**X SEPOPE**

21 a 25 de maio de 2006
May – 21st to 25th – 2006

FLORIANÓPOLIS (SC) – BRASIL

**X SIMPÓSIO DE ESPECIALISTAS EM PLANEJAMENTO DA OPERAÇÃO
E EXPANSÃO ELÉTRICA****X SYMPOSIUM OF SPECIALISTS IN ELECTRIC OPERATIONAL
AND EXPANSION PLANNING****SIGNIFICANCE OF LOAD MODELLING IN POWER SYSTEM DYNAMICS**

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SUMMARY

Load modelling is a perennial source of consternation. Postmortem analyses of disturbances frequently conclude that load models must be adjusted to achieve an adequate match between measured and simulated behaviour. However the development of load models that are accurate over a wide range of system conditions requires, 1) testing that is undesirable for customers, and/or 2) measurement of behaviour during infrequent disturbances. Furthermore, the stochastic nature of many loads implies load models that are accurate at one time may be inaccurate a short time later.

In terms of quantitative analysis, for example matching simulations with measurements, it is absolutely clear that accurate load modelling is vitally important. But for qualitative investigations, where the aim is to assess the likelihood of a certain disturbance scenario remaining stable or causing instability, then the need for accurate load modelling is much reduced. The paper explores this conjecture through the use of trajectory sensitivity concepts. It is shown that for qualitative assessment, it is more important to know the sensitivity of behaviour to load parameters, than to precisely know the parameter values.

However most power system failures are not initiated by instability, but rather by reactionary (unanticipated) protection operation. The ability to predict operation of protection devices, and hence potential protection-induced cascading failures, may be greatly affected by load model fidelity.

KEYWORDS

Load modelling; dynamic performance assessment; parameter uncertainty; trajectory sensitivity.

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1 Introduction

Postmortem analysis of power system disturbances invariably reveals discrepancies between measured and simulated behaviour. Load model inaccuracy is often found to be an important contributing factor. This is not a new issue, with many instances reported in the literature over the years. For example, the inadequacy of constant admittance load models was discussed in [1], in the context of analyzing a major system disturbance that resulted from a staged test. Other studies from that era also suggest load representation has an important influence on power system behaviour [2]. More recently, studies of the WECC power systems, which spans the western US, have identified load modelling as an important issue in replicating measurements from large disturbances [3, 4].

Recognition of the importance of load modelling motivated numerous task force reports [5, 6, 7]. A huge effort has been directed towards identifying load model parameters from measurements of natural and deliberately-induced disturbances [8, 9]. Yet even after extensive investment in load model improvement, analysis of subsequent disturbances invariably reveals errors in load models [10]. This should not be unexpected though, given that load models are meant to account for the aggregated effects of many time-varying composite loads [2]. Load composition, and hence behaviour, will generally differ from one disturbance to the next.

Expressing load models as a deterministic set of parameters would therefore seem pointless. Rather, parameters should be described via their statistical distributions [11]. But that raises the issue of how to use such information. A Monte-Carlo approach, whereby sets of load parameters are randomly sampled and a time-domain simulation performed for each set, is computationally infeasible. Statistical analysis of load effects is feasible though, by utilizing techniques that build on probabilistic collocation [12] and/or trajectory sensitivities [13]. This paper however takes a different line.

In terms of quantitative analysis, for example matching simulations with measurements, it is absolutely clear that accurate load modelling is vitally important. But for qualitative investigations, where the aim of dynamic simulation is to assess the likelihood of a certain disturbance scenario being stable or unstable, then the need for accurate load modelling is much reduced. In other words, if a system is stable (unstable) for a certain set of load model parameters, then it will most likely also be stable (unstable) for perturbed load models. The paper will explore this conjecture through the use of trajectory sensitivity concepts. It will be shown that for such qualitative assessment, it is more important to know the sensitivity of behaviour to load parameters, than to precisely know the parameter values.

A caveat is required though. Most power system failures are not initiated by instability [14], though instability is frequently a consequence. Rather, it is more common for an initiating (relatively minor) disturbance to escalate through reactionary protection operation. Examples of such reactionary effects include zone 3 distance protection unnecessarily tripping feeders, and volts/hz relays tripping generators. This subsequent weakening of the system may induce further protection operation, leading to a cascading system failure. It has been found from disturbance analysis that load modelling can be very important in predicting such reactionary protection behaviour [10].

Protection is binary; either the system encounters the operating characteristic initiating a trip, or it does not encounter the characteristic and the component remains in service. The bounding case, separating protection operation from non-operation, corresponds to the trajectory *gazing* (just touching) the operation characteristic [15]. Parameter sets that induce grazing are pivotal, in that they divide parameter space into regions that exhibit vastly different behaviour [16]. It follows that in potential grazing situations, where reactionary

protection operation may or may not occur, special care should be given to understanding the influence of load parameter variations.

The paper is organized as follows. Section 2 provides brief background to modelling concepts, and to trajectory sensitivities. The IEEE 39 bus system is used in Section 3 to illustrate various aspects of the influence and significance of load modelling. Conclusions are presented in Section 4.

2 Background

2.1 System model

In response to large disturbances, power systems typically exhibit periods of smooth behaviour, interspersed with discrete events. Smooth behaviour is driven by devices such as generators, that are well described analytically by differential-algebraic models. Discrete events, arising for example from operation of protection devices or enforcement of controller hard limits, are not so easy to describe analytically. Systems that exhibit intrinsic interactions between continuous dynamics and discrete events have become known generically as *hybrid systems* [17], or *piecewise smooth dynamical systems*.

Analysis of power system dynamics requires a non-restrictive model formulation that is capable of capturing the full range of continuous/discrete hybrid system dynamics, yet is computationally efficient. It is shown in [18, 19] that these specifications are met by a model that consists of a set of differential-algebraic equations, adapted to incorporate switching of the algebraic equations, and impulsive (state reset) action. The details of this model are not vital for later discussions, therefore concepts will be presented in terms of the simpler model,

$$\dot{x} = f(x; \lambda), \quad x(0) = x_0 \quad (1)$$

where $x \in \mathbb{R}^n$ describes the system state, and $\lambda \in \mathbb{R}^p$ are parameters. The behaviour of the system, as driven by (1), can be represented by the *flow*,

$$x(t) = \phi(x_0, t; \lambda). \quad (2)$$

The extension of this flow concept to more realistic hybrid system behaviour is straightforward.

2.2 Load models

It is common for aggregate load models to be composed of a static voltage dependent component together with an induction motor [4, 7]. This composition can be described parametrically by

$$S_{tot} = \nu S_v + (1 - \nu) S_{ind} \quad (3)$$

where S_{tot} is the total complex power of the load,

$$S_v = P_0 V^{\eta_p} + j Q_0 V^{\eta_q} \quad (4)$$

describes the voltage dependent part of the load, and S_{ind} is the complex power demanded by the induction motor component. The dynamics underlying S_{ind} are typically described by a third order differential equation model [20].

Each of the load components in (3) should be sized to match the total bus demand. The parameter ν provides the necessary scaling, with 100ν specifying the percentage of static load. The significance of the load composition, i.e., the weighting between S_v and S_{ind} , can be captured by the sensitivity of dynamic behaviour to variations in ν . Later examples will consider uncertainty in η_p , η_q and ν .

2.3 Trajectory sensitivities

Sensitivity concepts are generally associated with the linearization of an input-output relationship. Small changes in inputs map through the linearized relationship to small output changes. Trajectory sensitivities fit this framework by describing the changes in the trajectory (the output) resulting from perturbations in the underlying parameters and/or initial conditions (the inputs). They provide a linearization around the trajectory, as against small disturbance analysis which builds on linearization around the equilibrium point. Trajectory sensitivity concepts are not new [21], though until recently progress on practical applications was impeded by:

- Computational inefficiency. Sensitivity to each parameter or initial condition required an additional full simulation.
- Non-smooth behaviour. Sensitivities were not well defined for situations where events influenced behaviour.

However both these limitations have recently been overcome, with efficient computation of trajectory sensitivities now possible for large-scale, non-smooth systems [18].

Trajectory sensitivities are motivated by the Taylor series expansion of the flow (2), which can be expressed as

$$\phi(x_0, t; \lambda + \Delta\lambda) = \phi(x_0, t; \lambda) + \frac{\partial\phi(x_0, t; \lambda)}{\partial\lambda}\Delta\lambda + \mathcal{E}(x_0, t; \lambda, \Delta\lambda) \quad (5)$$

where \mathcal{E} captures the higher order terms. For small $\|\Delta\lambda\|$, the higher order terms may be neglected, giving

$$\begin{aligned} \Delta x(t) &= \phi(x_0, t; \lambda + \Delta\lambda) - \phi(x_0, t; \lambda) \approx \frac{\partial\phi(x_0, t; \lambda)}{\partial\lambda}\Delta\lambda \\ &\equiv \Phi(x_0, t; \lambda)\Delta\lambda \end{aligned} \quad (6)$$

where Φ is the *sensitivity transition matrix*, or *trajectory sensitivities*, associated with the flow [21]. Equation (6) describes the approximate change $\Delta x(t)$ in a trajectory, at time t along the trajectory, for a given small change in parameters $\Delta\lambda$.

Evolution of the sensitivity transition matrix Φ is described by variational equations that follow from differentiating (1) with respect to λ . This gives

$$\dot{\Phi} = f_x(t)\Phi + f_\lambda(t), \quad \Phi(0) = 0 \quad (7)$$

where $f_x \equiv \partial f/\partial x$, $f_\lambda \equiv \partial f/\partial \lambda$ and $\Phi(t) \equiv \partial x(t)/\partial \lambda$. Note that f_x and f_λ are evaluated along the trajectory, and hence are time varying matrices. The computational burden of numerically integrating this (potentially high order) linear time-varying system is minimal though. It is shown in [18, 22] that when an implicit numerical integration technique such as trapezoidal integration is used, the solution of (7) can be obtained as a by-product of computing the underlying trajectory.

2.4 Sensitivity behaviour under stressed conditions

Consider a trajectory of (1) which lies exactly on the stability boundary. (This is of course unachievable [23], but is conceptually possible.) Under certain (fairly generic) conditions, the trajectory must approach an unstable equilibrium point (UEP) in the limit [24]. But at that

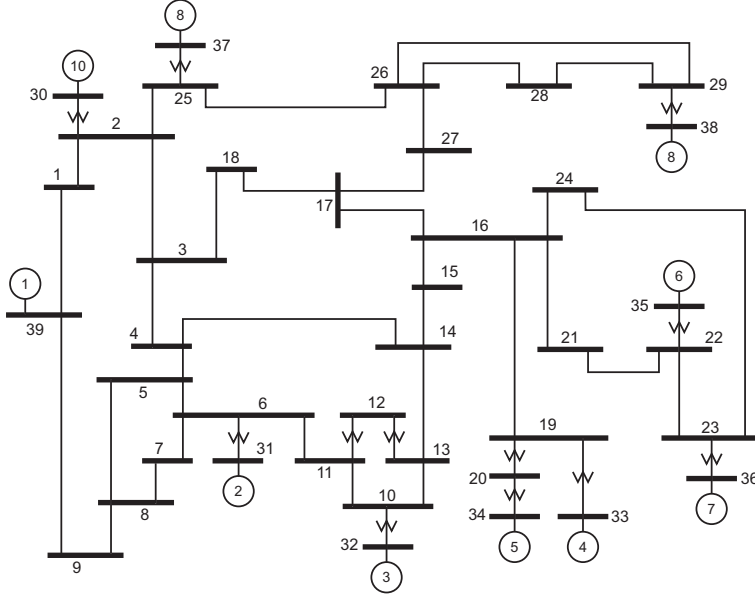


Figure 1: IEEE 39 bus system.

UEP, the matrix f_x driving the evolution of Φ in (7) is constant and unstable. Therefore $\|\Phi(t)\|$ will diverge to infinity as t increases.

It follows intuitively that if a trajectory approaches, but does not encounter, the stability boundary, then $\|\Phi(t)\|$ will grow very large, before contracting back to its steady state. Therefore larger values of $\|\Phi(t)\|$ along a trajectory suggest a small stability margin.

3 Examples

The examples throughout this section utilize the IEEE 39 bus system of Figure 1. All generators in this system were represented by a fourth order machine model [25], and were regulated by the IEEE standard AVR/PSS models AC4A and PSS1A [26]. All generator and network data were obtained from [27].

3.1 Parameter ranking

Trajectory sensitivities provide a basis for ranking the relative influence of parameters. Large sensitivities imply that parameter variations have a large effect on behaviour, whereas small sensitivities suggest behaviour changes very little with parameter variation. In this example, trajectory sensitivities are used to rank the importance of voltage indices at all loads throughout the IEEE 39 bus system. A three-phase fault was applied at bus 16 at 0.1 sec, and cleared (without any line tripping) 0.2 sec later. The static load model (4) was used for all loads, with $\eta_p = \eta_q = 2$ in all cases.

The sensitivities of bus 16 voltage V_{16} to load indices η_p and η_q at all buses were computed in conjunction with the nominal trajectory. These trajectory sensitivities are provided in Figure 2, where the vertical axis gives the change in the pu voltage for a unity change in load index values. It is immediately clear that the real power index η_p for bus 20 has a much greater influence on behaviour than all other indices. (The reason is that generator 5

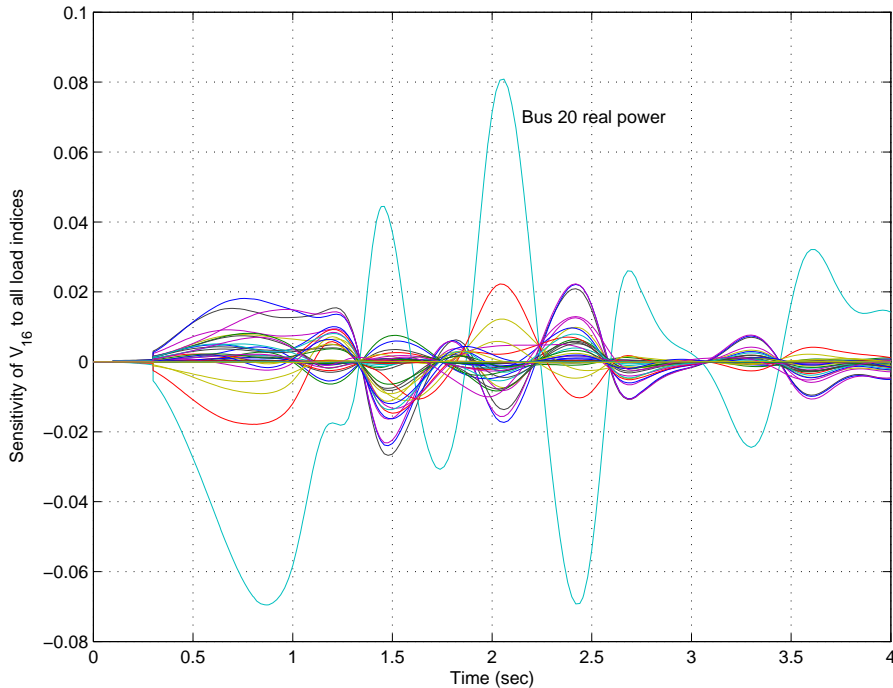


Figure 2: Trajectory sensitivities for all load indices.

is marginally stable for this disturbance scenario, and bus 20 lies on the corridor linking that generator to the rest of the system.) The loads at buses 4, 8 and 23 also display a reasonable, though certainly less pronounced, level of influence. Loads 4 and 8 are influential due to their large size. Load 23 has an important impact on the dynamics of generator 7. The influence of all other loads, for this disturbance scenario, is negligible. Of course a different disturbance could possible highlight some other set of loads.

Field testing loads to determine their (approximate) voltage dependence is an expensive exercise. However, by utilizing trajectory sensitivities, the most important loads can be immediately identified, and attention focused accordingly. This use of trajectory sensitivities relates to parameter identifiability, and was used in [28] to determine the most influential generator parameters.

3.2 Indicator of stressed conditions

It was shown in Section 2.4 that as systems become more heavily stressed, sensitivity to parameter variation increases significantly. This can be illustrated by continuing the previous example. The upper plot of Figure 3 shows the behaviour of generator 5 angle (relative to generator 10) for a range of fault clearing times. (The fault clearing time used in the previous example was 0.2 sec.) The critical clearing time is 0.213 sec; slower clearing results in generator 5 losing synchronism. Notice that the angular deviations do not show a great increase, even though instability is imminent.

The sensitivity of V_{16} to the bus 20 load index η_p , for the same range of fault clearing times, is shown in the lower plot of Figure 3. The deviations exhibited by these trajectory sensitivities grow dramatically as critical conditions are approached. This behaviour motivated the sensitivity related measures developed in [29, 30] to predict conditions that induce

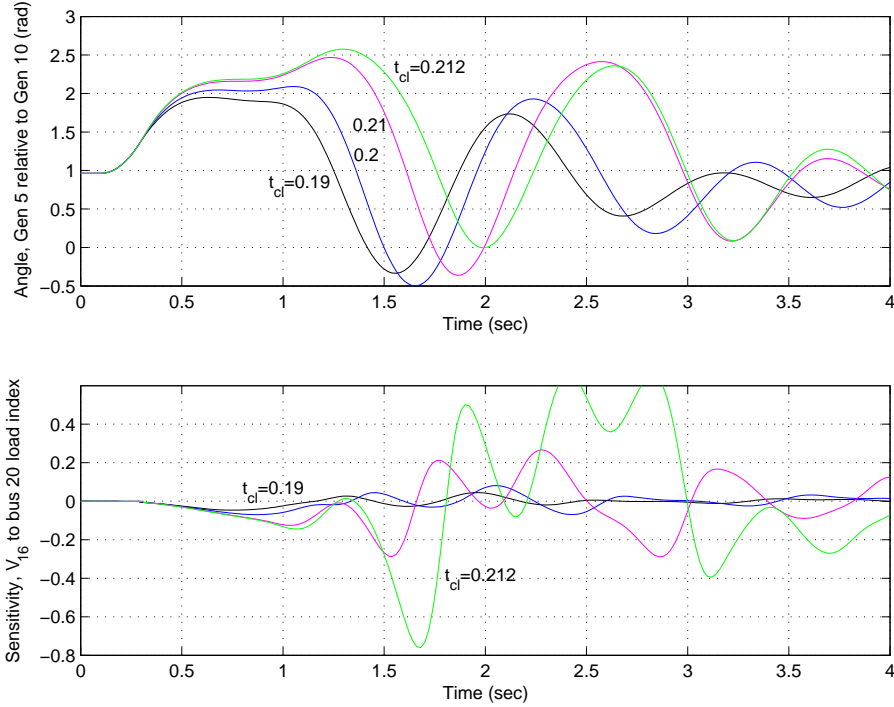


Figure 3: Trajectory and sensitivity variation for increasing system stress.

marginal stability. Further work is required though to fully exploit this phenomenon.

3.3 Load-induced variations in qualitative behaviour

Previous analysis and examples have suggested that load models have negligible qualitative influence on the behaviour of systems that are robustly stable. This will be further illustrated using the IEEE 39 bus system of Figure 1, though in this case the disturbance scenario involves a solid three-phase fault on line 16-21, at the bus 21 end. The fault was cleared after 0.15 sec by tripping the faulted line. That left buses 21 and 23, and generators 6 and 7, radially fed over line 23-24.

The loads at buses 23 and 24 were modelled according to (3), with ν_{23} and ν_{24} both nominally set to 0.5. In other words, both loads were composed of 50% static voltage dependent load and 50% induction motor load. The static load component was modelled as constant admittance, while the induction motor component used parameter values from [20, p. 305], with appropriate per unit scaling.

The response of generator 6 angle (relative to generator 1), under the nominal load conditions, is shown as a dashed line in Figure 4. The load composition parameters ν_{23} and ν_{24} were then varied between extremes of 0 and 1. The corresponding behaviour is shown as thick solid lines in Figure 4. Notice that this large variation in load composition, in the area directly affected by the disturbance, has negligible effect on the qualitative form of the response.

The fault clearing time was then increased to 0.18 sec, quite close to the critical clearing time of 0.18375 sec. Nominal behaviour is again shown as a dashed line, with behaviour corresponding to extremes in ν_{23} and ν_{24} shown as thinner solid lines. In this case, it turns

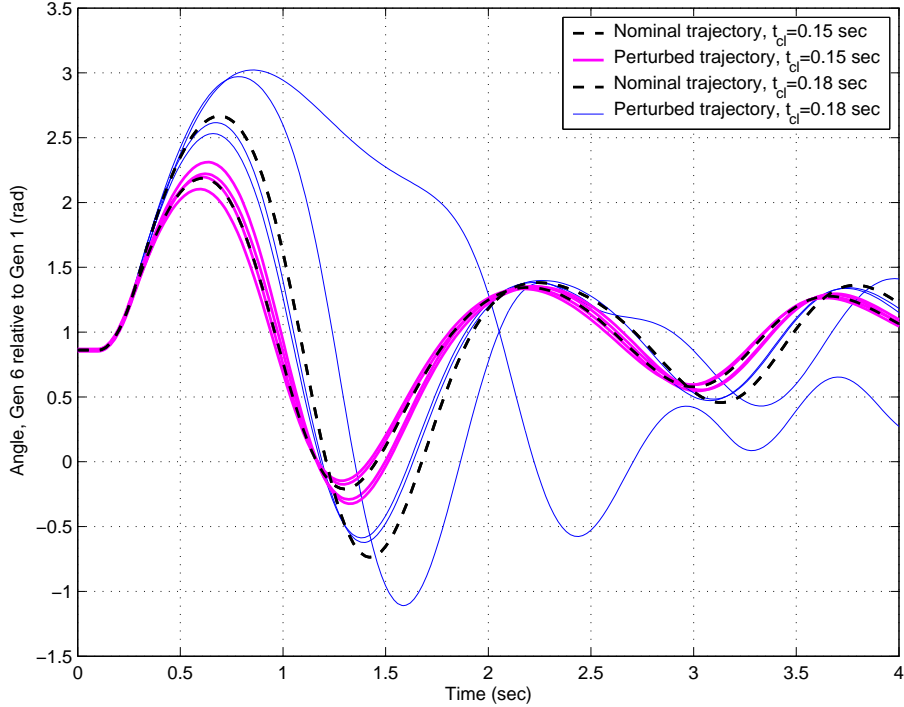


Figure 4: Influence of load parameter perturbations with increased system stress.

out that reduction of ν_{24} to 0 has a marked effect on the qualitative form of the response; the system is only just stable.

This example supports the hypotheses that load modelling only becomes important qualitatively when the system is close to instability, and that proximity to instability can be detected by high sensitivity.

3.4 Protection operation

The previous example showed that for unstressed systems, load composition has negligible effect on the qualitative form of behaviour. However that example did not take account of protection. In this example, zone 3 protection at the bus 23 end of line 23-24 will be considered. Figure 5 shows the separation² between the zone 3 mho characteristic [31] and the apparent impedance seen from bus 23. The dashed line was obtained for a fault clearing time of 0.15 sec, and using the nominal set of load parameters. It remains above zero, suggesting the zone 3 characteristic is not entered.

An uncertainty of ± 0.2 was assumed in both load composition parameters, so that

$$0.3 \leq \nu_{23}, \nu_{24} \leq 0.7. \quad (8)$$

Worst-case analysis [13] was used to explore bounds on behaviour, and in particular to determine whether this uncertainty could affect conclusions regarding protection operation.

Based on this nominal trajectory, sensitivities indicated that over the time frame of interest, where the trip signal approached zero, worst behaviour (lowest dip) occurred for load indices $\nu_{23} = 0.7$ and $\nu_{24} = 0.3$. Best behaviour (least dip) occurred for $\nu_{23} = 0.3$ and

²This distance goes negative when the apparent impedance enters the mho characteristic.

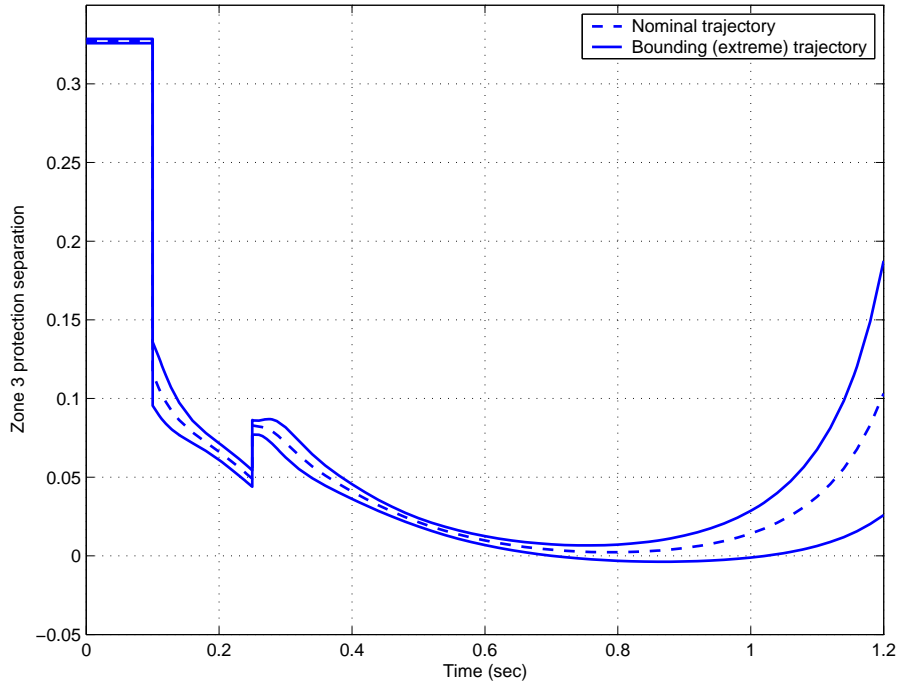


Figure 5: Zone 3 protection on line 23-24, worst-case bounds for $0.3 \leq \nu_{23}, \nu_{24} \leq 0.7$.

$\nu_{24} = 0.7$. The corresponding bounds on behaviour are shown as solid lines in Figure 5. Every selection of ν_{23} and ν_{24} from the range (8) results in a trajectory that lies within those bounds. Notice that the lower bound passes below zero, indicating the possibility of a zone 3 trip. Knowledge of the load composition is therefore very important in this case.

4 Conclusions

Quantitative analysis of power system dynamics, for example matching simulations to disturbance measurements, requires accurate load modelling. However load model accuracy is not so crucial for qualitative investigations, where the aim is to assess the likelihood of a certain disturbance scenario being stable or unstable. Though it should be kept in mind that most power system failures are not initiated by instability, but rather by reactionary (unanticipated) protection operation. Accurate load modelling can be very important in predicting such behaviour.

Trajectory sensitivities provide an efficient way of ranking the relative influence of parameters. Furthermore, as systems become more heavily stressed, sensitivity to parameter variation increases significantly. This characteristic can be used to predict disturbance scenarios that induce marginally stable behaviour.

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