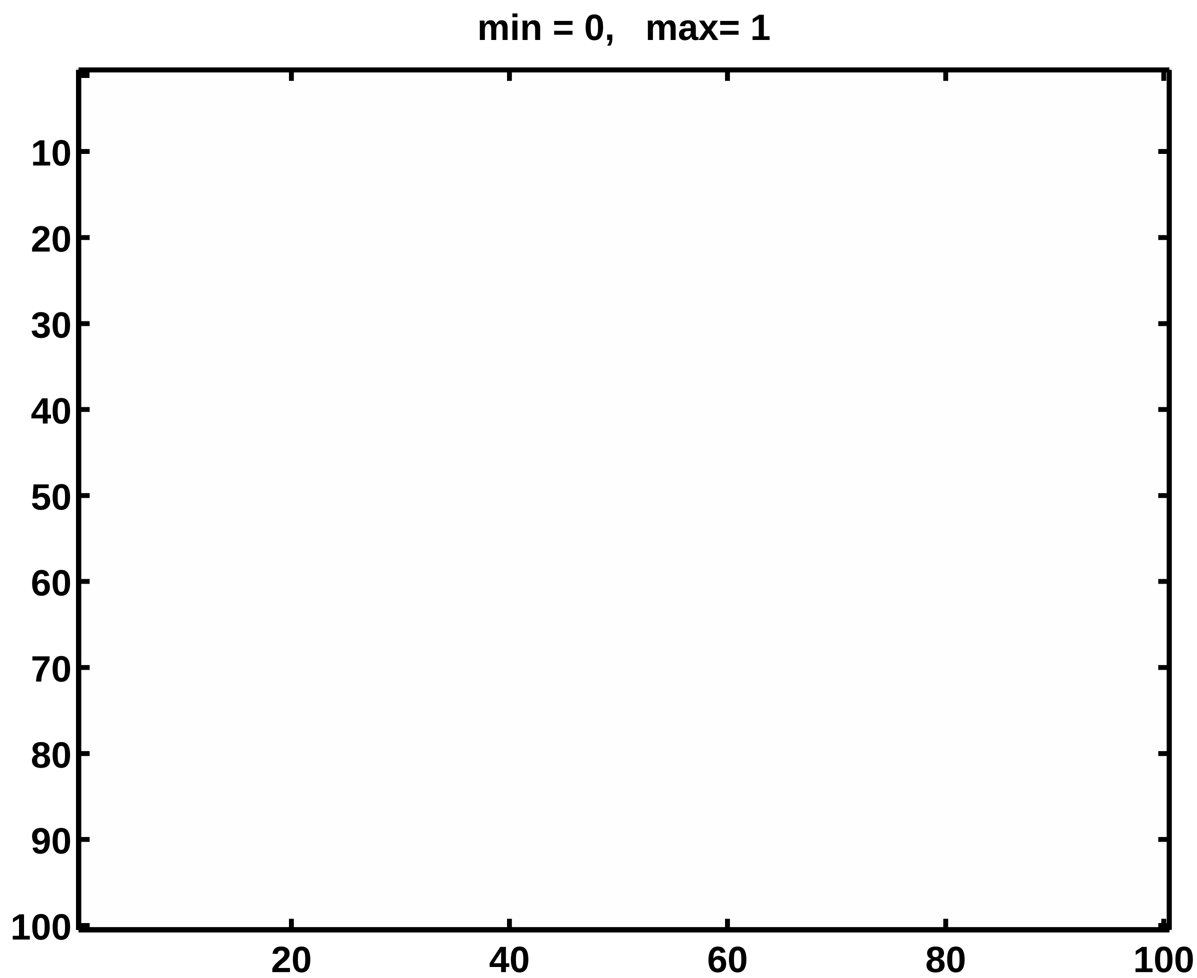


Figure 9. Transformed Target

$$\Theta_0 = \{\underline{s} = 0, \mathbf{R} > 0\}, \qquad \Theta_1 = \{\underline{s} \neq 0, \mathbf{R} > 0\}$$

Group of transformations leaving decision problem invariant:

$\Rightarrow G$ has group action: $g(\mathbf{X}) = \mathbf{FXH}$,

$\mathbf{F}$ invertible $m \times m$ and $\mathbf{H}$ unitary of form: $\mathbf{H} = \begin{bmatrix} 1 & \underline{0}^T \\ \underline{0} & \mathbf{U} \end{bmatrix}$

Note:

1. $g(\mathbf{X})$ remains multivariate Normal
2. Under $\overline{g}$:

$$\begin{aligned} E_\theta[\mathbf{X}] &= \underline{s} \cdot \underline{b}^T \rightarrow \mathbf{F}\underline{s} \cdot \underline{b}^T\mathbf{H} = \overline{\underline{s}} \cdot \underline{b}^T \\ \mathrm{cov}_\theta[\mathbf{X}] &= \mathbf{R} \bigotimes \mathbf{I}_n \rightarrow \mathbf{FRF}^T \bigotimes \mathbf{HH}^T = \overline{\mathbf{R}} \bigotimes \mathbf{I}_n \end{aligned}$$

3. $\overline{g}(\underline{s}, \mathbf{R}) = \{\mathbf{F}\underline{s}, \mathbf{FRF}^T\}$
   $\Rightarrow \overline{g}(\Theta_0) = \Theta_0$ and $\overline{g}(\Theta_1) = \Theta_1$.

<u>Maximal Invariant</u> and **Induced MI** are scalar :

$$z(\mathbf{X}) = \underline{x}_1^T [\mathbf{X}_2 \mathbf{X}_2^T]^{-1} \underline{x}_1, \qquad , \delta(\theta) = \underline{s}^T \mathbf{R}^{-1} \underline{s}, \qquad \text{where} \mathbf{X} = [\underline{x}_1, \mathbf{X}_2]$$

Using reduced data $z(\mathbf{X})$ we have equivalent hypotheses:

$$\Theta_0 = \{\delta = 0\}, \qquad \Theta_1 = \{\delta > 0\}$$

Density function of $z(\mathbf{X})$ is non-central F:

$$z(\mathbf{X}) \cdot \frac{n-m}{m(n-1)} \sim F_{m,n-m}(z, (n+1)\delta)$$

Most powerful invariant (MPI) test of level α for known δ:

$$z \underset{H_0}{\overset{H_1}{\gtrless}} \frac{(n-1)m}{n-m} \cdot F_{m,n-m}^{-1}(1-\alpha), \qquad (UMPI - CFAR)$$

Known Target Unknown Clutter

$$\mathbf{X} = a\underline{s} \cdot \underline{b}^T + \mathbf{N}$$

$\underline{s}$ *known* $m \times 1$, $\mathbf{R}$ unknown $m \times m$, $\underline{b}$ known.

Map to canonical form via $\mathbf{X} \to \mathbf{Q}_s^T \mathbf{X} \mathbf{Q}_b$:

$$\mathbf{X} = a \cdot \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \cdot [1, 0, \ldots, 0] + \mathbf{N}$$

$$\Theta_0 = \{a = 0, \mathbf{R} > 0\}, \qquad \Theta_1 = \{a \neq 0, \mathbf{R} > 0\}$$

Group of transformations leaving decision problem invariant:

$\Rightarrow G$ has group action: $g(\mathbf{X}) = \mathbf{FXH}$,

$\mathbf{F}$ invertible $m \times m$ and $\mathbf{H}$ unitary $n \times n$ of forms

$$\mathbf{F} = \begin{bmatrix} \beta_1 & \underline{\beta}^T \\ \underline{0} & \mathbf{M} \end{bmatrix}, \qquad \mathbf{H} = \begin{bmatrix} 1 & \underline{0}^T \\ \underline{0} & \mathbf{U} \end{bmatrix}$$

Note:

1. $g(\mathbf{X})$ remains multivariate Normal
2. $\bar{g}(a, \mathbf{R}) = \{\beta_1 a, \mathbf{FRF}^T\} \Rightarrow \bar{g}(\Theta_0) = \Theta_0$ and $\bar{g}(\Theta_1) = \Theta_1$.

Let

$$\mathbf{X} = [\underline{x}_1, \mathbf{X}_2] = \begin{bmatrix} x_{11} & \underline{x}_{12} \\ \underline{x}_{21} & \mathbf{X}_{22} \end{bmatrix}$$

Maximal Invariant is two dimensional:

$$z_1(\mathbf{X}) = \underline{x}_1^T [\mathbf{X}_2 \mathbf{X}_2^T]^{-1} \underline{x}_1$$

$$z_2(\mathbf{X}) = \underline{x}_{21}^T [\mathbf{X}_{22} \mathbf{X}_{22}^T]^{-1} \underline{x}_{21} \quad \text{multiple correlation factor}$$

Induced Maximal Invariant: $\delta = a^2 \, \underline{s}^T \mathbf{R}^{-1} \underline{s}$

Given invariant data $z(\mathbf{X})$ we have equivalent hypotheses:

$$\Theta_0 = \{\delta = 0\}, \qquad \Theta_1 = \{\delta > 0\}$$

Decision region of MPI test depends on unknown δ – no UMPI exists.

Some CFAR alternatives:

$$z_1 \overset{H_1}{\underset{H_0}{\gtrless}} \eta, \qquad \text{(Robey's AMF)}$$

$$\frac{z_1}{1 + z_1 + z_2} \overset{H_1}{\underset{H_0}{\gtrless}} \eta, \qquad \text{(Kelly's GLRT)}$$

Two Unknown Clutter Regions

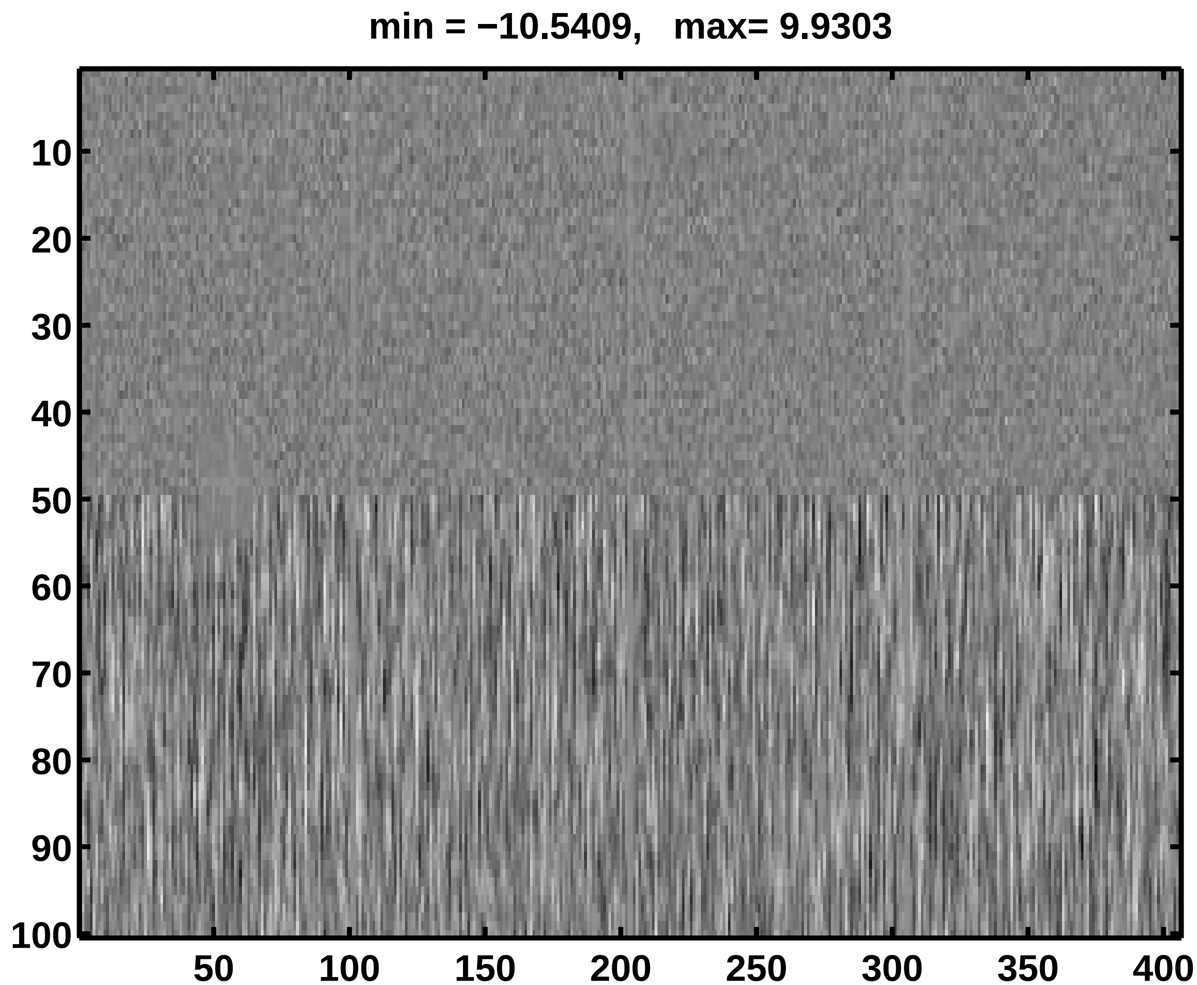


Figure 10. Deep hide target on clutter boundary

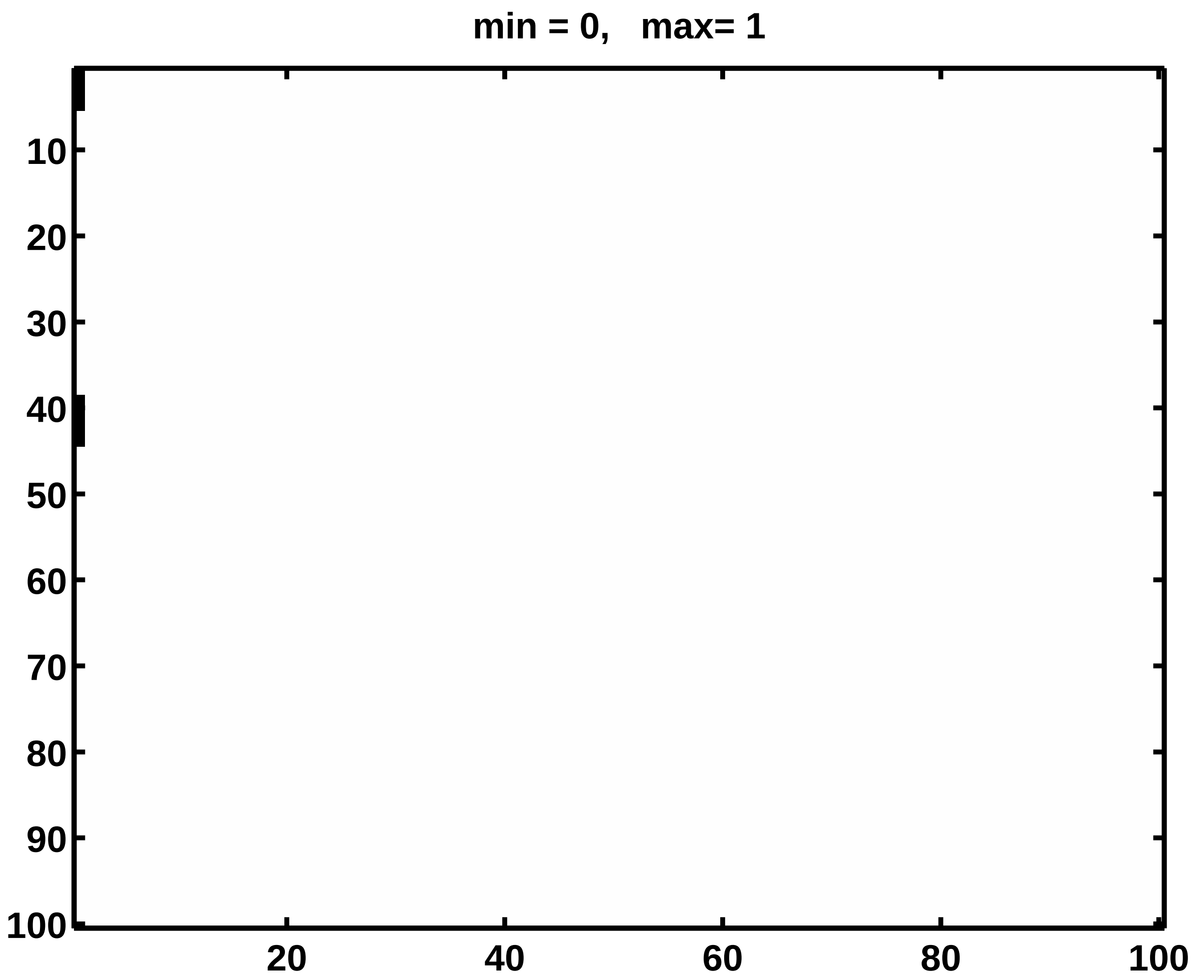


Figure 11. Transformed target matrix

Multivariate model:

$$\mathbf{X} = a \cdot \begin{bmatrix} \underline{s}_A \\ \underline{s}_B \end{bmatrix} \underline{b}^T + \begin{bmatrix} \mathbf{N}_A \\ \mathbf{N}_B \end{bmatrix}$$

$\Rightarrow$ canonical form:

$$\mathbf{X} = a \begin{bmatrix} 1 \\ \underline{0} \\ 1 \\ \underline{0} \end{bmatrix} \cdot [1, 0, \ldots, 0] + \begin{bmatrix} \mathbf{N}_A \\ \mathbf{N}_B \end{bmatrix}$$

$$\Theta_0 = \{a = 0, \mathbf{R}_A > 0, \mathbf{R}_B > 0\}, \qquad \Theta_1 = \{a \neq 0, \mathbf{R}_A > 0, \mathbf{R}_B > 0\}$$

Group of transformations leaving decision problem invariant:

G has group action: $g(\mathbf{X}) = \mathbf{FXH}$,

- $\mathbf{F}$ invertible $m \times m$ of form:

$$\mathbf{F} = \gamma \begin{bmatrix} \begin{matrix} \mathbf{I}_A & \mathbf{C}_A \\ \underline{0} & \Gamma_A \end{matrix} & \mathbf{O} \\ \mathbf{O} & \begin{matrix} \mathbf{I}_B & \mathbf{C}_B \\ \underline{0} & \Gamma_B \end{matrix} \end{bmatrix}$$

- $\mathbf{H}$ unitary $n \times n$ of form:

$$\mathbf{H} = \begin{bmatrix} 1 & \underline{0}^T \\ \underline{0} & \mathbf{U} \end{bmatrix}$$

Let

$$\mathbf{X} = \begin{bmatrix} \underline{x}_{A1} & \mathbf{X}_{A2} \\ \underline{x}_{B1} & \mathbf{X}_{B2} \end{bmatrix} = \begin{bmatrix} \underline{x}_{A11} & \mathbf{X}_{A12} \\ \underline{x}_{A21} & \mathbf{X}_{A22} \\ \underline{x}_{B11} & \mathbf{X}_{B12} \\ \underline{x}_{B21} & \mathbf{X}_{B22} \end{bmatrix}$$

Maximal Invariant consists of seven terms

$$z_{1A}(\mathbf{X}) = \underline{u}_A^T \mathbf{D}_A^{-1} \underline{u}_A, \qquad z_{1B}(\mathbf{X}) = \underline{u}_B^T \mathbf{D}_B^{-1} \underline{u}_B$$

$$z_{2A}(\mathbf{X}) = \underline{x}_{A21}^T [\mathbf{X}_{A22}\mathbf{X}_{A22}^T]^{-1} \underline{x}_{A21}, \quad z_{2B}(\mathbf{X}) = \underline{x}_{B21}^T [\mathbf{X}_{B22}\mathbf{X}_{B22}^T]^{-1} \underline{x}_{B21}$$

$$z_{3A}(\mathbf{X}) = \frac{\underline{u}_A \underline{u}_A^T}{\|\underline{u}_A\|^2}, \qquad z_{3B}(\mathbf{X}) = \frac{\underline{u}_B \underline{u}_B^T}{\|\underline{u}_B\|^2}$$

$$z_4(\mathbf{X}) = \frac{\underline{u}_A}{\underline{u}_B}$$

Partially Known (*Structured*) Clutter

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_A & \mathbf{O} \\ \mathbf{O} & \mathbf{R}_B \end{bmatrix}$$

- Case 1. : $\mathbf{R}_A > 0$, $\mathbf{R}_B > 0$
- Case 2. : $\mathbf{R}_A > 0$, $\mathbf{R}_B = \sigma^2\mathbf{I}$, $\sigma^2 > 0$
- Case 3. : $\mathbf{R}_A > 0$, $\mathbf{R}_B = \mathbf{I}$

$$\mathbf{X} = a \begin{bmatrix} \underline{s}_A \\ \underline{s}_B \end{bmatrix} \underline{e}_1^T + \begin{bmatrix} \mathbf{N}_A \\ \mathbf{N}_B \end{bmatrix}$$

GLR Test Statistics

$\mathbf{R}_A$	$\mathbf{R}_B$	Log GLR : $\frac{1}{n}\ln\Lambda = \max_a\{\cdot\}$
?	?	$\ln\left[\frac{1+p(0,\underline{s}_A,\mathbf{X}_A)}{1+p(a,\underline{s}_A,\mathbf{X}_A)}\right] + \ln\left[\frac{1+p(0,\underline{s}_B,\mathbf{X}_B)}{1+p(a,\underline{s}_B,\mathbf{X}_B)}\right]$
?	$\sigma^2\mathbf{I}$	$\ln\left[\frac{1+p(0,\underline{s}_A,\mathbf{X}_A)}{1+p(a,\underline{s}_A,\mathbf{X}_A)}\right] + m_B \cdot \ln\left[\frac{q(0,\underline{s}_B,\mathbf{X}_B)}{q(a,\underline{s}_B,\mathbf{X}_B)}\right]$
?	$\mathbf{I}$	$\ln\left[\frac{1+p(0,\underline{s}_A,\mathbf{X}_A)}{1+p(a,\underline{s}_A,\mathbf{X}_A)}\right] + \frac{1}{n}[q(0,\underline{s}_B,\mathbf{X}_B) - q(a,\underline{s}_B,\mathbf{X}_B)]$

where m_B = number of rows in $\mathbf{X}_B$, n = number of columns in $\mathbf{X}_B$, and

$$p(a,\underline{s}_A,\mathbf{X}_A) = (\underline{x}_{A1} - a\underline{s}_A)^H (\mathbf{X}_{A2}\mathbf{X}_{A2}^H)^{-1} (\underline{x}_{A1} - a\underline{s}_A)$$

$$q(a,\underline{s}_B,\mathbf{X}_B) = tr\left\{(\mathbf{X}_B - a\underline{s}_B\underline{e}_1^T)^H (\mathbf{X}_B - a\underline{s}_B\underline{e}_1^T)\right\}$$

MI Test Statistics

$\mathbf{R}_A$	$\mathbf{R}_B$	MI Test : $T = \dfrac{\left| \left[\underline{s}_A^H \; \underline{s}_B^H \right] \begin{bmatrix} \mathbf{K}_A & \mathbf{O} \\ \mathbf{O} & \mathbf{K}_B \end{bmatrix}^{-1} \begin{bmatrix} \underline{x}_{A1} \\ \underline{x}_{B1} \end{bmatrix} \right|^2}{\left[\underline{s}_A^H \; \underline{s}_B^H \right] \begin{bmatrix} \mathbf{K}_A & \mathbf{O} \\ \mathbf{O} & \mathbf{K}_B \end{bmatrix}^{-1} \begin{bmatrix} \underline{s}_A \\ \underline{s}_B \end{bmatrix}}$
?	?	$\mathbf{K}_A = q_A \mathbf{X}_{A2} \mathbf{X}_{A2}^H, \quad \mathbf{K}_B = q_B \mathbf{X}_{B2} \mathbf{X}_{B2}^H$
?	$\sigma^2 \mathbf{I}$	$\mathbf{K}_A = q_A \mathbf{X}_{A2} \mathbf{X}_{A2}^H, \quad \mathbf{K}_B = v_2 \mathbf{I}$
?	$\mathbf{I}$	$\mathbf{K}_A = q_A \mathbf{X}_{A2} \mathbf{X}_{A2}^H, \quad \mathbf{K}_B = v_3 \mathbf{I}$

where

$$q_A = 1 + \underline{x}_{A1}^H (\mathbf{X}_{A2} \mathbf{X}_{A2}^H)^{-1} \underline{x}_{A1} \quad , \quad v_2 = \frac{1}{m_B} \mathrm{tr}\{\mathbf{X}_B^H \mathbf{X}_B\}$$

$$q_B = 1 + \underline{x}_{B1}^H (\mathbf{X}_{B2} \mathbf{X}_{B2}^H)^{-1} \underline{x}_{B1} \quad , \quad v_3 = n$$

$\mathbf{R}_A$	$\mathbf{R}_B$	MI Tests in the Maximal Invariant Form
?	?	$T_1 = \frac{z_{A1}}{1+z_{A1}+z_{A2}} + \frac{z_{B1}}{1+z_{B1}+z_{B2}} - coupling$
?	$\sigma^2\mathbf{I}$	$T_2 = \frac{z_{A1}}{1+z_{A1}+z_{A2}} + m_B \cdot \frac{\lvert x_{B11}\rvert^2}{\mathrm{tr}\{\mathbf{X}_B^H \mathbf{X}_B\}} - coupling$
?	$\mathbf{I}$	$T_3 = \frac{z_{A1}}{1+z_{A1}+z_{A2}} + \frac{1}{n} \cdot \lvert x_{B11}\rvert^2 - coupling$

- Case 1. : Structured Kelly's test

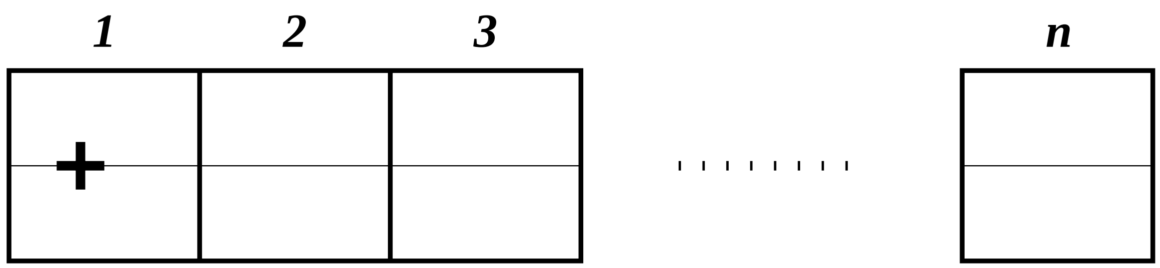
$$T_{Ks} = \frac{z_{A1} + z_{B1} - coupling}{1 + z_{A1} + z_{A2} + z_{B1} + z_{B2}}$$

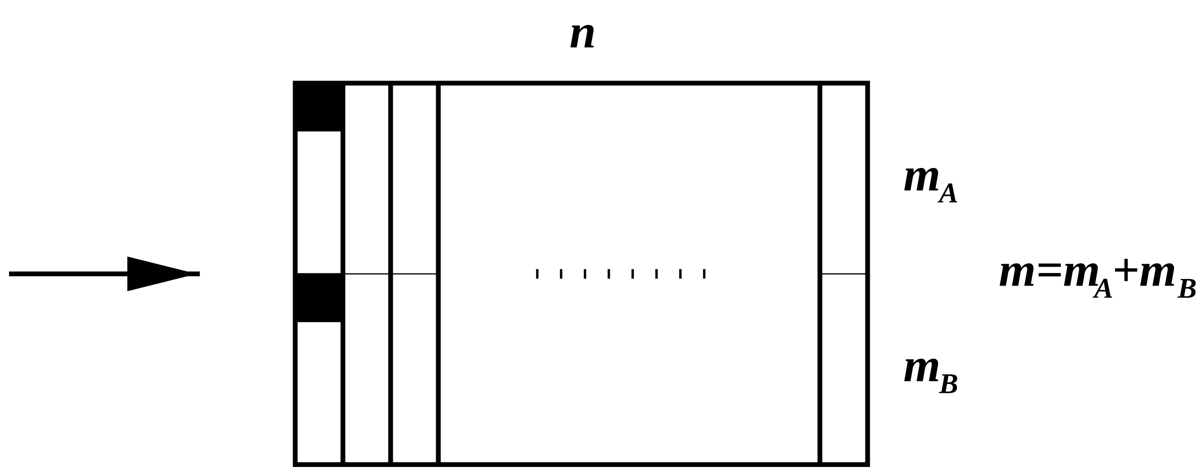
- Case 2. : Bose and Steinhardt MI test

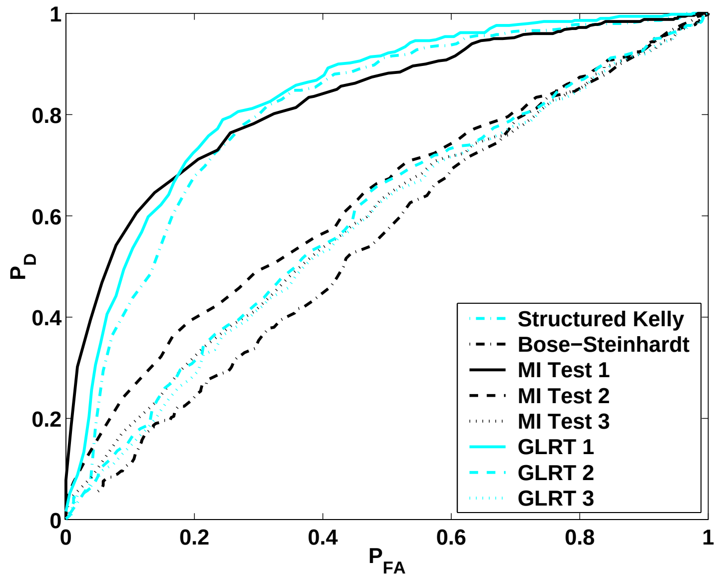
$$T_{BS} = (n - m_A) z_{A1} (1 + z_{A2}) + \frac{(m_B n - 1)\lvert x_{B11}\rvert^2}{\mathrm{tr}\{\mathbf{X}_B^H \mathbf{X}_B\} - \lvert x_{B11}\rvert^2} - coupling$$

Simulation : ROC performance (P_D vs. P_{FA})

- Case 1. : GLRT 1, MI Test 1, Structured Kelly
- Case 2. : GLRT 2, MI Test 2, Bose–Steinhardt
- Case 3. : GLRT 3, MI Test 3

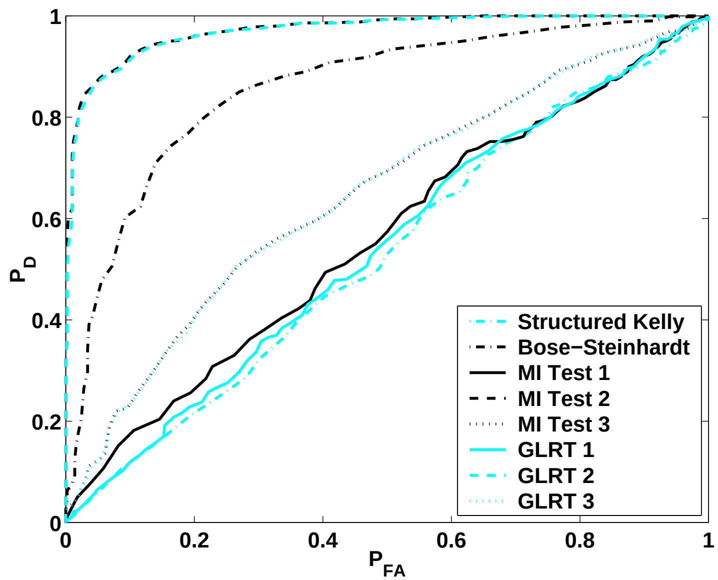


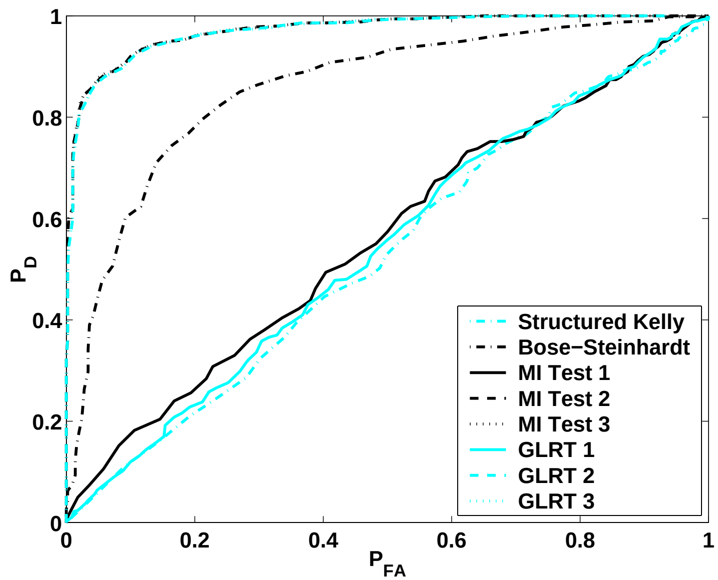




Case 1.

$$SNR = 22dB\ (SNR_A = 11dB\ /\ SNR_B = 22dB)$$
$$m_A = 50\ /\ m_B = 50\ ,\ n = 51$$



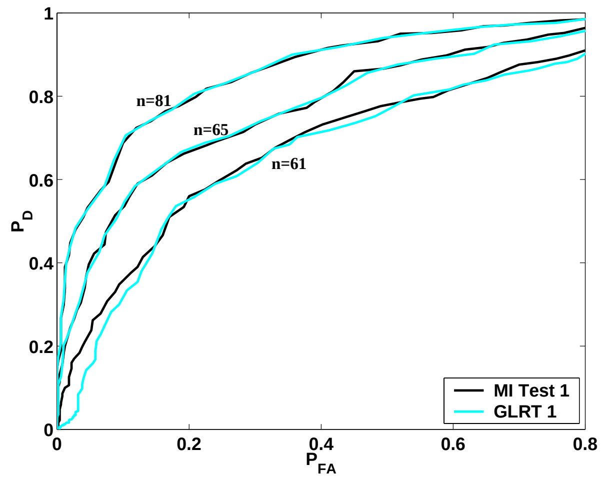


Case 2. $(\sigma^2 = 0.1)$

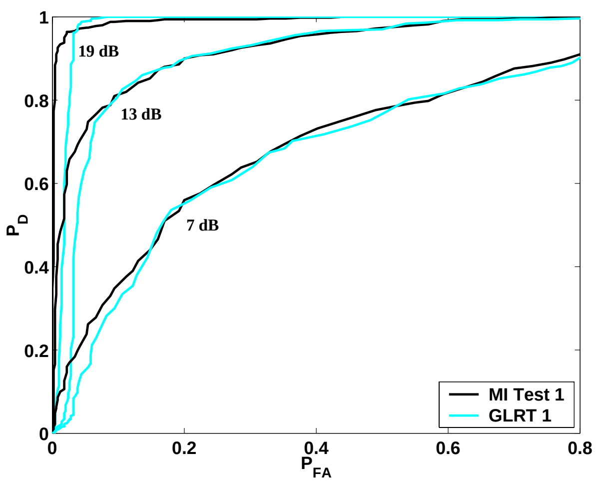
Case 3. $(\sigma^2 = 1)$

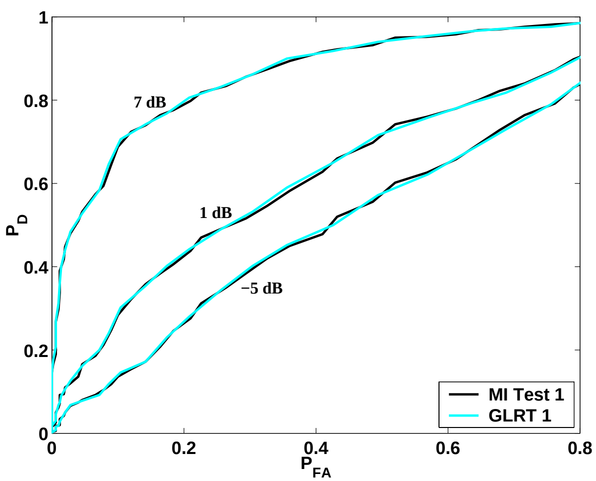
$$SNR = 10dB\ (SNR_A = 3dB\ /\ SNR_B = 8dB)$$

$$m_A = 40\ /\ m_B = 60\ ,\ n = 61$$



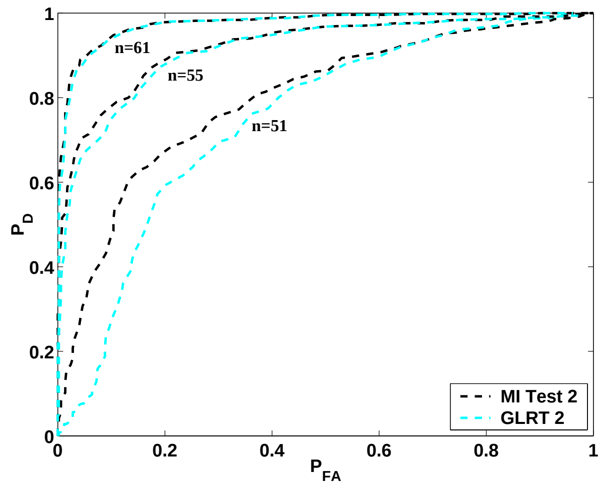
Case 1. ($SNR = 7dB$)



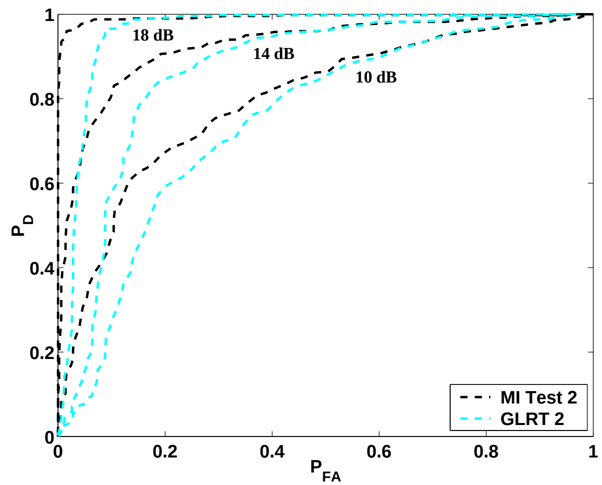


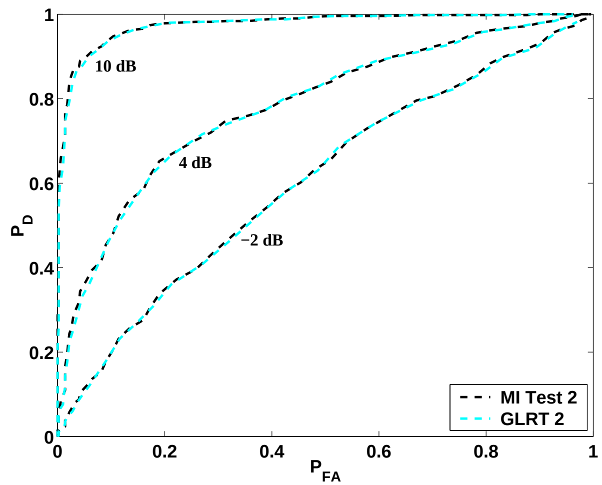
Case 1. ($n = 61$)

Case 1. ($n = 81$)



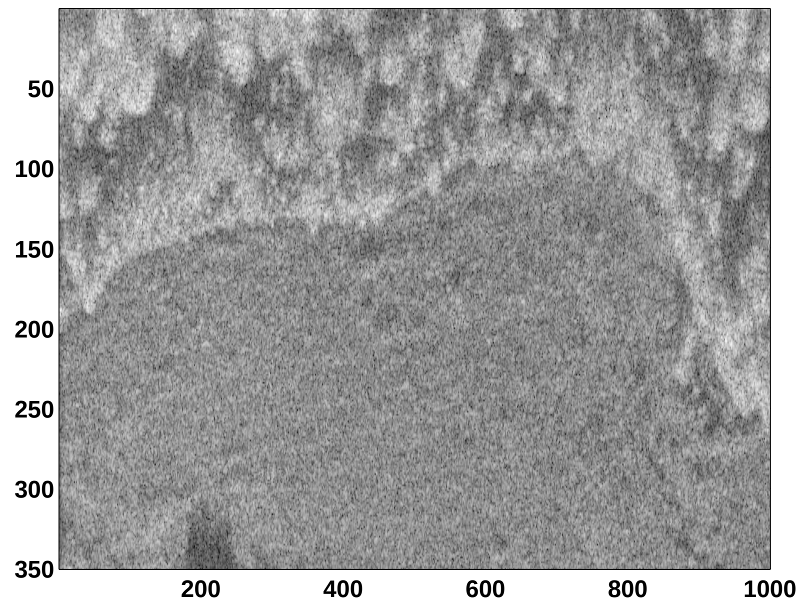
Case 2. $(SNR = 10dB)$



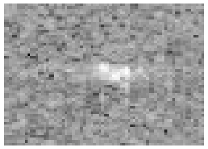


Case 2. $(n = 51)$

Case 2. $(n = 61)$



SAR Clutter image with a target in the boundary at column 300



Target image at azimuth = 163° and elevation = 39°

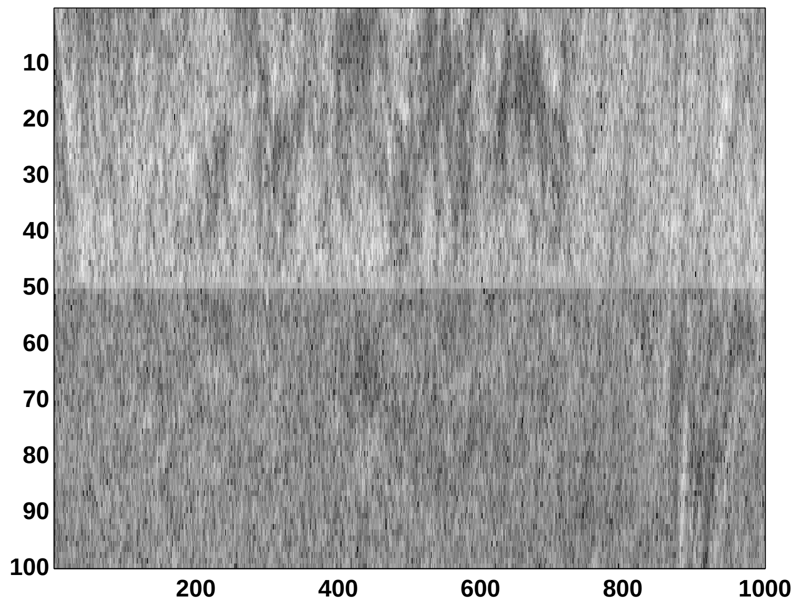
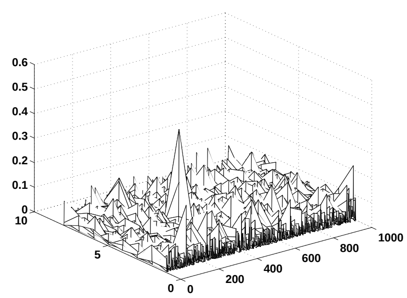
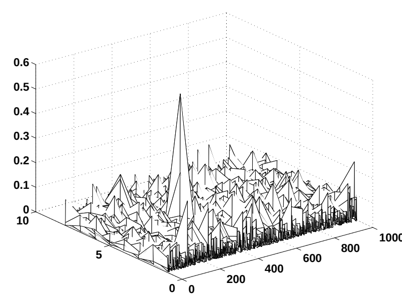


Image realigned along the extracted boundary

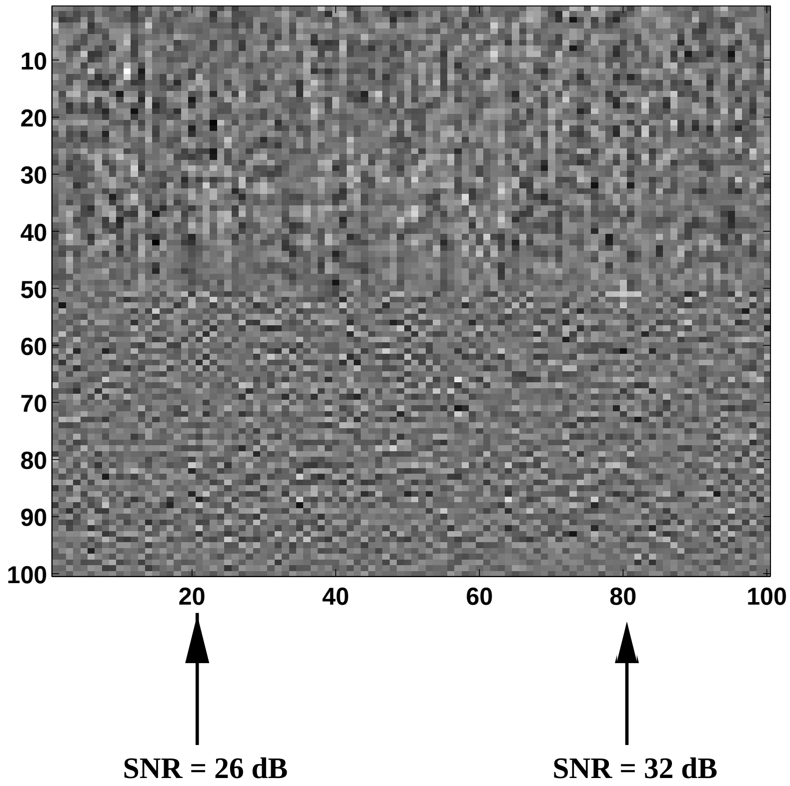




MI test values

GLR test values

Typical Realizations



CONCLUSION

- Detection algorithms for inhomogeneous clutter
  - GLR extended to case of block structured covariance
  - MI test proposed as an alternative
- GLR/MI significantly outperform existing tests (Kelly's test, Bose and Steinhardt's test)
- GLR and MI tests are complementary:
  1. GLRT is asymptotically optimal (UMP) for large n.
  2. MI test ensures robust detection.
  3. For small n and low SNR: MI outperforms GLRT for low P_{FA}
- Suggests hybrid GLR/MI for optimal performance
- Sensitivity to boundary errors $\Rightarrow$ need reliable boundary estimates.