OBSERVED HYBRID OSCILLATIONS IN AN ELECTRICAL DISTRIBUTION SYSTEM

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ABSTRACT

Slow oscillations, with a period of around 15 minutes, were observed in an 11kV electrical distribution system. Investigations determined that the oscillations were a result of interactions between tap-changing transformers and switched capacitors. Analysis required a hybrid systems framework, due to the non-smooth (switched) nature of these interactions. It was found that the oscillations could be suppressed by adjusting the lower limit of the capacitor deadband. A grazing bifurcation separated oscillatory from steady-state behaviour.

1. INTRODUCTION

A number of years ago, the voltage of an 11kV distribution system exhibited oscillatory behaviour. A recording taken at the time is shown in Fig. 1. Two features set this event apart from usual power system oscillations [1]: 1) behaviour was quite non-smooth, and 2) the oscillation period of approximately 15 minutes was much larger than that associated with electromechanical oscillations. It was clear that the event was related to distribution system interactions, involving tap-changing transformers, switched capacitors and loads.

The oscillations occurred at a time when the transmission system serving the area was in an abnormal state. The local electricity utility was keen to fully understand the factors contributing to these self-excited oscillations. The initiating circumstances were of particular interest, and why the oscillations abruptly ended. Investigations at the time were limited by available analytical tools [2]. However the insights offered by newer tools [3] allow a more conclusive understanding of the events.

Inspection of the voltage plot of Fig. 1 reveals significant non-smoothness, due presumably to the discrete behaviour of transformer tap changing and capacitor switching.
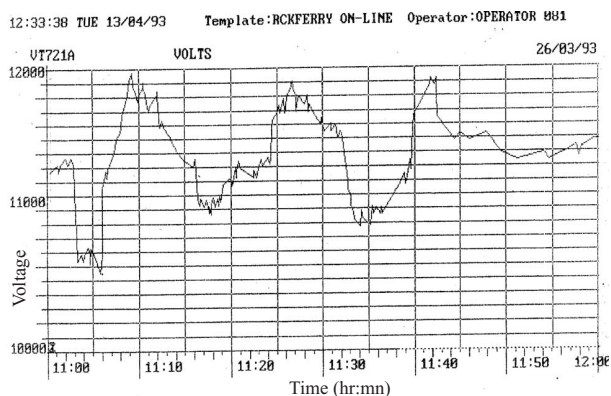

Fig. 1. Oscillations observed in a distribution system.

Both types of devices introduce step changes into the network due to the discreteness of transformer taps and capacitors respectively. Accurate modelling must take such behaviour into account. Just as importantly though, the time-delays and deadbands of the controllers associated with these devices must be considered. The resulting behaviour involves intrinsic interactions between discrete events and the continuous dynamics of other components such as motor loads. Systems that exhibit such interactions have become known as *hybrid systems* [4, 5]. The oscillations of Fig. 1 therefore provide an example of a *hybrid limit cycle*.[1]

Analysis of power system large disturbance behaviour has traditionally focussed on electromechanical swing dynamics. Apart from fault initiation and clearance, such behaviour is predominantly smooth. Therefore analysis packages have paid less attention to the requirements for modelling discrete events. This limitation provided a major impediment to the study reported in [2].

The paper therefore provides, in Section 2, a brief discussion of modelling extensions necessary for capturing hybrid system behaviour in a more systematic way. This is followed in Section 3 by analysis of the limit cycle phe-

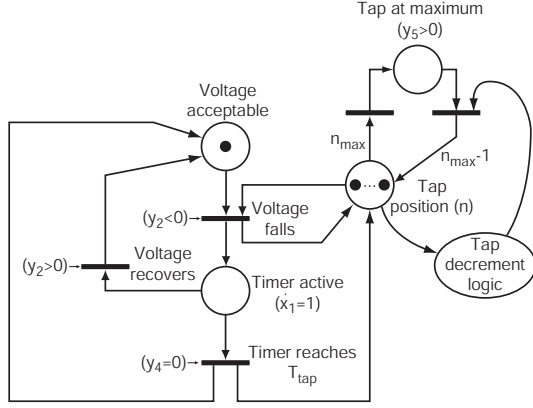[1]A formal definition of limit cycles, though in the context of smooth systems, is provided in [6].

Fig. 2. Petri net representation of tap-changing transformer AVR.

Fig. 3. Supply system (simplified) one-line diagram.

nomenon of Fig. 1. Conclusions are presented in Section 4.

2. HYBRID SYSTEM MODEL

The voltage regulator of a tap-changing transformer monitors the regulated-bus voltage for deviations from a desired deadband. Transitions outside the deadband enable a timer, while transitions back to within the deadband reset and disable the timer. If the timer reaches its maximum, a discrete tap change occurs and the timer is reset. Fig. 2 provides a Petri net representation [7] of behaviour. Clearly discrete dynamics play a major role. Yet voltage behaviour is also influenced by the continuous dynamics of many devices, such as loads.

It is shown in [3, 8] that the continuous/discrete dynamic behaviour of hybrid systems can be captured by a model that consists of a set of differential-algebraic equations, adapted to incorporate impulsive (state reset) action and switching of the algebraic equations. This <u>DA</u> <u>I</u>mpulsive <u>S</u>witched (DAIS) model can be written in the form,

$$\dot{x} = f(x, y) \tag{1}$$

$$x^+ = h_j(x^-, y^-) \qquad \text{when } y_{r[j]} = 0 \tag{2}$$

$$0 = g(x, y) \equiv g^{(0)}(x, y) + \sum_{i=1}^{s} g^{(i)}(x, y) \tag{3}$$

where

$$g^{(i)}(x, y) = \begin{cases} g^{(i-)}(x, y) & y_{s[i]} < 0 \\ g^{(i+)}(x, y) & y_{s[i]} > 0 \end{cases} \quad i = 1, ..., s \tag{4}$$

and

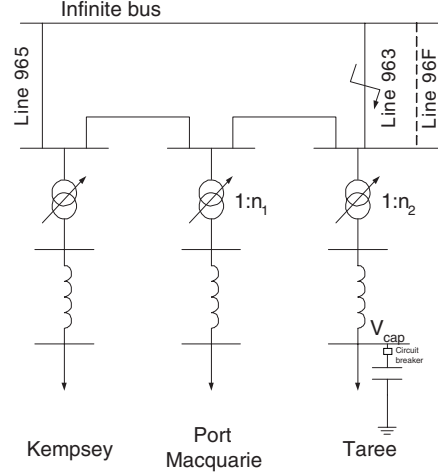- $x \in \mathbb{R}^n$ are dynamic states, and $y \in \mathbb{R}^m$ are algebraic states;

- x^+ denotes the value of x just after a reset event, while x^- and y^- refer to the values of x and y just prior to the event.

- subscripts $r[j], s[i]$ reference the elements of y that trigger the j-th reset (impulsive) event and the i-th algebraic switching event respectively.

Away from events, system dynamics evolve smoothly according to the familiar differential-algebraic model

$$\dot{x} = f(x, y) \tag{5}$$

$$0 = g(x, y) \tag{6}$$

where g is composed of $g^{(0)}$ together with appropriate choices of $g^{(i-)}$ or $g^{(i+)}$, depending on the signs of the corresponding trigger variables $y_{s[i]}$. At switching events (4), some component equations of g change. To satisfy the new $g = 0$ constraints, algebraic variables y may undergo a step change. Impulse events (2) force a discrete change in elements of x. Algebraic variables may again step to ensure $g = 0$ is always satisfied.

3. DISTRIBUTION SYSTEM ANALYSIS

3.1. Background

The voltage oscillation of Fig. 1 occurred in a part of the system that consisted primarily of three 132kV substations, Kempsey, Port Macquarie and Taree. A simplified representation is shown in Fig. 3. Those 132kV substations were normally connected to the rest of the grid via feeders 965, 963 and 96F. Each substation included tap-changing transformers for supplying the 66/33/11kV sub-transmission and distribution systems. Switched capacitors were also installed on the distribution system for supporting the voltage.

Limited quantitative information was available for transformers, capacitors, loads, and distribution system feeders. However [2] documents a thorough data collection process that pieced together a good overview of the prevailing system conditions.

In subsequent analysis, tap-changers and switched capacitors are modelled in detail as event-driven switching devices. It was found that their time-delay elements were particularly important. The discrete-event dynamics of tap-changer controls were discussed earlier in Section 2, and presented schematically in Fig. 2. Switched capacitors have similar deadband/timer interactions, though typically the timer is set for a much longer delay.

3.2. Results

At the time of the voltage oscillation event, feeder 96F was out of service for maintenance. The loads being supplied by the three substations were each approximately $(10 + j2)$ MVA. These loads were modelled as voltage dependent dynamic loads [9]. The tap positions were initialized to $n_1 = n_2 = 1$ pu. Feeder 963 was outaged at 5 seconds, leaving the system radially fed through feeder 965.

Line outaging initiated sustained oscillations (limit cycle behaviour) in distribution system voltages, as shown in Figs. 4 and 5. These oscillations show the same qualitative behaviour as the measured response in Fig. 1. Clearly the real system exhibits much richer dynamic behaviour, with many effects unmodelled in the simplified representation. However the general behavioural trends and oscillation periods match well.

Referring to Fig. 4, following the outage of feeder 963 the capacitor and tap-changer voltages fell outside their deadbands. This activated all respective timers. Tap-changing transformers increased their tap positions until about 400 seconds, when the capacitor controller timed out and the capacitor switched into service. However, this switching action drove voltages beyond deadband upper limits instantaneously. Consequently tap-changers started decrementing their tap positions until the capacitor switched out of service at about 800 seconds. Although capacitor switching is meant to aid tap changing, it continually resulted in voltage overshoot. Steady-state could not be achieved, with the tap-changer/capacitor interactions giving rise to a hybrid limit cycle. This limit cycle behaviour is clearly evident in Fig. 5.

The voltage overshoot due to capacitor switching is a direct consequence of the reduced fault level resulting from the outage of feeders 963 and 96F. When the system is stronger, with at least one of those feeders in service, capacitor switching causes much smaller voltage steps, relieving the overshoot problem.
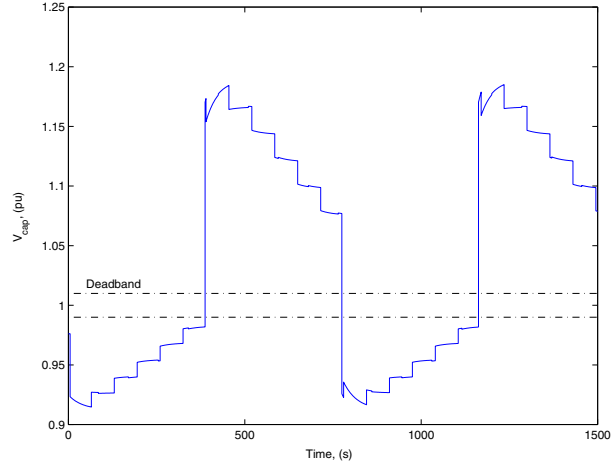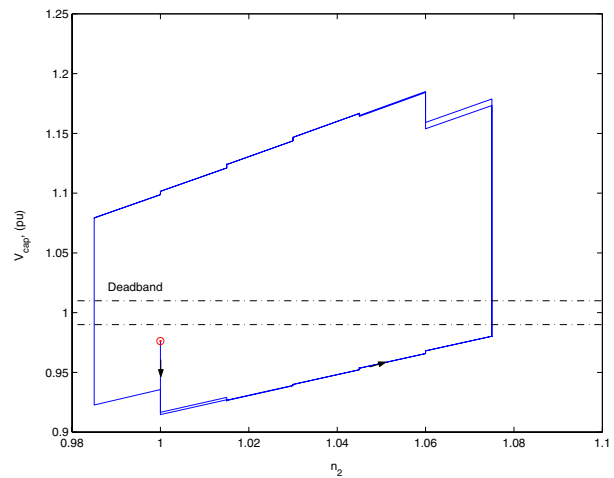

Fig. 4. Capacitor voltage trajectory.

Fig. 5. Trajectory in $n_2 - V$ plane.

3.3. Pivotal parameter values

Limit cycle existence is sensitive to the lower limit of the deadband, V_{min}. Figure 6 provides an illustration. By lowering V_{min} to 0.95 pu, no more switching operations occurred after 250 seconds, and the capacitor voltage reached a steady-state value of $V_{cap} = 0.955$ pu. Although the undesirable oscillations were suppressed, the load voltage profile was unacceptably low. Sizing the deadband is therefore a trade-off between maintaining a good post-feeder-outage voltage profile and suppressing sustained oscillations. The pivotal case corresponds to raising the value of V_{min} until capacitor switching is only just initiated.

This situation is closely related to *grazing bifurcations* [10, 11], where a system trajectory just touches a switching hypersurface, triggering the switching event.[2] In the present

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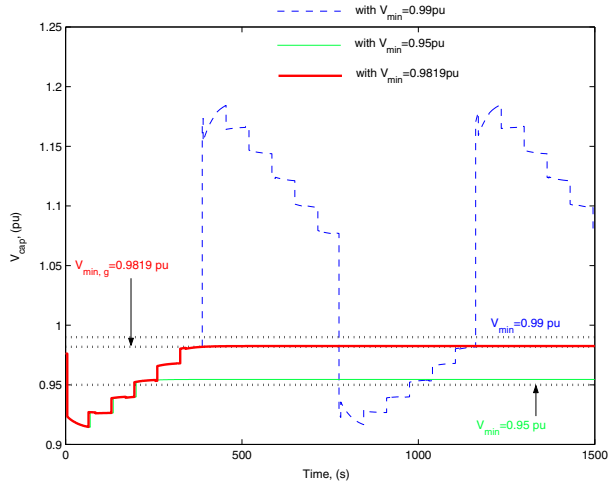[2]The implication is that for an incremental parameter change, there is

Fig. 6. Influence of deadband lower limit V_{min} on behaviour.

case of capacitor switching, a grazing bifurcation separates the hybrid limit cycle from steady-state (equilibrium) behaviour, i.e., a structurally different form of response. The bifurcation in this case corresponds to the *time-difference* form of grazing [11], with the trajectory just touching the deadband $V_{min} = 0.9819$ pu at the exact moment the capacitor timer reaches its trigger value. An incremental delay in reaching the deadband would result in capacitor energization. This situation is depicted in Fig. 6, where the grazing value of the parameter is identified as $V_{min,g}$.

4. CONCLUSIONS

The controls associated with distribution system devices such as tap-changing transformers and switched capacitors aim to maintain an appropriate load voltage profile. However these devices introduce discrete events into system dynamics, providing an example of a hybrid system. Control design for such systems is notoriously difficult. Simultaneous action of these devices may give rise to sustained oscillations (hybrid limit cycles.) This is particularly so when the supply system has been weakened by feeder outages.

The paper reports on such a situation that was observed in practice. By carefully modelling the tap-changer and switched-capacitor controls, it was possible to obtain a good match between measured and simulated behaviour. This investigation highlighted the importance of establishing appropriate deadbands for discrete devices.

With a wide deadband, oscillatory behaviour was suppressed, but distribution bus voltages tended to be too low. As the deadband was reduced, by raising the lower limit, a pivotal value was reached beyond which oscillatory be-

haviour was reinstated. That pivotal value corresponds to a form of grazing bifurcation, where the trajectory just touches the deadband at the exact moment the capacitor timer reaches its trigger condition. Shooting methods can be formulated to identify parameter values corresponding to such bounding situations.

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no contact with the switching hypersurface, so no event triggering occurs.