CFAR TARGET DETECTION IN IMAGING RADAR

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Reed Symposium
Nov. 6, 1998

Selected Imaging Contributions of Irving Reed

Image texture via fBM models:

"Spectral representations of fBM in n dimensions and its properties", Reed, Lee, Truong, IEEE T-IT, Sept. 1995

Reconstruction from projections:

"Generalized $|\omega|$-filter for 3-D reconstruction," Kwoh, Reed, Truong, IEEE T-NS, Oct. 1977

"3-D reconstruction for diverging X-ray beams," Reed, Truong, Chang, Kwoh, IEEE T-NS, June 1978

"Modified reconstruction filter for diverging X-ray beams," Truong, Reed, Jonckheere, Kwoh, IEEE T-BME, Jul 1983

CFAR radar imaging:

"A detection algorithm for optical targets in clutter," Chen, Reed, IEEE T-AES, Jan. 1987

"Adaptive multiple band CFAR detection of an optical pattern with unknown spectral distribution," Reed, Yu, IEEE T-SP, Oct 1990

"Comparative performance analysis of adaptive multi-spectral detectors," Yu, Reed, IEEE T-SP, Aug. 1993

"Adaptive detection of signals with linear feature mappings and representations," Yu, Reed, IEEE T-SP, Dec. 1995

"Automatic target detection and recognition in multi-band imagery: a unified ML detection and estimation approach," Yu, Hoff, Reed, Chen, Stotts, IEEE T-IP, Jan. 1997

Detection of Deep Hide Targets

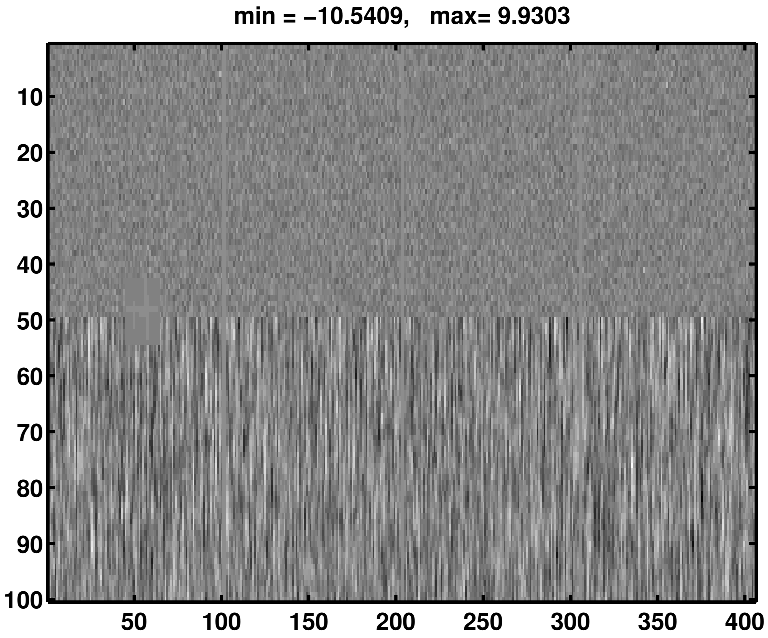


Figure 1: Known deep hide target on clutter boundary

Objective: Adaptive target detection in structured unknown clutter.

Methods: GLR and maximal invariant tests for structured MANOVA

Statistical measurement model:

$$\begin{aligned}\mathbf{X} &= \sum_{i=1}^{p} a_i \, \underline{s}_i \underline{b}_i^T + \mathbf{N} \\ &= \mathbf{S} \, \mathrm{a} \, \mathbf{B} + \mathbf{N}\end{aligned}$$

- $\mathbf{S} = [\underline{s}_1, \ldots, \underline{s}_p] = m \times p$ matrix of known spatial target signatures
- $\mathbf{B} = p \times n$ known temporal target signatures
- a = unknown $p \times p$ matrix
- $\mathrm{vec}\{\mathbf{N}\} \sim \mathcal{N}_{mn}(\underline{0}, \mathbf{R} \otimes \mathbf{I}_n)$
- $\mathbf{R}$ unknown **block diagonal** spatial clutter covariance

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_A & O \\ O & \mathbf{R}_B \end{bmatrix}$$

Target detection problem

Define $\underline{\theta} = \{\mathrm{a},\ \mathbf{R}\}$

$$
\begin{aligned}
H_0 &: \underline{\theta} \in \Theta_0 \stackrel{\mathrm{def}}{=} \{\mathrm{a} = 0,\ \mathbf{R} > 0\} \\
H_1 &: \underline{\theta} \in \Theta_1 \stackrel{\mathrm{def}}{=} \{\mathrm{a} \neq 0,\ \mathbf{R} > 0\}
\end{aligned}
$$

Two approaches can be considered:

1. **GLR Test**:

$$
\Lambda(\mathbf{X}) = \frac{\max_{\underline{\theta} \in \Theta_1} f(\mathbf{X}|\ \underline{\theta})}{\max_{\underline{\theta} \in \Theta_0} f(\mathbf{X}|\ \underline{\theta})} \begin{array}{c} H_1 \\ > \\ < \\ H_0 \end{array} \eta
$$

2. **MPI test**: $\mathbf{Z} = g(\mathbf{X})$ is *maximal invariant*

$$
\Lambda(\mathbf{Z}) = \frac{f(\mathbf{Z}|\ \underline{\theta} \in \Theta_1)}{f(\mathbf{Z}|\ \underline{\theta} \in \Theta_0)} \begin{array}{c} H_1 \\ > \\ < \\ H_0 \end{array} \eta
$$

where η is selected to ensure:

$$P_{FA} = \alpha, \quad \forall \underline{\theta} \in \Theta_0$$

Single known target in homogeneous clutter

Multiple dwell acquisition model:

$$\mathbf{X} = a\ \underline{s}\ \underline{1}^T + \mathbf{N}$$

Target detection problem:

$$\Theta_0 = \{a = 0, \mathbf{R} > 0\}, \qquad \Theta_1 = \{a \neq 0, \mathbf{R} > 0\}$$

GLR Test (Yu&Reed&Stocker, SP93)

$$2/n\ \ln(\Lambda)\ =\ \frac{|\hat{M}_0|}{|\hat{M}_{\hat{a}\underline{s}}|}\ \begin{array}{c} H_1 \\ > \\ < \\ H_0 \end{array}\ \eta$$

$$\hat{M}_{\underline{u}} = \frac{1}{m}\Sigma([\mathbf{X}]_{*i} - \underline{u})([\mathbf{X}]_{*i} - \underline{u})^H, \quad \text{shifted correlation matrix}$$

Equivalent form: **minimum Mahalanobis distance test**

$$\ln\left(\frac{1 + q(0, \hat{R})}{1 + q(\hat{a}_0\underline{s}, \ \hat{R})}\right) \quad \begin{array}{c} H_1 \\ > \\ < \\ H_0 \end{array} \quad \eta$$

$$\hat{R} = \hat{M}_{\underline{\bar{x}}} = \frac{1}{m}\mathbf{X}\mathbf{X}^H - \underline{\bar{x}}\ \underline{\bar{x}}^H, \quad \text{sample covariance matrix}$$

- $q(\underline{u}, R)$ is quadratic form:

$$q(\underline{u}, R) = (\underline{\bar{x}} - \underline{u})^H \mathbf{R}^{-1}(\underline{\bar{x}} - \underline{u})$$

Single known target on clutter boundary

$\mathbf{R}_{AB}$	$\mathbf{R}_A$	$\mathbf{R}_B$	log GLR
?	?	?	$2/n \ \ln(\Lambda) = \ln\left(\frac{1+q(0,\hat{R})}{1+q(\underline{s}\hat{a}_0,\ \hat{R})}\right)$
0	?	?	$2/n \ \ln(\Lambda) = \ln\left(\frac{1+q(0,\hat{R}_A)}{1+q(\hat{a}_1\underline{s}_A,\ \hat{R}_A)}\right) + \ln\left(\frac{1+q(0,\hat{R}_B)}{1+q(\hat{a}_1\underline{s}_B,\ \hat{R}_B)}\right)$
0	$\sigma^2\mathbf{C}_A$	?	$2/n \ \ln(\Lambda) = \ln\left(\frac{1+q(0,\hat{R}_B)}{1+q(\hat{a}_2\underline{s}_B,\ \hat{R}_B)}\right) + \ln\left(\frac{q(0,C_A)}{q(\hat{a}_2\underline{s}_A,\ C_A)}\right)$
0	$\mathbf{C}_A$	?	$2/n \ \ln(\Lambda) = \ln\left(\frac{1+q(0,\hat{R}_B)}{1+q(\hat{a}_3\underline{s}_B,\ \hat{R}_B)}\right) + q(0,C_A) - q(\hat{a}_3\underline{s}_A,\ C_A)$

Table 1: GLR Test statistics for various block structured clutter covariance

- $\hat{a}_1, \ldots, \hat{a}_3$ are solutions to cubic equations

Homogeneous clutter: Group Invariance approach

:

Map to canonical form via $\mathbf{X} \to \mathbf{Q}_s^T \mathbf{X} \mathbf{Q}_b$:

$$\mathbf{X} = a \cdot \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \cdot [1, 0, \ldots, 0] + \mathbf{N}$$

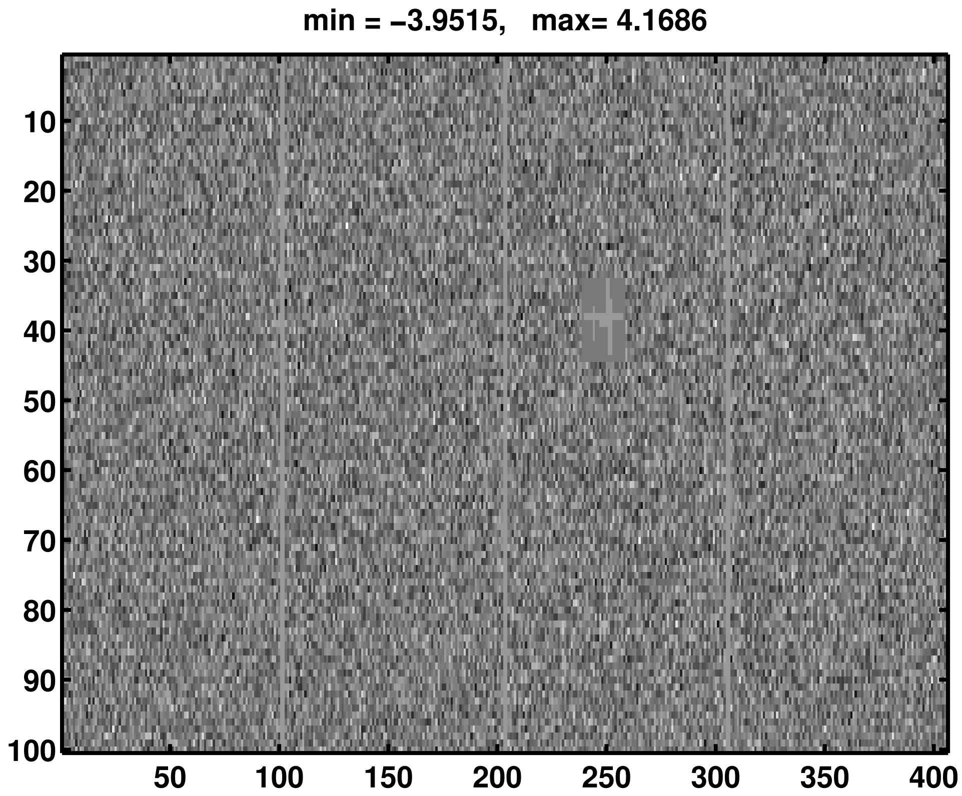


Figure 2: Target in unknown clutter

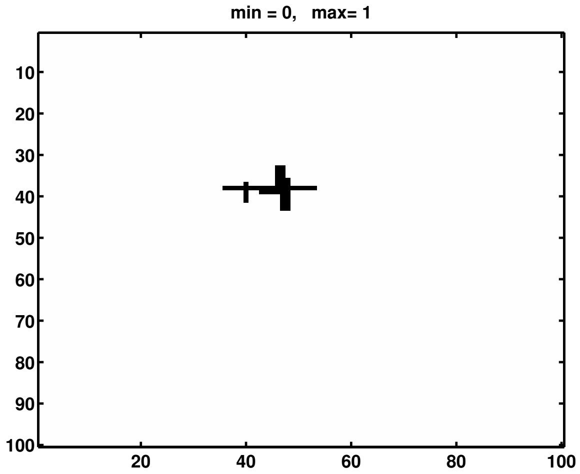


Figure 3: Target alone

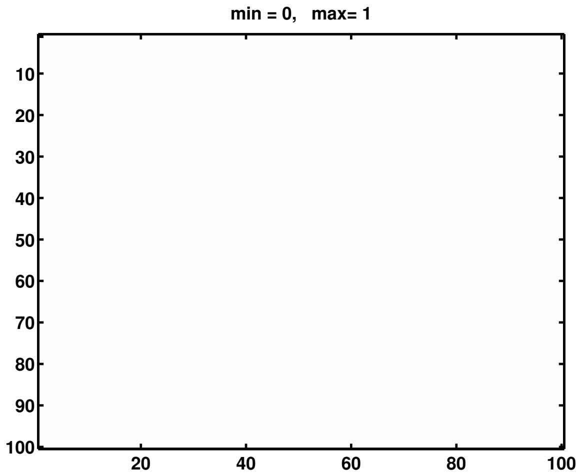


Figure 4: Transformed Target

$$\mathbf{X} = a \cdot \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \cdot [1, 0, \ldots, 0] + \mathbf{N}$$

Group of transformations leaving decision problem invariant:

$\Rightarrow \mathcal{G}$ has group action: $g(\mathbf{X}) = \mathbf{FXH}$,

$\mathbf{F}$ invertible $m \times m$ and $\mathbf{H}$ unitary $n \times n$

$$\mathbf{F} = \begin{bmatrix} \beta_1 & \underline{\beta}^T \\ \underline{0} & \mathbf{M} \end{bmatrix}, \qquad \mathbf{H} = \begin{bmatrix} 1 & \underline{0}^T \\ \underline{0} & \mathbf{U} \end{bmatrix}$$

Note: $g(\mathbf{X})$ obeys identical model under H_0 and H_1 as $\mathbf{X}$:

Let

$$\mathbf{X} = [\underline{x}_1, \mathbf{X}_2] = \begin{bmatrix} x_{11} & \underline{x}_{12} \\ \underline{x}_{21} & \mathbf{X}_{22} \end{bmatrix}$$

Maximal Invariant is two dimensional:

$$\begin{aligned} z_1(\mathbf{X}) &= \underline{x}_1^T [\mathbf{X}_2 \mathbf{X}_2^T]^{-1} \underline{x}_1 \\ z_2(\mathbf{X}) &= \underline{x}_{21}^T [\mathbf{X}_{22} \mathbf{X}_{22}^T]^{-1} \underline{x}_{21} \quad \text{multiple correlation factor - (Ancillary)} \end{aligned}$$

Note:

- LR for z_1, z_2 is invariant to changes in a, $\mathbf{R}^{-1}$
- LR test is CFAR
- Power depends on $\delta = a^2 \underline{s}^T \mathbf{R}^{-1} \underline{s}$.

Spatial prediction interpretation of maximal invariant

Using partioned matrix inverse identities, can show that

$$z_1(\mathbf{X}) - z_2(\mathbf{X}) = \frac{\left| x_{11} - \underline{x}_{12}\mathbf{X}_{22}^T[\mathbf{X}_{22}\mathbf{X}_{22}^T]^{-1}\underline{x}_{21} \right|^2}{\underline{x}_{12}\left[\mathbf{I} - \mathbf{X}_{22}^T[\mathbf{X}_{22}\mathbf{X}_{22}^T]^{-1}\mathbf{X}_{22}\right]\underline{x}_{12}^T}$$

$$= \frac{\left| x_{11} - \hat{x}_{11|21} \right|^2}{\|\underline{x}_{12} - \hat{\underline{x}}_{12|22}\|^2} \qquad (\underline{s}\text{-prediction SNR})$$

$$= \frac{|\underline{x}_1^H \hat{\mathbf{M}}_{22}^{-1} \underline{e}_1|^2}{\underline{e}_1^T \hat{\mathbf{M}}_{22}^{-1} \underline{e}_1} \qquad (\text{AMF})$$

Now: GLR can be represented by:

$$\Lambda(\mathbf{X}) = \frac{z_1 - z_2}{1 + z_1} = \frac{\text{AMF}}{\text{AMF} + \text{ANC} + 1}$$

Target at unknown position/pose on clutter boundary

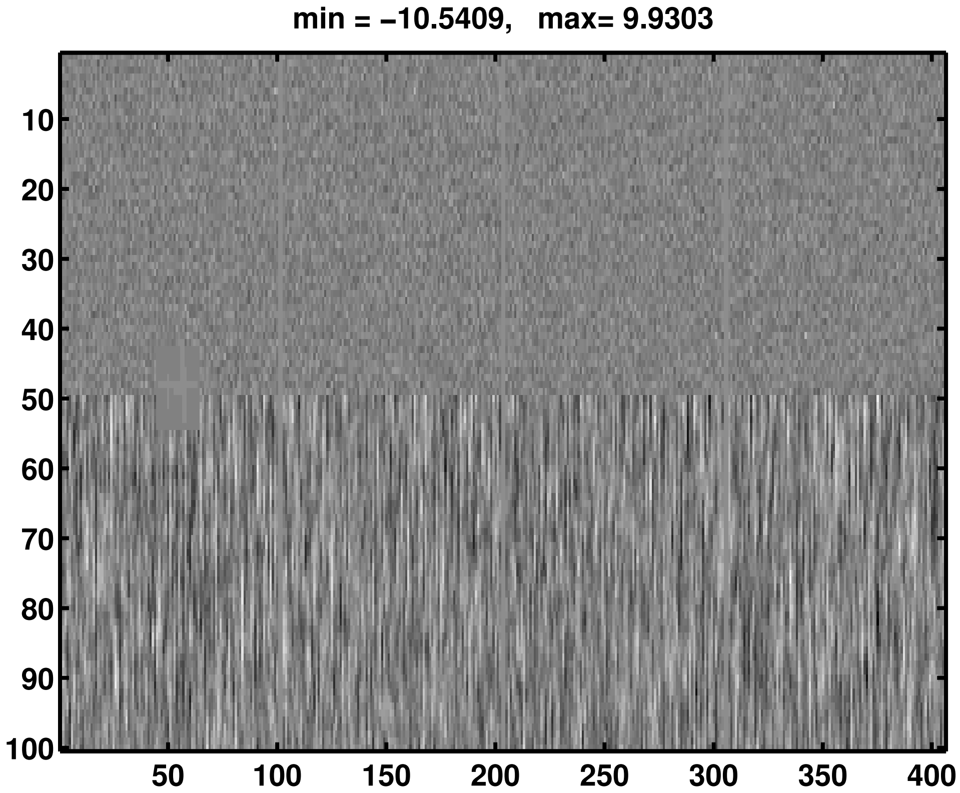


Figure 5: Deep hide target on clutter boundary

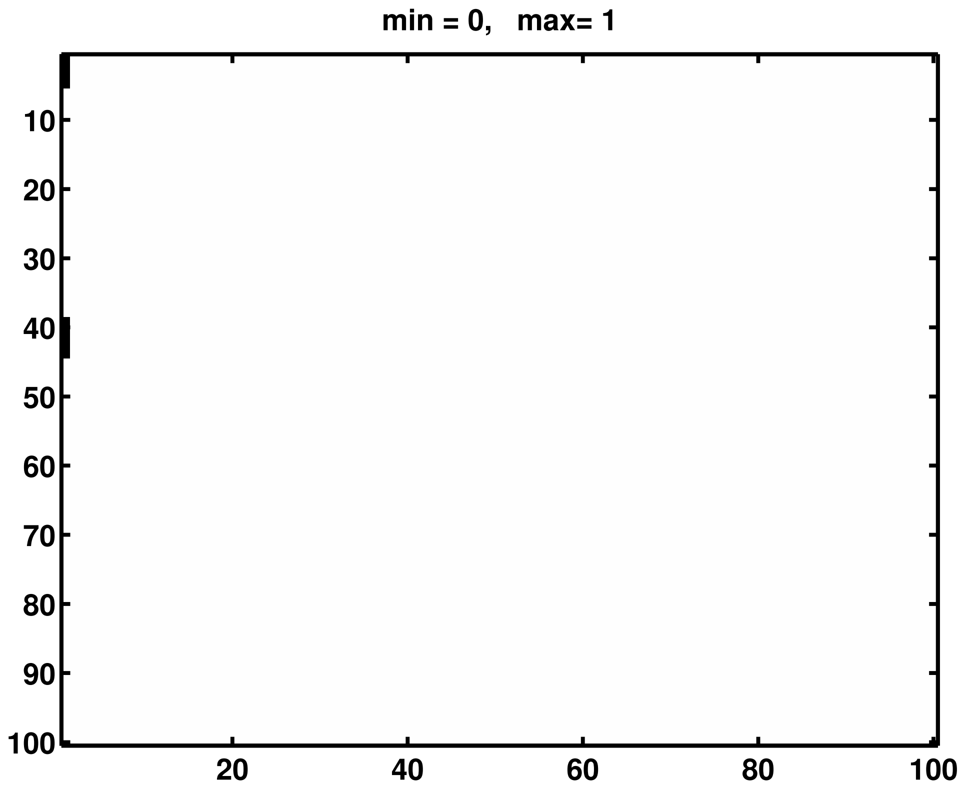


Figure 6: Transformed target matrix

Canonical form:

$$\tilde{\mathbf{X}} = a \begin{bmatrix} \underline{e} \\ \underline{0} \\ \underline{e} \\ \underline{0} \end{bmatrix} \cdot [1, 0, \ldots, 0] + \begin{bmatrix} \mathbf{N}_A \\ \mathbf{N}_B \end{bmatrix}$$

$$\Theta_0 = \{a = 0, \mathbf{R}_A > 0, \mathbf{R}_B > 0\}, \qquad \Theta_1 = \{a \neq 0, \mathbf{R}_A > 0, \mathbf{R}_B > 0\}$$

$\mathcal{G}$ has group action: $g(\tilde{\mathbf{X}}) = \mathbf{F}\tilde{\mathbf{X}}\mathbf{H}$,

$$\mathbf{F} = \begin{bmatrix} \begin{matrix} \mathbf{D} & \mathbf{G}_A \\ \underline{0} & \mathbf{\Gamma}_A \end{matrix} & \mathrm{O} \\ \mathrm{O} & \begin{matrix} \mathbf{D} & \mathbf{G}_B \\ \underline{0} & \mathbf{\Gamma}_B \end{matrix} \end{bmatrix} \qquad \mathbf{H} = \begin{bmatrix} 1 & \underline{0}^T \\ \underline{0} & \mathbf{U} \end{bmatrix}$$

Maximal Invariant

$$z_{1A}(\tilde{\mathbf{X}}) = \underline{u}_A^H \mathbf{C}_A^{-1} \underline{u}_A, \qquad z_{1B}(\tilde{\mathbf{X}}) = \underline{u}_B^H \mathbf{C}_B^{-1} \underline{u}_B$$

(total s-prediction SNR's)

$$z_{2A}(\tilde{\mathbf{X}}) = \underline{x}_{A21}^H [\mathbf{X}_{A22}\mathbf{X}_{A22}^H]^{-1} \underline{x}_{A21}, \quad z_{2B}(\tilde{\mathbf{X}}) = \underline{x}_{B21}^H [\mathbf{X}_{B22}\mathbf{X}_{B22}^H]^{-1} \underline{x}_{B21}$$

(ancillary statistics)

$$z_{3A}(\mathbf{X}) = [\mathrm{diag}(\mathbf{C}_A)]^{-1} \; |\underline{u}_A|^2, \qquad z_{3B}(\mathbf{X}) = [\mathrm{diag}(\mathbf{C}_B)]^{-1} \; |\underline{u}_B|^2$$

(marginal s-prediction SNR's)

$$z_4(\mathbf{X}) = \underline{u}_A \; ./ \; \underline{u}_B$$

(relative amplitude predictor)

$$\underline{u} \;=\; \underline{x}_{11} - \mathbf{X}_{12}\mathbf{X}_{22}^T[\mathbf{X}_{22}\mathbf{X}_{22}^T]^{-1}\underline{x}_{21}$$

$$\mathbf{C} \;=\; \mathbf{X}_{12}[\mathbf{I}_{n-1} - \mathbf{X}_{22}^T[\mathbf{X}_{22}\mathbf{X}_{22}^T]^{-1}\mathbf{X}_{22}]\mathbf{X}_{12}^T$$

Proposed Test

$$\max_k \left\{ \frac{|\underline{s}_k^H \tilde{\mathbf{C}}^{-1} \underline{u}|^2}{\underline{s}_k^H \tilde{\mathbf{C}}^{-1} \underline{s}_k} \right\} \quad \begin{array}{c} H_1 \\ > \\ < \\ H_0 \end{array} \quad \eta$$

$$\underline{s}_k = \begin{bmatrix} \underline{e}_k \\ \underline{e}_k \end{bmatrix}, \; \underline{u} = \begin{bmatrix} \underline{u}_A \\ \underline{u}_B \end{bmatrix}, \; \tilde{\mathbf{C}} = \begin{bmatrix} (1+z_{1A})\mathbf{C}_A & O \\ O & (1+z_{1B})\mathbf{C}_B \end{bmatrix}$$

Numerical Example

Assumptions

- presegmented clutter regions
- MSTAR SLICY Target

Target detection algorithms implemented:

- Maximal invariant tests
- Mismatched GLR using global clutter covariance matrix R
- Block covariance constrained GLR's implemented with cubic rooting

Legends for graphs

- Reed(uns) is GLRT which ignores the boundary information and uses the full covariance matrix estimate.
- Reed(str) is the Reed(uns) test implemented with a block diagonal covariance estimator.
- Max-inv is maximal invariant test for known target which accounts for block diagonal clutter covariance.
- GLRT1 is GLR test which assumes unknown clutter covariance on each side of boundary
- GLRT2 is GLR test which assumes clutter on one side of boundary is known white ($\mathbf{R}_A$ = identity matrix).
- GLRT3 is GLR test which assumes clutter on one side of boundary is white with unknown power ($\mathbf{R}_A = \sigma^2\mathbf{I}$)

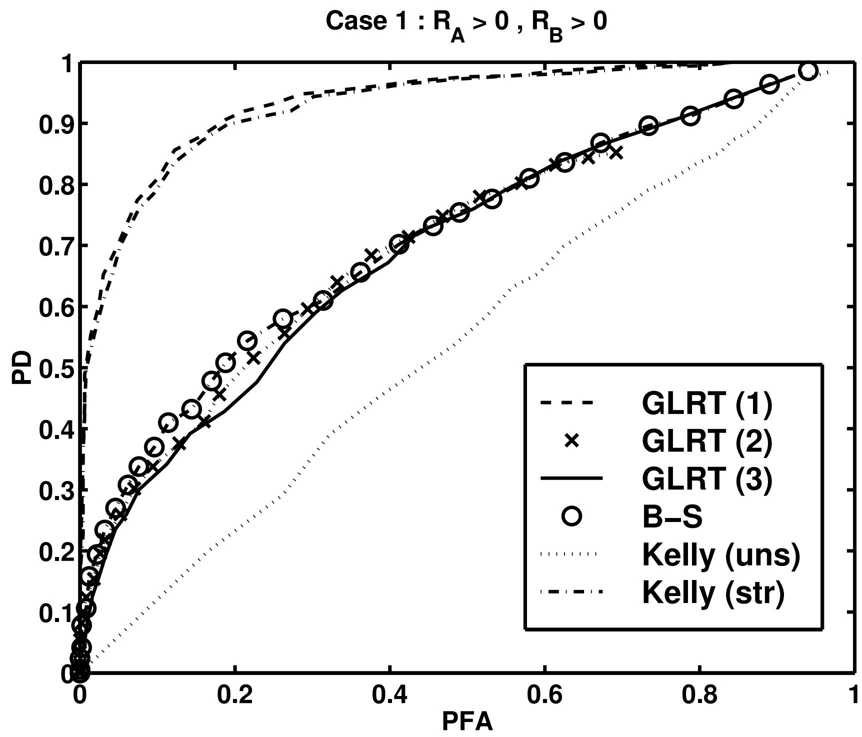


Figure 7: ROC comparisons for the case of a known target at known location on boundary between lowpass and highpass clutter regions. Note GLRT1 performs slightly better than Reed(uns).

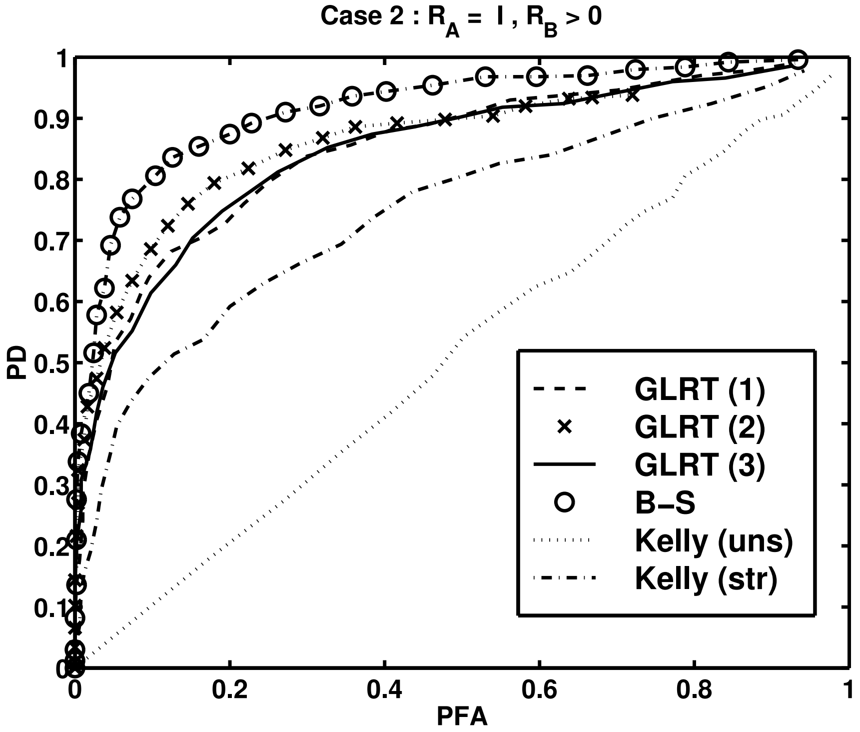


Figure 8: ROC comparisons for the case of a known target at known location on boundary between lowpass clutter region and a white clutter regionwith known power. Note B-S performs better than GLRT2.

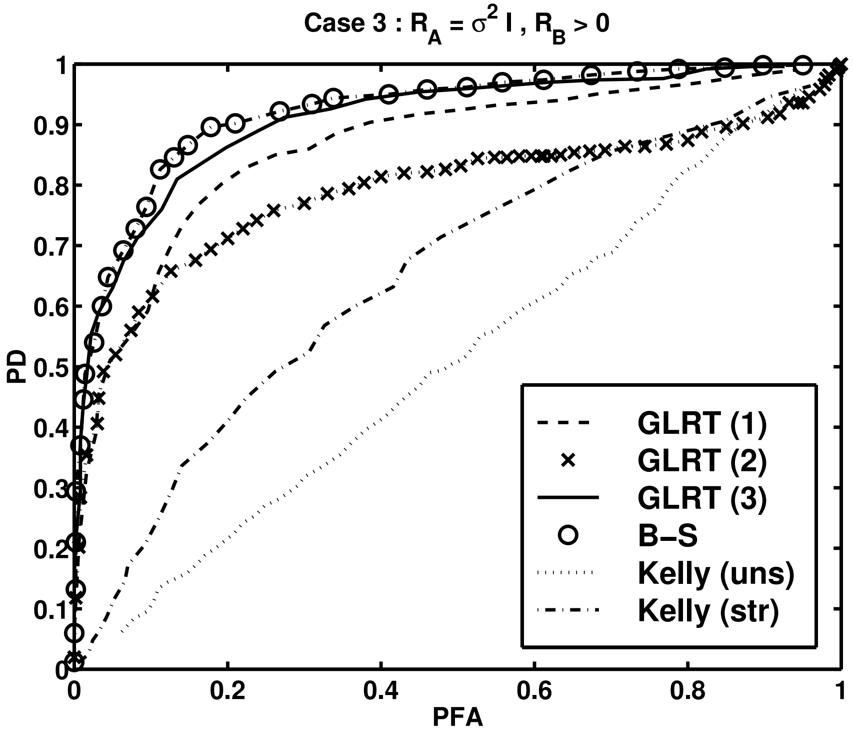
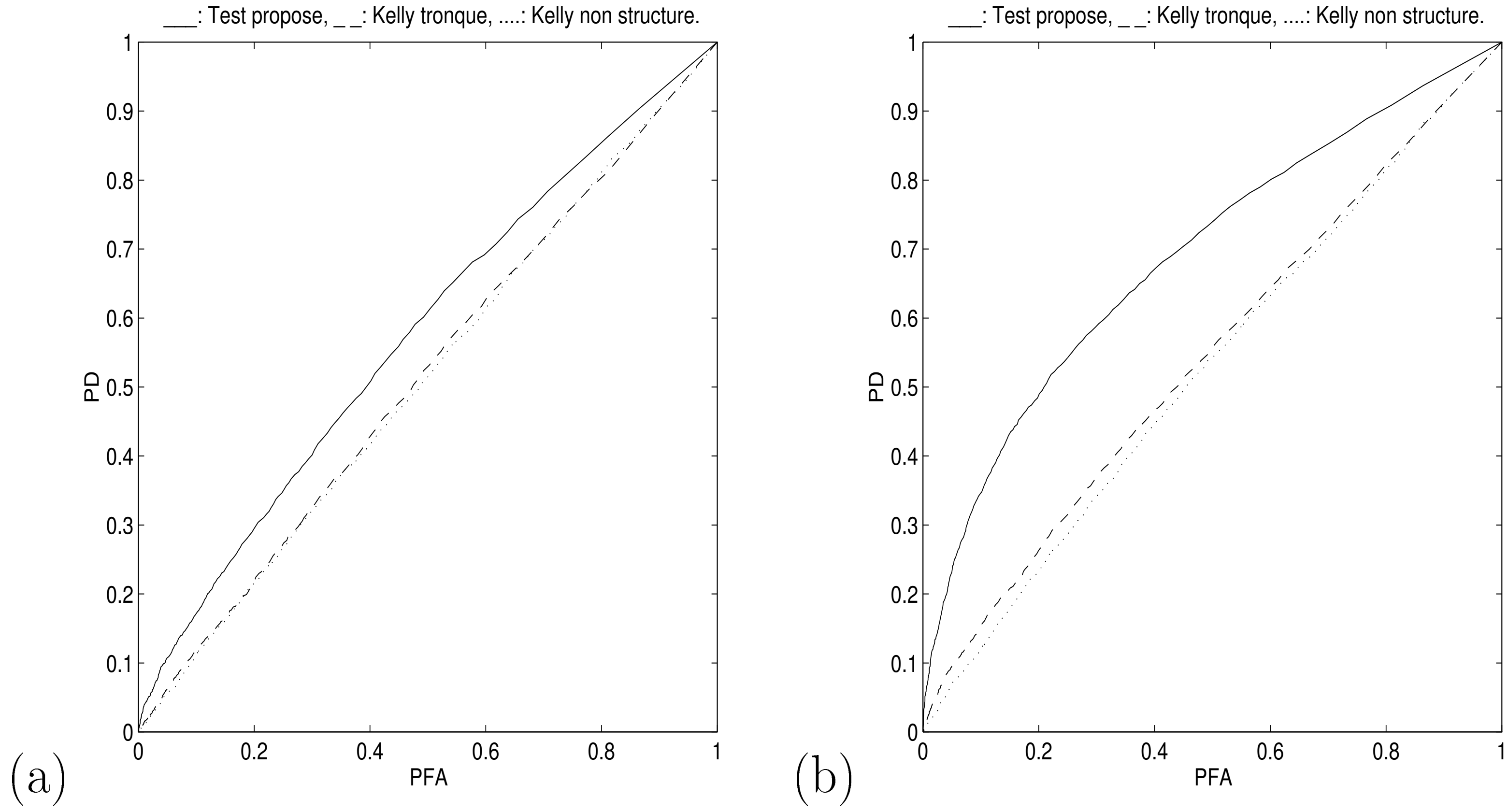


Figure 9: ROC comparisons for the case of a known target at known location on boundary between lowpass clutter region and a white clutter region with unknown power. Note Max-inv performs slightly better than GLRT3.

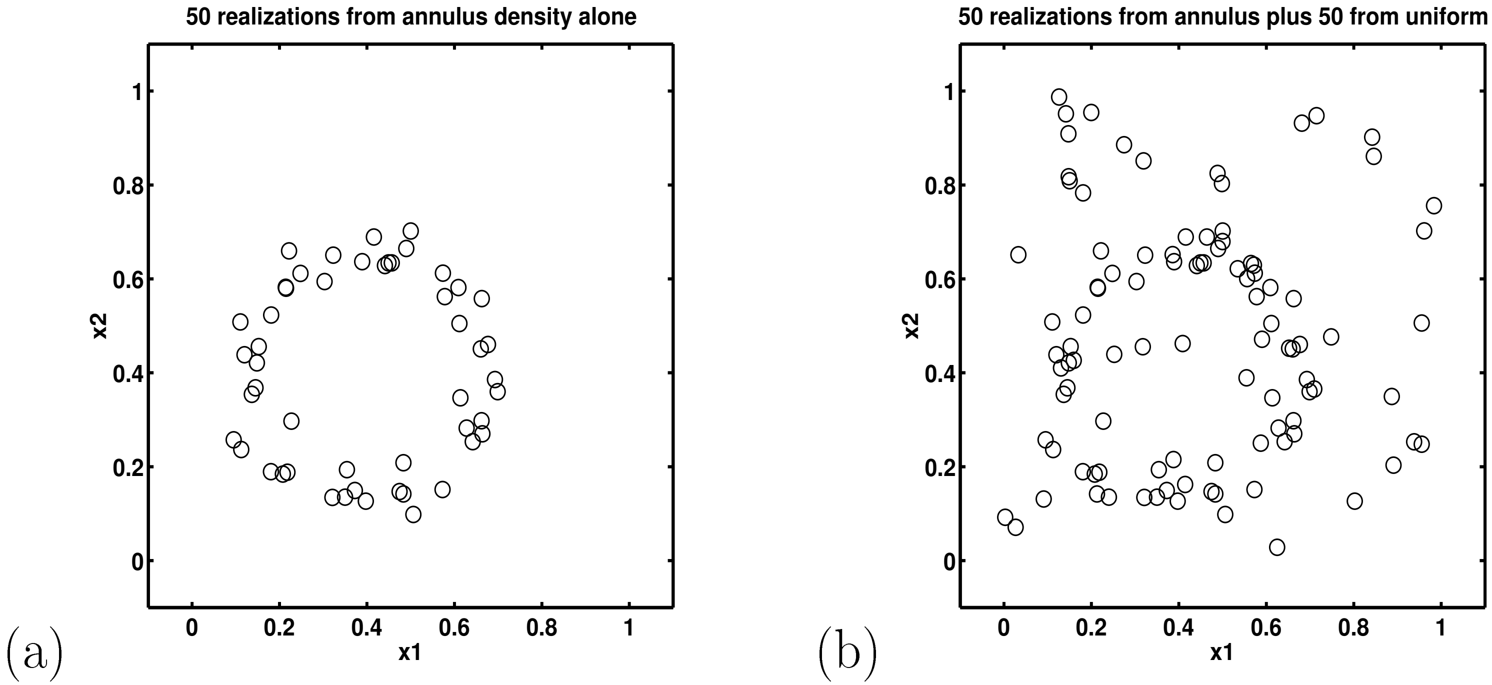


Single target at **unknown** location. ROC curves for Reed's GLRT (structured and unstructured) and new max invariant test: (a) SNR = 0dB, (b) SNR = 20dB (From Hero&Guillouet:ICIP97).

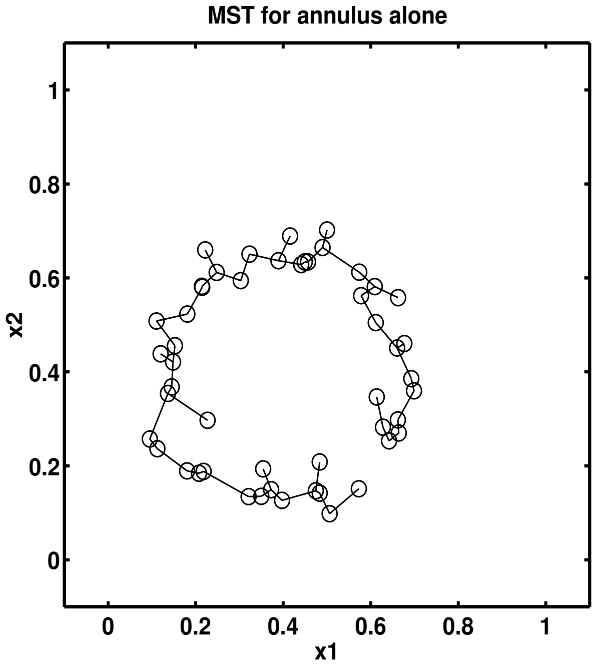
Extensions

- Sensitivity to boundary segmentation errors
- Multiple regions: b-block structured maximal invariant
- Non-Gaussian clutter: elliptical family of densities, heavy-tailed mixture models
- Unknown pose/location: **GLR via iterative optimization**

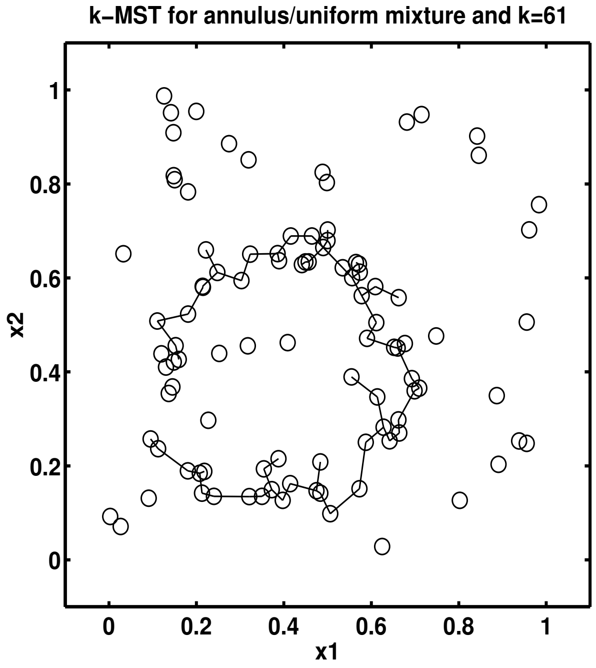
Theory and Application of k-shortest Graphs



Definition: MST over $\{x_i\}_{i=1}^n$ in $\mathbb{R}^n$ is **shortest tree spanning the** n **points**



(a)



(b)

Definition: k-shortest MST over $\{x_i\}_{i=1}^n$ in $\mathbb{R}^n$ is **shortest MST spanning any k points**

Applications:

- Robust entropy estimation - extension of Ahmad&Lin:IT76, Vasicek:JRSS76, Hall&Morton:AnnInstStatMath93
- Clustering/classification with false alarms - extension of Toussaint:PR80, Zahn:TC71, Banks:Interface92
- Robust image registration and two-sample matching - extension of Viola&Wells:MIT95, Friedman&Rafsky:AnnStat79)

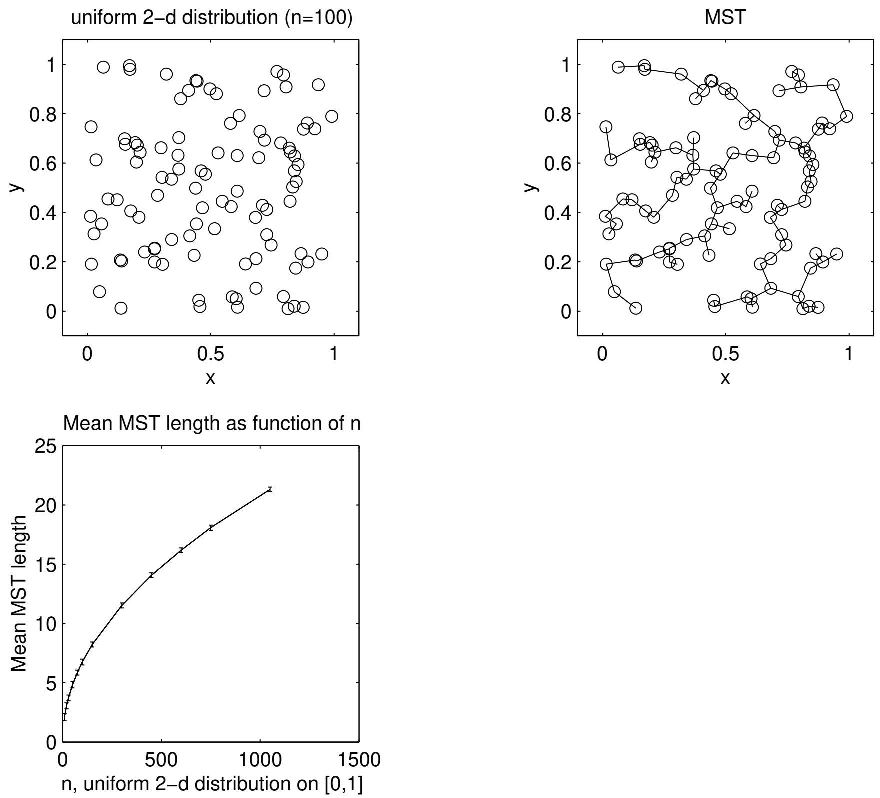


Figure 13: 2D Uniform sample study.

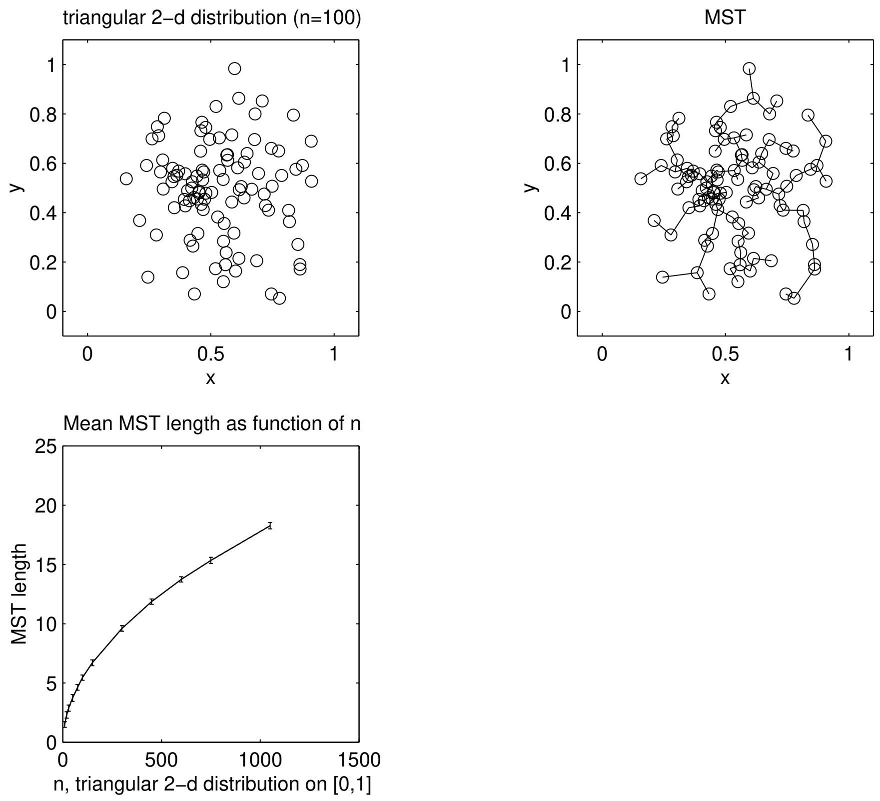


Figure 14: 2D Triangular sample study.

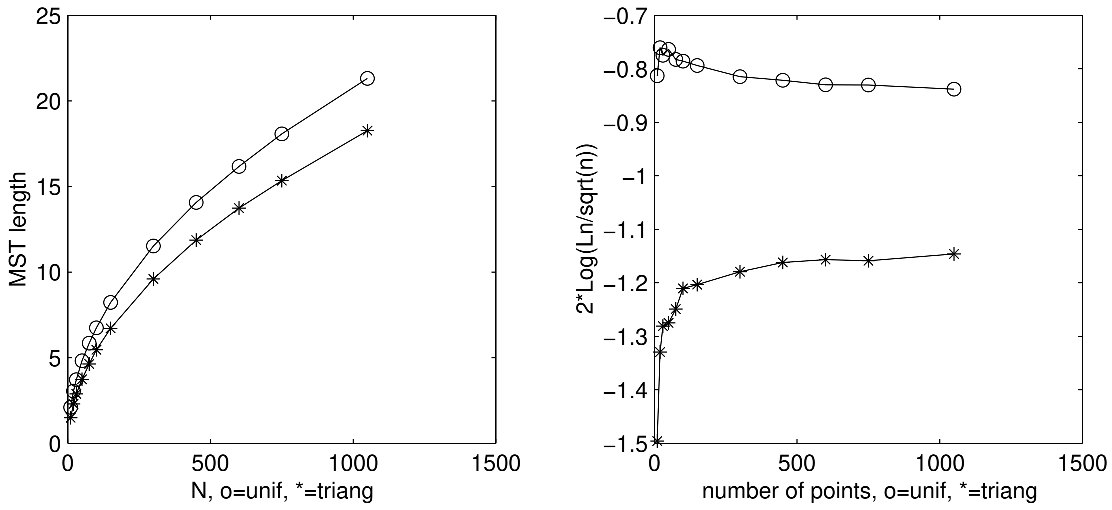


Figure 15: MST Length and log length comparisons.

Principal asymptotic result: extension of Steele:AnnAppProb1988, Redmond&Yukich:AnnAppProb94:

Theorem (Hero&Michel:IT98): For any $\alpha \in (0,1]$ the sum $L_{n,k} = \Sigma\,|e|^\gamma$ of the power-weighted edges of the k-MST satisfies

$$\frac{1}{1-\nu} \ln\left(L_{n,\lfloor \alpha n \rfloor}/(\lfloor \alpha n \rfloor)^\nu\right) \;\to\; \min_{A:P(A)\geq\alpha} R_\nu(X|A) + \ln \beta_{d,\gamma} \quad (a.s.)$$

where $R_\nu(X|A)$ is the Renyi entropy of the conditional distribution $f(x|X \in A)$ of order $\nu = (d-\gamma)/d$

$$R_\nu(X|A) = \frac{1}{1-\nu}\, \ln \int f^\nu(x|X \in A)dx.$$

Application to Image Registration

Minimum entropy property:

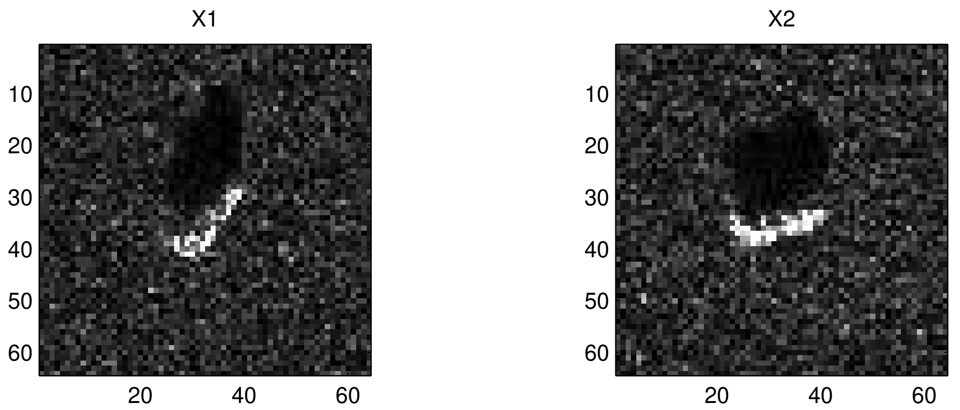
$$R_\nu([X,Y]) \geq R_\nu(X)$$

Two step registration process:

1. Select α, $\nu \in [0,1]$
2. Implement outlier rejection on X, Y via k-MST
3. Entropy minimization via MST over coordinate transformations on Y

Applications:

1. **MSTAR targets in clutter**
2. MRI brain image registration

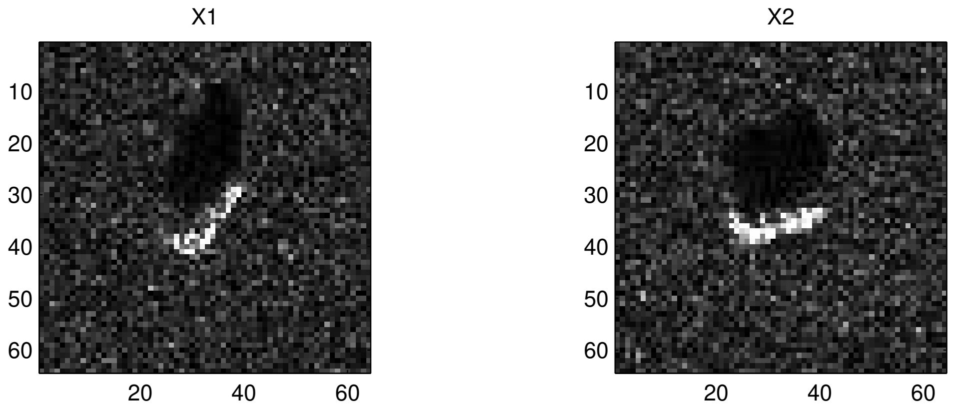


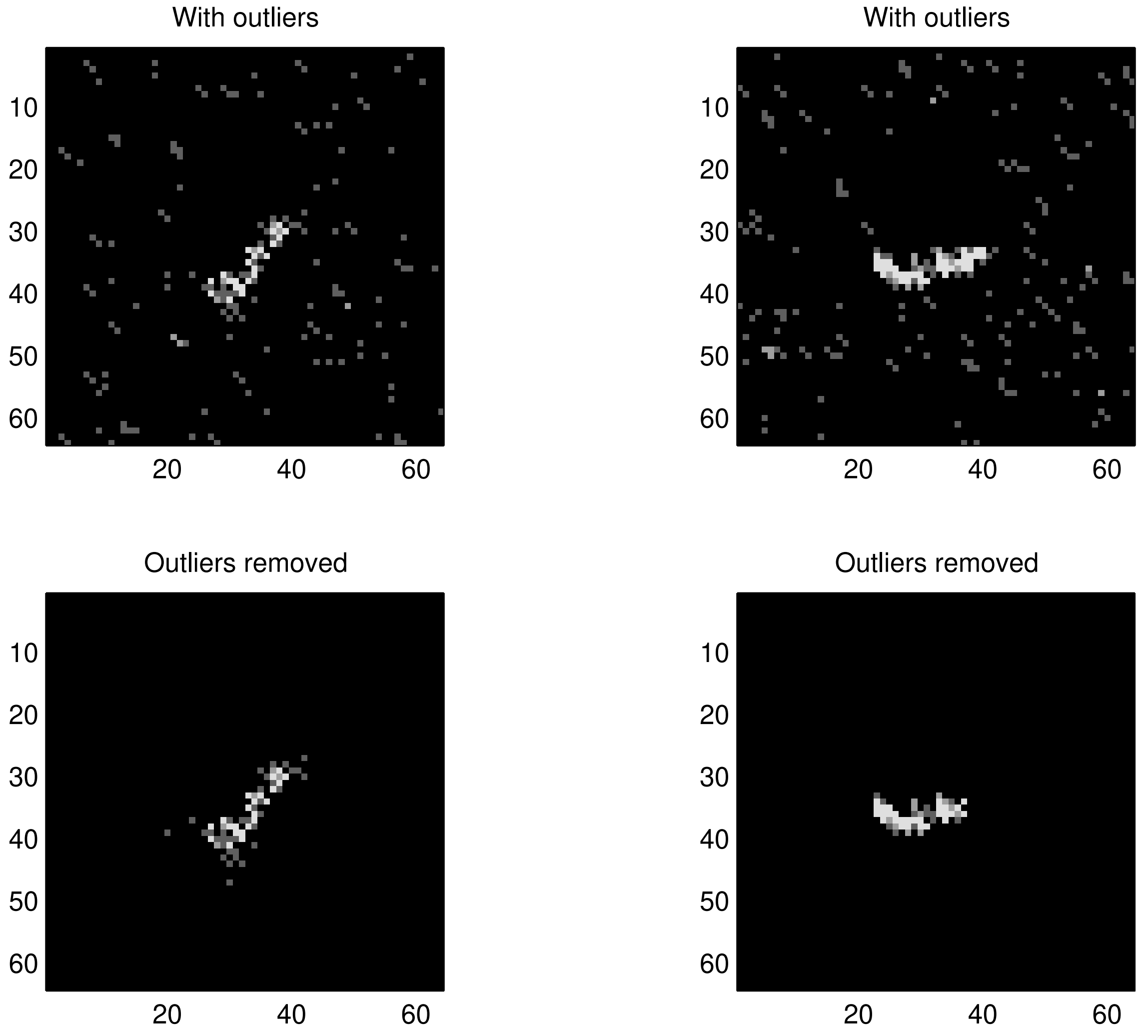
BTR-70 Personnel Carrier - Quadrant 3 ($180° - 270°$)

Targets at 214° and 255° (spotlight mode)

Depression angle 14.95° and 15.12°

Images from MBVLab's MSTAR Radar Image Gallery
(http://www.mbvlab.wpafb.af.mil/public/MBVDATA/img_intro.htm)


X1
X2
10
20
30
40
50
60
20
40
60


With outliers
With outliers
Outliers removed
Outliers removed
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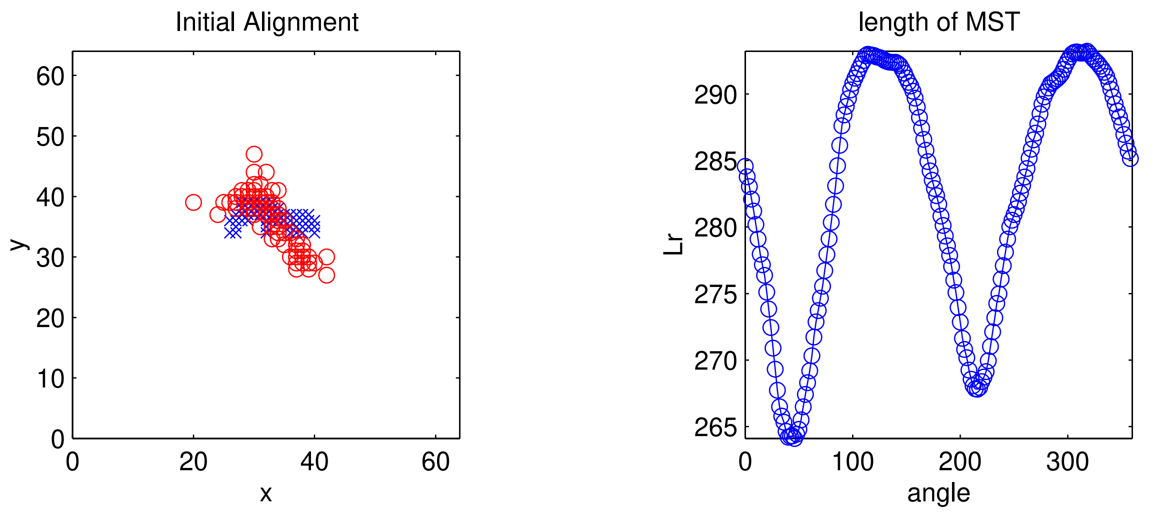
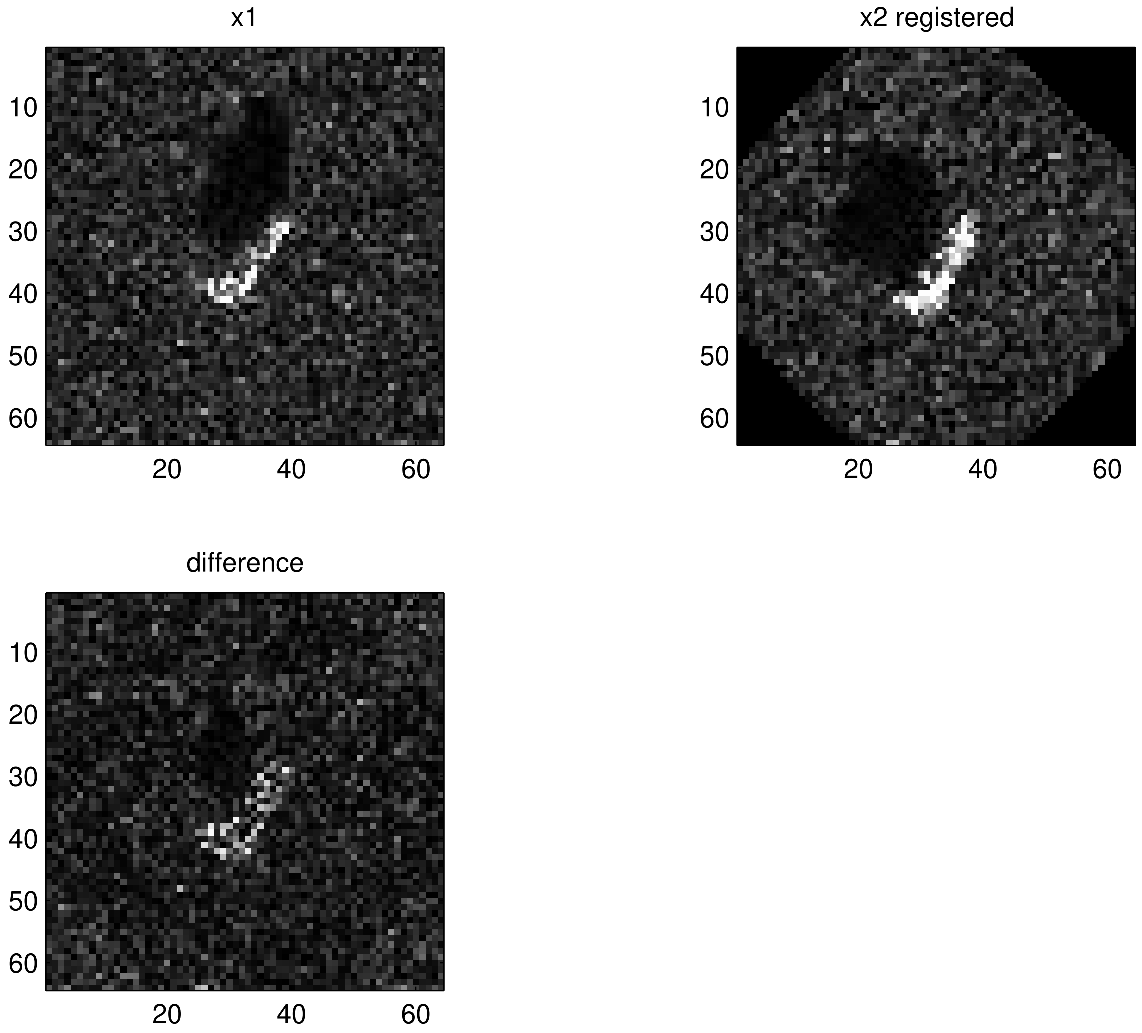

Initial Alignment
60
50
40
30
20
10
0
y
0
20
40
60
x
length of MST
290
285
280
275
270
265
Lr
0
100
200
300
angle


x1
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x2 registered
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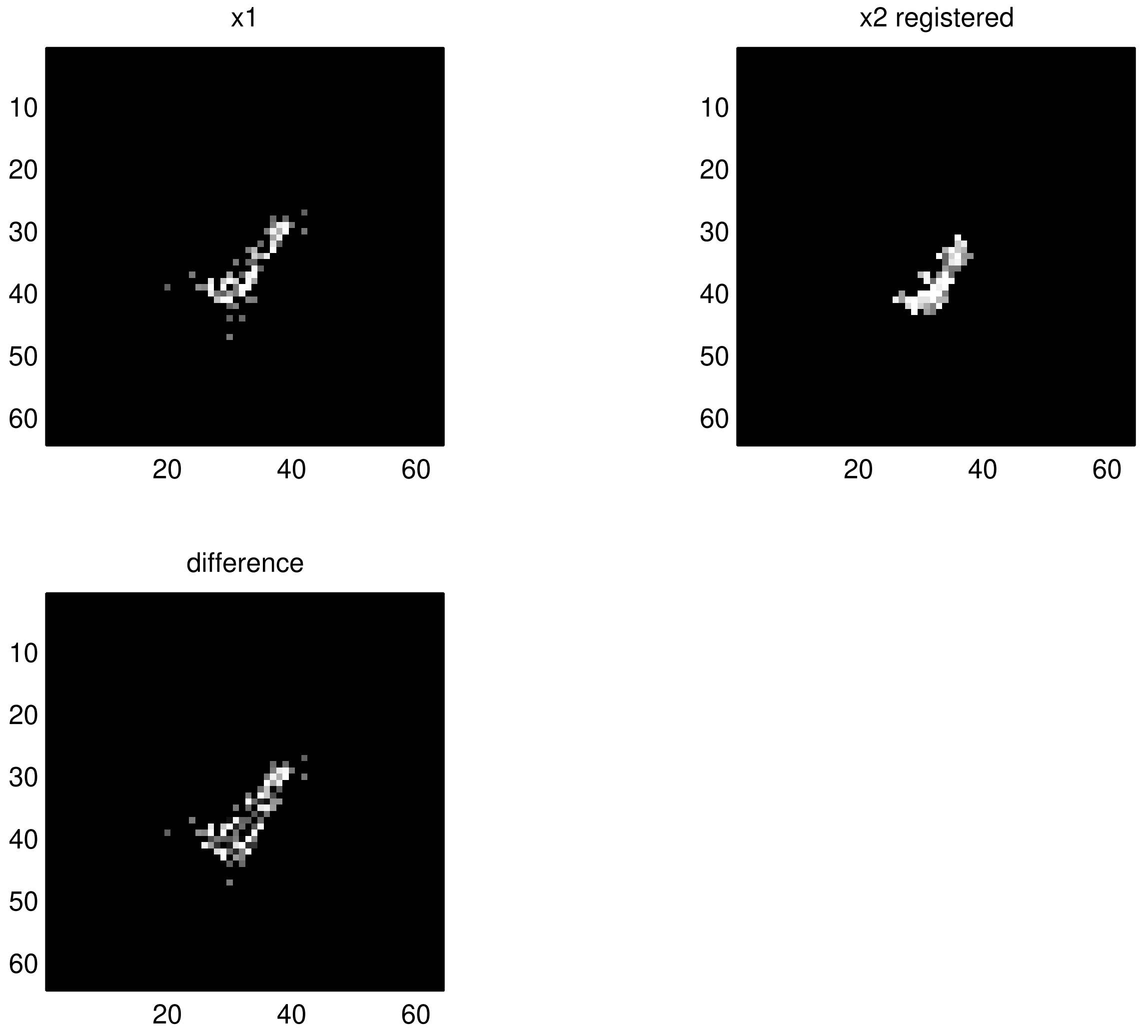
x1
x2 registered
difference
10
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60

Future Work:

- Selection of α for outlier rejection, e.g. linear prediction criteria.
- Quantitative comparison to histogram, RBF, kernel estimators of entropy.
- Convergence analysis of k-MST for spatially correlated feature vectors.