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Decentralized Coordination for Large-scale Plug-in Electric Vehicles in Smart Grid: An Efficient Real-time Price Approach

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***Abstract*— It has been a hot research topic to research the incorporation of large-scale PEVs into smart grid, such as the valley-fill strategy. However high charging rates under the valley-fill behavior may result in high battery degradation cost. Consequently in this paper, we novelly setup a framework to study a class of charging coordination problems which deals with the tradeoff between total generation cost and the accumulated battery degradation costs for all PEVs during a multi-time interval. Due to the autonomy of individual PEVs and the computational complexity for the system with large-scale PEV populations, it is impractical to implement the solution in a centralized way. Alternatively we propose a novel decentralized method such that each individual submits a charging profile, with respect to a given fixed price curve, which minimizes its own cost dealing with the tradeoff between the electricity cost and battery degradation cost over the charging interval; the price curve is updated based upon the aggregated PEV charging profiles. We show that, following the proposed decentralized price update procedure, the system converges to the unique efficient (in the sense of social optimality) solution under certain mild conditions.**

***Index Terms*— Plug-in electric vehicle (PEV); battery degradation cost; decentralized method; real-time price; generation marginal cost; efficiency.**

I. INTRODUCTION

Along with the rapid consumption of exhaustible nonrenewable petroleum energy resources and high emission of green gas, the plug-in electric vehicles (PEVs) achieve a high-speed development. However the charging behaviors of high-penetration PEVs have significant negative impacts on power grid, see [1], [2]. In order to support the accommodation of high-penetration PEVs in power grid, it is important to study how to coordinate the PEV charging behaviors.

Quite a few studies have explored the potential impacts of high-penetration PEVs on power grid, e.g., [3]–[6]. In general, these studies assume that PEV charging patterns "fill the valley" of night-time demand, that is to say, the overnight demand valley can be filled by the charging demand of large-scale plugged vehicles. In [7]–[9] and references therein, centralized methods are implemented to schedule the charging power of PEVs to minimize theirs effects on utilities. However due to the autonomy of individual PEVs and the computational complexity for large-scale PEVs, it is impractical to implement the solution in centralized way. As an alternative to centralized approach, decentralized methods may preserve individual authority and only requires limited individual information, e.g., [10], [11] and so on.

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As discussed above, the work in the literature mainly deals with how to minimize the impact on the power grid by scheduling the charging behaviors of high-penetration PEVs, and ignores the negative effects of the charging behavior of a PEV on itself. A few of research works, e.g., [12], [13], studied the optimal charging behavior for a single PEV in a way that takes into account both total energy cost and state of health of battery. In this paper we study the decentralized charging coordination for large-scale PEVs in power grid dealing with the total generation and accumulated battery degradation costs over multi-time interval.

The challenge of coordinating large-scale autonomous individuals in a decentralized way to achieve an optimal or near-optimal outcome is non-trivial. Because time-based and fixed schedule price based charging behaviors have difficulty effectively filling the night-time valley as studied in [14], in this paper we adopt the real-time price model which has been widely applied in the literature, e.g., [15], [16] for demand response management, and [17]–[19] for electric vehicle charging and discharging coordination. The adopted real-time electricity price represents the generation marginal cost which is determined by the total demand.

We propose a novel decentralized charging coordination method in this paper such that all of PEVs simultaneously update their own best charging behaviors with respect to a given price curve, which is updated with respect to the generation marginal cost associated to the charging behavior implemented at last step. More specifically, the system set a lower price at an instant to encourage PEVs to raise their charging demand at this instant in case the price at this instant was higher than the generation marginal cost at this instant; and set a higher price, otherwise. The price is not updated any longer only if the price curve is coincident with the generation marginal cost over the whole charging interval. As a consequence, the implemented charging behavior is efficient (or socially optimal). We specify a sufficient condition and show that under this condition the system converges to the unique efficient solution following the proposed decentralized algorithm.

Due to the cross-elastic correlation, as studied in [20], over the whole charging interval, the best updated charging behavior of an individual PEV, at an instant, is determined by both the demand at this instant and the total demand, over the whole interval, which is related to the given price curve. Moreover it is worth to note that, in the decentralized framework proposed in the work, each of individual PEVs deals with the tradeoff between battery degradation cost and electricity price cost. The battery degradation cost plays a same role as congestion pricing used in the traffic control in communication networks, see [21], which has been adopted by Fan in [19] to schedule PEV charging behavior in smart grid.

The paper is organized as follows. We firstly formulate a class of PEV charging coordination problems in power grid in Section II; then, in Section III, we propose a novel decentralized charging update algorithm, and show that the system converges to the unique efficient charging behavior under certain mild conditions. Simulations are studied in Section IV to demonstrate the results developed in the paper. Section V presents conclusions of the paper.

II. Formulations of PEV charging coordination problems

We consider charging control of a population of PEVs with size N over a horizon $\mathcal{T}$. For each PEV n, we call $\boldsymbol{u}_n \equiv (u_{nt}; t \in \mathcal{T})$ an admissible charging control of PEV n, if

$$u_{nt} \begin{cases} \geq 0, & t \in \mathcal{T}_n \\ = 0, & t \in \mathcal{T}/\mathcal{T}_n \end{cases}, \quad ||\boldsymbol{u}_n||_1 \equiv \sum_{t\in\mathcal{T}} u_{nt} \leq \Gamma_n, \tag{1}$$

where $\mathcal{T}_n$, with $\mathcal{T}_n \subset \mathcal{T}$ and Γ_n represent respectively the charging interval, the charging energy capacity of individual PEV n. The set of admissible charging controls for PEV n is denoted by $\mathcal{U}_n$.

Due to the wide application of the $LiFePO_4$ battery, we will analyze the battery degradation cost for $LiFePO_4$ battery, with respect to the analysis given in [22] where a degradation cost model for $LiFePO_4$ battery cell is formulated based upon the evolution of charging and discharging behaviors.

Denote $g_n(u_{nt})$ as the battery degradation cost of PEV n during the charging interval t under the charging power u_{nt}; then we can verify that $g_n(u_{nt})$ has a quadratic form such that

$$g_n(u_{nt}) = a_n u_{nt}^2 + b_n u_{nt} + c_n, \tag{2}$$

which represents the total monetary loss on the battery over charging interval t under charging power u_{nt} on PEV n, and where a_n, b_n, c_n are individual parameters related to the battery characteristics.

A. Formulations of PEV charging coordination concerning battery degradation costs

We suppose that the system deals the tradeoff between the *total cost* composed of the generation cost and the PEV battery degradation cost and the *benefit* to supply the energy for PEV populations over the charging intervals.

Denote by $J(\boldsymbol{u})$ the *system cost function* under a collection of admissible charging behaviors $\boldsymbol{u} \in \mathcal{U}$, such that

$$J(\boldsymbol{u}) \triangleq \sum_{t\in\mathcal{T}} \left\{ c(y_t) + \sum_{n\in\mathcal{N}} g_n(u_{nt}) \right\} - \sum_{n\in\mathcal{N}} \left\{ h_n(||\boldsymbol{u}_n||_1) \right\}, \tag{3}$$

where (i) $c(y_t)$, with $y_t \equiv D_t + ||\boldsymbol{u}_t||_1$, represents the generation cost with respect to the total demand $D_t + ||\boldsymbol{u}_t||_1$ with D_t denoting the aggregated inelastic base demand in power grid at instant t; (ii) $g_n(\cdot)$ is the battery degradation cost of PEV n formulated in (2); and (iii) $h_n(||\boldsymbol{u}_n||_1)$ denotes the benefit function of PEV n with respect to the total charged energy to PEV n over the whole interval.

In [23], a specific quadratic form is specified for $h_n\left(||\boldsymbol{u}_n||_1\right)$, such that

$$h_n(||\boldsymbol{u}_n||_1) = -\delta_n(||\boldsymbol{u}_n||_1 - \Gamma_n)^2, \tag{4}$$

with δ_n representing a proportional factor which reflects the relative importance of the desire to drive the individual PEV n to be fully charged over the whole charging interval.

In the literature, e.g., [24], [25] and references therein, the electricity generation cost, $c(\cdot)$, has been widely considered in a quadratic form on the supply, say

$$c(y_t) = \frac{1}{2}\mathsf{a} y_t^2 + \mathsf{b} y_t + \mathsf{c}, \tag{5}$$

with properly valued parameters a, b and c; then the marginal generation cost evolves linearly with respect to the total demand, say $p_t(y_t) \triangleq c'(y_t) = \mathsf{a} y_t + \mathsf{b}$.

We denote by $v_n(\boldsymbol{u}_n)$ the *utility function* of PEV n under $\boldsymbol{u}_n \in \mathcal{U}_n$, such that

$$v_n(\boldsymbol{u}_n) \triangleq h_n(||\boldsymbol{u}_n||_1) - \sum_{t\in\mathcal{T}} g_n(u_{nt}). \tag{6}$$

We formally formulate a class of centralized PEV charging coordination problems as follows:

Problem 1:

$$\min_{\boldsymbol{u}\in\mathcal{U}} \left\{ J(\boldsymbol{u}) \right\}, \tag{7}$$

that is to say, the objective of system is to implement a collection of socially optimal charging behaviors for PEVs, denoted $\boldsymbol{u}^{**}$, minimizing the system cost (3). ■

We consider the following assumptions in the paper:

- (A1). $c(y)$ is monotonically increasing, strictly convex and differentiable on y;
- (A2). $g_n(x)$, for all $n \in \mathcal{N}$, is monotonically increasing, strictly convex and differentiable on x. ■

Based on Assumptions (A1,A2), the efficient (socially optimal) charging behavior is unique and can be characterized by its associated KKT conditions [26]. When the benefit function takes the form (4), the optimal solution $\boldsymbol{u}^{**}$ obtained by minimizing $J(\boldsymbol{u})$ is therefore given by:

$$p_t^{**} \begin{cases} = \frac{\partial}{\partial u_{nt}} v_n(\boldsymbol{u}_n^{**}), & \text{when } u_{nt}^{**} > 0, \\ \geq \frac{\partial}{\partial u_{nt}} v_n(\boldsymbol{u}_n^{**}), & \text{when } u_{nt}^{**} = 0, \end{cases} \tag{8}$$

where $p_t^{**} = c'\big(d_t + \sum_{n\in\mathcal{N}} u_{nt}^{**}\big)$ is the generation marginal cost over the charging horizon with respect to the efficient allocation $\boldsymbol{u}^{**}$.

B. Numerical simulation

As discussed in (5), we firstly suppose that the generation cost function $c(\cdot)$ is in a quadratic form, such that

$$c(y_t) = 2.9 \times 10^{-7} \cdot y_t^2 + 0.06 \cdot y_t, \quad (9)$$

where $y_t = D_t + ||\boldsymbol{u}_t||_1$, and the base demand $\boldsymbol{D}$, which is a typical demand in hot summer season, is illustrated in Fig. 1.

We consider the charging coordination for a population of PEVs with population size of $N = 5,000$ over a common charging interval from 12:00AM on a day to 12:00AM on next day.

As an example, in this paper we suppose that the battery package in the PEVs is composed of a collection of ANR26650M1-B type battery cells from the A123 system. The nominal voltage and energy capacity of this type of battery cell is 3.3 volts and 2.5Ah (Amp $\times$ Hour) respectively, and the price of a single battery cell is about 15$. We further consider that the battery capacity $\varphi_n = 40$kWh for all n. Thus, the battery degradation cost can be approximately specified as below:

$$g_n(u_{nt}) = 0.003u_{nt}^2 + 0.075u_{nt}. \quad (10)$$

We consider that all of the PEVs share an identical minimal and maximum soc such that $soc_n^{\min} = 15\%$ and $soc_n^{\max} = 90\%$ for all n; then we can get the maximum charging energy of each PEV, $\Gamma_n = \varphi_n \cdot (soc_n^{\max} - soc_{n0}) = 30$kWh with $soc_{n0} = 15\%$ for all n. We consider the valuation function of individual PEVs following (4) with $\delta_n = 0.03$.

The efficient charging behaviors $\boldsymbol{u}^{**}$ deals with the trade-off between the total generation cost of system and the battery degradation cost of PEV populations; then as illustrated in Figure 1, $\boldsymbol{u}^{**}$ is much different with the valley-fill strategy under which the total generation cost is minimized. That is to say, the valley-fill charging behavior may be penalized with high battery degradation cost of populations of PEVs.

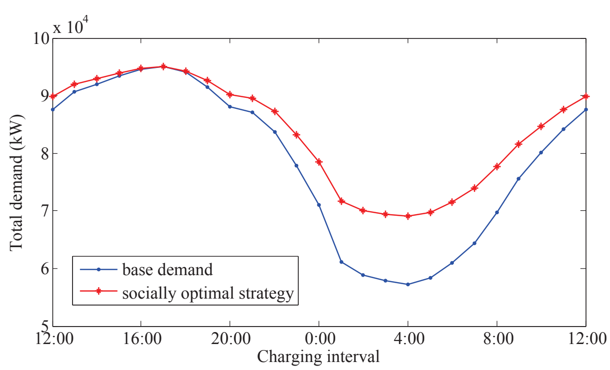

Fig. 1. The aggregated efficient charging behavior which is much distinct from the valley-fill one.

As studied, the centralized coordination can be effectively implemented in case the system has complete information and can directly schedule the charging behaviors of all PEVs, which in practice are not feasible. Thus in the rest of the paper we propose a decentralized coordination method in this paper, such that each PEV updates its charging behavior with respect to a common electricity price.

III. Decentralized Charging Coordination Method for PEV Populations

In this section, we will propose a decentralized method and study its convergence and efficiency.

A. Best response of individual PEVs with respect to a fixed price curve

We denote by $J_n(\boldsymbol{u}_n; \boldsymbol{p})$ the individual cost function of PEV n with respect to a given price curve $\boldsymbol{p}$, such that

$$J_n(\boldsymbol{u}_n; \boldsymbol{p}) \triangleq \boldsymbol{p}^\tau \boldsymbol{u}_n - v_n(\boldsymbol{u}_n), \quad (11)$$

where $\boldsymbol{p}^\tau$ represents the transposition of the column vector $\boldsymbol{p} \equiv (p_t; t \in \mathcal{T})$.

By (6), the individual cost function defined in (11) can be rewritten as the following:

$$J_n(\boldsymbol{u}_n; \boldsymbol{p}) = \sum_{t \in \mathcal{T}} \Big\{ p_t u_{nt} + g_n(u_{nt}) \Big\} - h_n \left(\sum_{t \in \mathcal{T}} u_{nt} \right).$$

The best charging behavior of PEV n with respect to $\boldsymbol{p}$, denoted by $\boldsymbol{u}_n^*(\boldsymbol{p})$, minimizing the individual cost function given in (11), such that

$$\boldsymbol{u}_n^*(\boldsymbol{p}) = \underset{\boldsymbol{u}_n \in \mathcal{U}_n}{\text{argmin}} \; \{J_n(\boldsymbol{u}_n; \boldsymbol{p})\}. \quad (12)$$

We first specify a set of charging behaviors of PEV n as below:

$$\mathcal{U}_n(\omega) \triangleq \Big\{ \boldsymbol{u}_n \in \mathcal{U}_n; \text{s.t. } ||\boldsymbol{u}_n||_1 = \omega \Big\}; \quad (13)$$

then we have $\mathcal{U}_n = \bigcup_{\omega \in [0, \Gamma_n]} \mathcal{U}_n(\omega)$. To examine constant total charging $||\boldsymbol{u}_n||_1 = \omega$ scenarios, we begin with an individual cost excluding h_n for PEV n, such that

$$F_n(\boldsymbol{u}_n; \boldsymbol{p}) \triangleq \boldsymbol{p}^\tau \boldsymbol{u}_n + \sum_{t \in \mathcal{T}} g_n(u_{nt}). \quad (14)$$

Lemma 3.1: Consider a fixed ω with $0 \leq \omega \leq \Gamma_n$, and a fixed $\boldsymbol{p}$; then the charging behavior of PEV n minimizing (14) under $\mathcal{U}_n(\omega)$ is unique and specified as follows:

$$u_{nt}(\boldsymbol{p}, A) = \max \Big\{ 0, [g_n']^{-1}(A - p_t) \Big\}, \quad (15)$$

for some A, where $[g_n']^{-1}$ represents the inverse function of the derivative of g_n. ■

However to determine the best behavior minimizing the individual cost function (11), the ω and A need to be specified. We will address this issue in Theorem 3.1. Before that we will firstly define some notion and give some results.

For any PEV n, the pair of values $A_n^-(\boldsymbol{p})$ and $A_n^+(\boldsymbol{p})$ are defined explicitly as follows:

$$A_n^-(\boldsymbol{p}) = \max \Big\{ A, \;\; \text{such that } ||\boldsymbol{u}_n(\boldsymbol{p}, A)||_1 = 0 \Big\} \quad (16)$$

$$A_n^+(\boldsymbol{p}) = A, \;\; \text{such that } ||\boldsymbol{u}_n(\boldsymbol{p}, A)||_1 = \Gamma_n. \quad (17)$$

Note that, although we include a subscript n on A_n^- for consistency, it is independent of n.

Lemma 3.2: Consider a fixed price curve $\boldsymbol{p}$; then (i) both $u_{nt}(\boldsymbol{p}, A)$, and $||\boldsymbol{u}_n(\boldsymbol{p}, A)||_1$ are increasing on $A \in \mathbb{R}$, and strictly increasing on $A \in [A_n^-(\boldsymbol{p}), +\infty)$, and (ii) $\boldsymbol{u}_n(\boldsymbol{p}, A)$ is admissible with $A \in [A_n^-(\boldsymbol{p}), A_n^+(\boldsymbol{p})]$, and not admissible for any $A > A_n^+(\boldsymbol{p})$. ■

Lemma 3.2 guarantees that, for a fixed $\boldsymbol{p}$, $||\boldsymbol{u}_n(\boldsymbol{p}, A)||_1$ strictly increases from 0 to Γ_n on the interval $A \in [A_n^-(\boldsymbol{p}), A_n^+(\boldsymbol{p})]$. This implies that $||\boldsymbol{u}_n(\boldsymbol{p}, A)||_1$ is invertible on $[A_n^-(\boldsymbol{p}), A_n^+(\boldsymbol{p})]$, and we denote the inverse as follows:

$$\mathcal{A}_n(\boldsymbol{p}, .) : [0, \Gamma_n] \to [A_n^-(\boldsymbol{p}), A_n^+(\boldsymbol{p})]. \tag{18}$$

Clearly, $\mathcal{A}_n(\boldsymbol{p}, \omega)$ is strictly increasing on ω. In general, we can write the inverse as follows:

$$\mathcal{A}_n(\boldsymbol{p}, \omega) = A, \qquad \text{in case } ||\boldsymbol{u}_n(\boldsymbol{p}, A)||_1 = \omega. \tag{19}$$

We will denote the individual charging control that satisfies (15) with total charging quantity equal to ω as $u_{nt}(\boldsymbol{p}, \mathcal{A}_n(\boldsymbol{p}, \omega))$.

Because of the non-negativity constraint on u_{nt} and the corresponding complementary slackness requirement from Lemma 3.1, it is not straightforward to determine the function $\mathcal{A}_n(\boldsymbol{p}, \omega)$. This is the purpose of Lemma 3.3.

Lemma 3.3: For any fixed price curve $\boldsymbol{p}$,

$$\frac{d}{d\omega} F_n^*(\boldsymbol{p}, \omega) = \mathcal{A}_n(\boldsymbol{p}, \omega), \qquad \text{with } \omega \in [0, \Gamma_n], \tag{20}$$

where $F_n^*(\boldsymbol{p}, \omega) \triangleq \inf_{\boldsymbol{u}_n \in \mathcal{U}_n(\omega)} F_n(\boldsymbol{u}_n; \boldsymbol{p})$. ■

Lemma 3.3 follows by Lemma 3.2 and the definition of $\mathcal{A}_n(\boldsymbol{p}, \omega)$ given in (19). This result allows us to interpret the Lagrange multiplier A as the marginal increase in total instantaneous costs as a function of the total charge quantity. At this point we have all the information necessary, with the exception of the optimal value of ω, to fully specify the individual optimal control for a given $\boldsymbol{p}$. Theorem 3.1 below brings all these results together and implicitly defines the optimal ω in the process.

Theorem 3.1: Assume $h_n(\omega)$ is continuously differentiable, increasing and concave on ω; then the control $\boldsymbol{u}_n(\boldsymbol{p}, A_n^*(\boldsymbol{p}))$ defined in (15) uniquely infimizes the cost function (11) with respect to a given $\boldsymbol{p}$, where $A_n^*(\boldsymbol{p})$ is defined as follows:

$$A_n^*(\boldsymbol{p}) = \begin{cases} \mathcal{A}_n(\boldsymbol{p}, \Gamma_n), & \text{in case } f_n(\Gamma_n) \le 0 \\ \mathcal{A}_n(\boldsymbol{p}, 0), & \text{in case } f_n(0) \ge 0 \\ \mathcal{A}_n(\boldsymbol{p}, \omega^*), & \text{in case } f_n(\omega^*) = 0 \end{cases} \tag{21}$$

for all $0 < \omega^* < \Gamma_n$, where $f_n(\omega) \triangleq \mathcal{A}_n(\boldsymbol{p}, \omega) - h_n'(\omega)$, with $\mathcal{A}_n(\boldsymbol{p}, \cdot)$ defined in (18). ■

B. Price curve update mechanism

By (8) and Lemma 3.1, in case $\boldsymbol{p} = \boldsymbol{p}^{**}$, with $\boldsymbol{p}^{**}$ representing the efficient generation marginal cost specified in (8), the collection of associated best response of individual PEVs, denoted $\boldsymbol{u}^*(\boldsymbol{p})$, is the efficient solution. However the system can not presumably set the system price equal to $\boldsymbol{p}^{**}$ in advance. Hence we will design a price update procedure in (22) below, by applying which the price may converge to the efficient marginal cost $\boldsymbol{p}^{**}$.

We specify, in (22) below, the updated price curve, denoted $\widehat{\boldsymbol{p}}(\boldsymbol{p})$, with respect to a given price curve $\boldsymbol{p}$

$$\widehat{p}_t(\boldsymbol{p}) = p_t + \eta \cdot (c' (D_t + ||\boldsymbol{u}_t^*(\boldsymbol{p})||_1) - p_t), \tag{22}$$

for all $t \in \mathcal{T}$, where η is a fixed positive valued parameter, and $\boldsymbol{u}_n^*(\boldsymbol{p})$, defined in (12), represents the best updated behavior of PEV n with respect to $\boldsymbol{p}$.

C. Decentralized PEV charging coordination algorithm

Here we are ready to formalize a decentralized charging coordination method for PEV populations in Algorithm 1 below.

Algorithm 1: (Decentralized method)

- Specify an aggregated base demand $\boldsymbol{D}$;
- Initialize a positive ε and a given initial price $\boldsymbol{p}^{(0)}$;
- Define an $\varepsilon_{\rm stop}$ to terminate iterations;
- Set $k = 0$;
- While $\varepsilon > \varepsilon_{\rm stop}$
  - Implement a best individual charging profile $\boldsymbol{u}_n^{(k+1)}$ w.r.t. $\boldsymbol{p}^{(k)}$ for all n simultaneously by minimizing the individual cost function (11), such that
  $$\boldsymbol{u}_n^{(k+1)}\left(\boldsymbol{p}^{(k)}\right) \triangleq \underset{\boldsymbol{u}_n \in \mathcal{U}_n}{\text{argmin}} \left\{\boldsymbol{p}^{(k),\tau} \boldsymbol{u}_n - v_n(\boldsymbol{u}_n)\right\};$$
  - Implement $\boldsymbol{p}^{(k+1)}$ w.r.t. $\left(\boldsymbol{p}^{(k)}, \boldsymbol{u}^{(k+1)}\left(\boldsymbol{p}^{(k)}\right)\right)$ following (22), i.e.,
  $$p_t^{(k+1)} = p_t^{(k)} + \eta \cdot \left(c'\left(D_t + ||\boldsymbol{u}_t^{(k+1)}||_1\right) - p_t^{(k)}\right),$$
  for all $t \in \mathcal{T}$;
  - Update $\varepsilon := ||\boldsymbol{p}^{(k+1)} - \boldsymbol{p}^{(k)}||_1$;
  - Update $k := k + 1$. ■

In case the system converges following the decentralized method proposed in Algorithm 1, the system reaches the efficient solution. However the oscillation may occur in the price update procedure.

We define some notion as below:

$$\nu \triangleq \sup_{n \in \mathcal{N}} \left\{\sup_{\epsilon > 0} \frac{1}{\epsilon} \left[[g_n']^{-1}(a + \epsilon) - [g_n']^{-1}(a)\right]\right\}, \tag{23a}$$

$$\kappa \triangleq \sup_{\epsilon > 0} \frac{1}{\epsilon} \Big(c'(u_s + \epsilon; D) - c'(u_s; D)\Big). \tag{23b}$$

Lemma 3.4: Assume the terminal valuation function h_n is increasing and strictly concave; then

$$||\boldsymbol{u}_n^*(\boldsymbol{p}) - \boldsymbol{u}_n^*(\boldsymbol{\rho})||_1 \le 2\nu ||\boldsymbol{p} - \boldsymbol{\rho}||_1 \tag{24}$$

where $||.||_1$ denotes the l_1 norm of the associated vector. ■

Corollary 3.1: (Convergence of Algorithm)

Suppose $|1 - \eta| + 2N\kappa\nu\eta < 1$ and consider any initial charging price $\boldsymbol{p}^{(0)}$; then the system converges to the efficient solution $\boldsymbol{u}^{**}$ which is specified in (8). Moreover, for any $\varepsilon > 0$, the system converges to a price curve $\boldsymbol{p}$, such that $||\boldsymbol{p} - \boldsymbol{p}^{**}||_1 \le \varepsilon$, in $K(\varepsilon)$ iteration steps, with

$$K(\varepsilon) = \left\lceil \frac{1}{\ln(\alpha)} \Big(\ln(\varepsilon) - \ln(T) - \ln(\rho_{max})\Big) \right\rceil, \tag{25}$$

with $\alpha \equiv |1-\eta|+2N\kappa\nu\eta$, where $\lceil x \rceil$ represents the minimal integer value larger than or equal to x, and ρ_{max} denotes the maximum price under the regulation. ■

Remarks: (i). Corollary 3.1 states that the system can reach a price curve $\boldsymbol{p}$, such that $||\boldsymbol{p}-\boldsymbol{p}^{**}||_1 \leq \varepsilon$ in iteration steps specified in (25), however in practice, the system may reach that price curve $\boldsymbol{p}$ in much less iteration steps than specified in (25), see simulations illustrated in Section IV; (ii). From (25), we obtain that the system converges to a price curve $\boldsymbol{p}$ such that $||\boldsymbol{p}-\boldsymbol{p}^{**}||_1 \leq \varepsilon$ for any small valued ε in $K(\varepsilon)$ iteration steps, where $K(\varepsilon)$ is in the order of $O(|\ln(\varepsilon)|)$, and is independent upon the size of PEV populations.

IV. Numerical Examples

We apply the decentralized charging coordination in Algorithm 1 on the problem. In this section, unless specified, we adopt the parameters considered in the example in Section II-B.

By (9), we have $p(y_t) = c'(y_t) = 5.8 \times 10^{-7} \cdot y_t + 0.06$; then $\kappa = 5.8 \times 10^{-7}$ by (23b). By (10) and (23a), $\nu = \frac{1}{2a_n} = 166.7$. With the given specifications, we have

$$|1-\eta| + 2N\kappa\nu\eta = |1-\eta| + 0.967\eta, \qquad (26)$$

which is always less than 1 for any η with $0 < \eta < 1.013$. Thus by Corollary 3.1, the system can always converge to the sufficient solution for all $\eta \in (0, 1.017)$.

In this simulation, we consider $\eta = 1 \in (0, 1.017)$ under which $\alpha \equiv |1-\eta| + 2N\kappa\nu\eta = 0.967 < 1$; then by (25), by applying Algorithm 1, we have the system converges to a price curve $\boldsymbol{p}^*$, such that $||\boldsymbol{p}^* - \boldsymbol{p}^{**}||_1 < \varepsilon$, in k iteration steps, with $K(\varepsilon) = \left\lceil \frac{1}{\ln(\alpha)}\Big(\ln(\varepsilon) - \ln(T) - \ln(\rho_{max})\Big)\right\rceil = 334$, given $\varepsilon = 0.0001$, $\alpha = 0.967$ in case $\eta = 1.0$, $T = 24$ and $\rho_{max} = 0.3$.

Fig. 2 displays the evolution of $||\boldsymbol{p}^{(k)} - \boldsymbol{p}^{**}||_1$ for the charging coordination of PEV populations following Algorithm 1, with an initial price curve $\boldsymbol{p}^{(0)}$ specified as $p_t^{(0)} = c'(D_t)$ for all t; then as illustrated in Fig. 2, $||\boldsymbol{p}^{(k)} - \boldsymbol{p}^{**}||_1$ converges to a value less than 0.0001 in about 10 iteration steps, which is much less than $K(\varepsilon) = 334$.

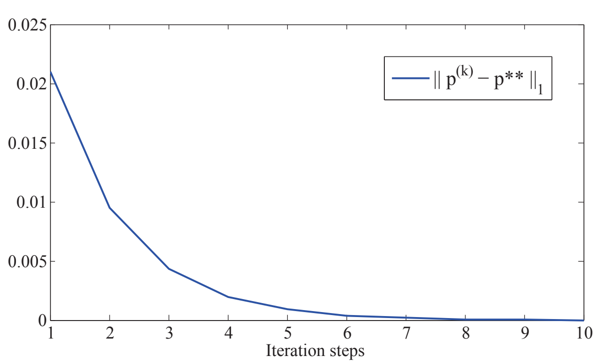

Fig. 2. Update of $||\boldsymbol{p}^{(k)} - \boldsymbol{p}^{**}||_1$ by applying Algorithm 1.

Fig. 3 displays the iteration updates of the PEV charging behavior following Algorithm 1, with the initial price curve $\boldsymbol{p}^{(0)}$ specified as $p_t^{(0)} = c'(D_t)$ for all t, such that the system converges to the efficient coordination solution. This is consistent with Corollary 3.1.

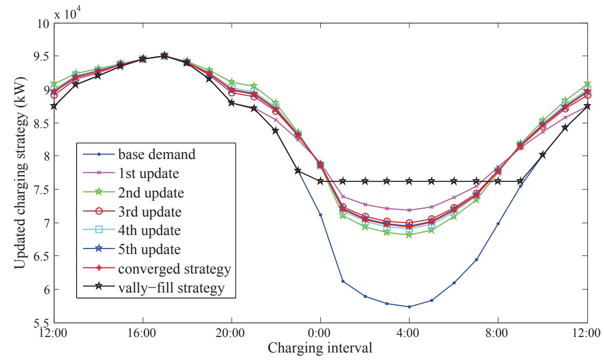

Fig. 3. Update procedure of PEV charging coordination following Algorithm 1.

To compare with the PEV charging coordination methods in the literature, we also illustrate, in Fig. 3, the implemented charging behavior, by applying the methods proposed in [10], [11], which fill the valley of the base demand and are quite different from the implemented socially optimal solution under the proposed method in this paper.

We denote by $\boldsymbol{u}^{\mathsf{vf}}$ the valley-fill charging behavior by applying the method proposed in [10], [11]. we have

$$\sum_{t\in\mathcal{T}} c\left(D_t + ||\boldsymbol{u}_t^{**}||_1\right) - \sum_{t\in\mathcal{T}} c\left(D_t + ||\boldsymbol{u}_t^{\mathsf{vf}}||_1\right) = 211.7;$$
$$\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}} g_n\left(u_{nt}^{**}\right) - \sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}} g_n\left(u_{nt}^{\mathsf{vf}}\right) = -671.3;$$

that is to say, compared with $\boldsymbol{u}^{\mathsf{vf}}$ implemented in [10], [11], the benefit under $\boldsymbol{u}^{**}$ with a save of 671.3\$ on battery degradation cost is only compromised with a small increased generation cost of 211.7\$. As a consequence, the system can benefit with a total cost decrease with 459.6\$ per day, which is about 167,000\$ per year, by adapting $\boldsymbol{u}^{**}$ instead of $\boldsymbol{u}^{\mathsf{vf}}$.

In the simulation illustrated above, we suppose that $||\boldsymbol{u}_n^{\mathsf{vf}}||_1 = ||\boldsymbol{u}_n^{**}||_1$, that is to say $h_n(||\boldsymbol{u}_n^{\mathsf{vf}}||_1) = h_n(||\boldsymbol{u}_n^{**}||_1)$ for all $n \in \mathcal{N}$. If following the case of $||\boldsymbol{u}_n^{\mathsf{vf}}||_1 = \Gamma_n$ as considered in [10], [11], the system can benefit with a higher cost decrease of 1640.2\$ per day, which is about 600,000\$ per year, by adapting $\boldsymbol{u}^{**}$ instead of $\boldsymbol{u}^{\mathsf{vf}}$.

For approaching a realistic situation for the charging coordination problems of PEV populations, we suppose that the initial value of state of charge of PEV batteries, denoted soc_{n0}, ahead of the PEV charging interval approximately satisfies a Gaussian distribution $N(\mu, \gamma)$, see [27], [28]. We further consider in the following simulation that $\mu = 50\%$ and $\gamma = 0.1$.

The update procedure of aggregated best charging profiles of all PEVs, with a typical base demand in spring season, is illustrated in Fig. 4 where we can observe that by adapting the proposed decentralized algorithm, the system converges to the efficient solution, see the solid line marked with asterisks, in a few iteration update steps.

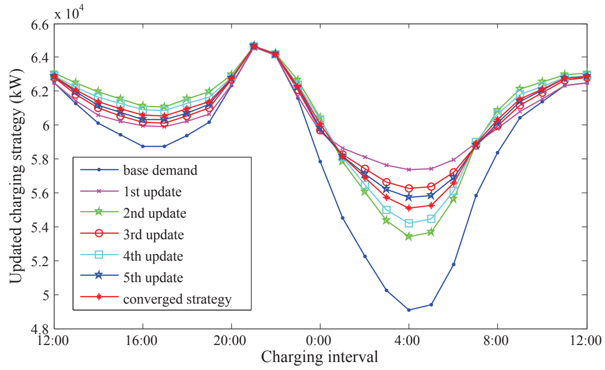

Fig. 4. Converged efficient charging behaviors for heterogeneous PEV populations in a typical spring season.

V. Conclusions

We formulated a class of charging coordination of large-scale PEVs in power grid, such that the system deals with the generation and battery degradation costs during the multi-time charging interval. A decentralized method is proposed such that all of individual PEVs simultaneously update their own best charging behaviors with respect to a common price curve, which is updated with respect to the generation marginal cost related to the charging schedule implemented at last step. The iteration procedure stops in case the price curve coincident with the marginal cost. Sufficient condition is specified under which the system converges to the unique efficient charging behavior by applying the proposed algorithm.

References

[1] L. Fernandez, T. Roman, R. Cossent, and C. Domingo, "Assessment of the impact of plug-in electric vehicles on distribution networks," *IEEE Transactions on Power Systems*, vol. 26, no. 1, pp. 206–213, 2011.

[2] A. Maitra, J. Taylor, D. Brooks, M. Alexander, and M. Duvall, "Integrating plug-in-electric vehicles with the distribution system," in *20th International Conference and Exhibition on Electricity Distribution – Part 1*, June 8-11 2009.

[3] P. Denholm and W. Short, "An evaluation of utility system impacts and benefits of optimally dispatched plug-in hybrid electric vehicles," National Renewable Energy Labortary, Technical Report NREL/TP-620-40293, October 2006.

[4] S. Rahman and G. Shrestha, "An investigation into the impact of electric vehicle load on the electric utility distribution system," *IEEE Transactions on Power Delivery*, vol. 8, no. 2, pp. 591–597, 1993.

[5] F. Koyanagi and Y. Uriu, "Modeling power consumption by electric vehicles and its impact on power demand," *Electrical Engineering in Japan*, vol. 120, no. 4, pp. 40–47, 1997.

[6] F. Koyanagi, T. Inuzuka, Y. Uriu, and R. Yokoyama, "Monte Carlo simulation on the demand impact by quick chargers for electric vehicles," in *IEEE Power Engineering Society Summer Meeting*, vol. 2, Istanbul, Turkey, July 18-22 1999, pp. 1031–1036.

[7] O. Sundstrom and C. Binding, "Planning electric-drive vehicle charging under constrained grid conditions," IBM - Zurich, Switzerland, Technical Report, August 2010.

[8] K. Clement-Nyns, E. Haesen, and J. Driesen, "The impact of charging plug-in hybrid electric vehicles on a residential distribution grid," *IEEE Transactions on Power Systems*, vol. 25, no. 1, pp. 371–380, 2010.

[9] M. Galus and G. Andersson, "Demand management of grid connected plug-in hybrid electric vehicles (PHEV)," in *IEEE Energy 2030*, Atlanta, Georgia, November 17-18 2008, pp. 1–8.

[10] Z. Ma, D. Callaway, and I. Hiskens, "Decentralized charging control of large populations of plug-in electric vehicles," *IEEE Transactions on Control Systems Technology*, vol. 21, no. 1, pp. 67–78, 2013.

[11] L. Gan, U. Topcu, and S. H. Low, "Optimal decentralized protocol for electric vehicle charging," *IEEE Transactions on Power Systems*, vol. 28, no. 2, pp. 940–951, 2013.

[12] S. Bashash, S. Moura, J. Forman, and H. Fathy, "Plug-in hybrid electric vehicle charge pattern optimization for energy cost and battery longevity," *Journal of Power Sources*, vol. 196, no. 1, pp. 541–549, 2011.

[13] K. Cheng, B. Divakar, H. Wu, K. Ding, and H. F. Ho, "Battery-management system and SOC development for electrical vehicles," *IEEE Transactions on Vehicular Technology*, vol. 60, no. 1, pp. 76–88, 2011.

[14] D. Callaway and I. Hiskens, "Achieving controllability of electric loads," *Proceedings of the IEEE*, vol. 99, no. 1, pp. 184–199, 2011.

[15] A. Mohsenian-Rad and A. Leon-Garcia, "Optimal residential load control with price prediction in real-time electricity pricing environments," *IEEE Transactions on Smart Grid*, vol. 1, no. 2, pp. 120–133, 2010.

[16] P. Samadi, A. Mohsenian-Rad, R. Schober, V. Wong, and J. Jatskevich, "Optimal real-time pricing algorithm based on utility maximization for smart grid," in *1st IEEE International Conference on Smart Grid Communications*, Gaithersburg, October 4-6 2010, pp. 415–420.

[17] R. Waraich, M. Galus, C. Dobler, M. Balmer, G. Andersson, and K. Axhausen, "Plug-in hybrid electric vehicles and smart grid: Investigations based on a micro-simulation," Institute for Transport Planning and Systems, ETH Zurich, Switzerland, Technical Report 10.3929/ethza-005916811, 2009.

[18] C. Wu, H. Mohsenian-Rad, and J. Huang, "Vehicle-to-aggregator interaction game," *IEEE Transactions on Smart Grid*, vol. 3, no. 1, pp. 434–442, 2012.

[19] Z. Fan, "A distributed demand response algorithm and its application to PHEV charging in smart grids," *IEEE Transactions on Smart Grid*, vol. 3, no. 3, pp. 1280–1290, 2012.

[20] D. S. Kirschen, G. Strbac, P. Cumperayot, and D. de Paiva Mendes, "Factoring the elasticity of demand in electricity prices," *IEEE Transactions on Power Systems*, vol. 15, no. 2, pp. 612–617, 2000.

[21] F. Kelly, A. Maulloo, and D. Tan, "Rate control for communication networks: Shadow prices, proportional fairness and stability," *Journal of the Operational Research society*, vol. 49, no. 3, pp. 237–252, 1998.

[22] J. C. Forman, J. L. Stein, and H. K. Fathy, "Optimization of dynamic battery paramter characterization experiments via differential evolution," in *American Control Conference (ACC)*, Washington, DC, June 17-19 2013, pp. 867–874.

[23] S. Han, S. Han, and Sezaki, "Development of an optimal vehicle-to-grid aggregrator for frequency regulation," *IEEE Transactions on Smart Grid*, vol. 1, no. 1, pp. 65–72, 2010.

[24] E. Bompard, Y. Ma, R. Napoli, and G. Abrate, "The demand elasticity impacts on the strategic bidding behavior of the electricity producers," *IEEE Transactions on Power Systems*, vol. 22, no. 1, pp. 188–197, 2007.

[25] V. Gountis and A. Bakirtzis, "Bidding strategies for electricity producers in a competitive electricity marketplace," *IEEE Transactions on Power Systems*, vol. 19, no. 1, pp. 356–365, 2004.

[26] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[27] Z. Luo, Z. Hu, Y. Song, Z. Xu, and H. Lu, "Optimal coordination of plug-in electric vehicles in power grids with cost-benefit analysis – part i: Enabling techniques," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 3546–3555, 2013.

[28] ——, "Optimal coordination of plug-in electric vehicles in power grids with cost-benefit analysis – part ii: A case study in china," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 3556–3565, 2013.