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Distributed Output Feedback MPC for Power System Control

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Abstract— **In this paper, a distributed output feedback model predictive control (MPC) framework with guaranteed nominal stability and performance properties is described. Distributed state estimation strategies are developed for supporting distributed output feedback MPC of large-scale systems, such as power systems. It is shown that under certain (easily verifiable) conditions, local measurements are sufficient for observer stability. More generally, stable observers can be designed by exchanging measurements between adjacent subsystems. Both estimation strategies are suboptimal, but the estimates generated converge exponentially to the optimal estimates. A disturbance modeling framework for achieving zero-offset control in the presence of nonzero mean disturbances and modeling errors is presented. Automatic generation control (AGC) provides a practical example for contrasting the performance of centralized and distributed controllers.**

I. Introduction

Control of large, networked systems has traditionally been achieved by designing local, subsystem-based controllers that ignore the interactions between the different subsystems. It is well known that such a decentralized control philosophy may result in poor system-wide control performance if the subsystems interact significantly. Centralized MPC, on the other hand, is impractical for control of large-scale, geographically expansive systems, such as power systems. A distributed MPC framework becomes necessary. Such a control strategy was established in [1], where iterative exchange of information between subsystems allowed the performance benefits of centralized MPC to be realized. An overview of this state feedback controller is provided in Sections III and IV .

The state information required by the distributed MPC strategy is not available in many applications. A state estimation process is therefore required. Centralized state estimation is inconsistent with the goal of distributed control. The paper addresses this issue by establishing two distributed estimation procedures, one that relies on local measurements only, and the other that requires limited exchange of measurement information with adjacent subsystems.

Developing techniques to integrate subsystem-based MPCs is both a challenge and an opportunity. The potential requirements and benefits of cross-integration within the MPC framework has been discussed in [2], [3]. A distributed MPC algorithm for unconstrained, linear time-invariant (LTI) systems in which the dynamics of the subsystems are influenced by the states of interacting subsystems has been described in [4], [5]. In the above mentioned distributed MPC framework, the only information transferred between subsystem-based MPCs (agents) are their current policies. Competing agents have no knowledge of each others cost/utility functions. It is known that such strategies in which competing agents have no knowledge of each others cost functions converge to the Nash equilibrium (NE) [6], which is usually suboptimal in the Pareto sense [7], [8].

Automatic generation control (AGC) provides a topical example for illustrating the performance of distributed MPC in a power system setting. The purpose of AGC is to regulate the real power output of generators, with the aim of controlling system frequency and tie-line interchange [9]. AGC must account for various limits, including restrictions on the amount and rate of generator power deviations.

Flexible AC transmission system (FACTS) devices allow control of the real power flow over selected paths through a transmission network [10] . As transmission systems become more heavily loaded, such controllability offers economic benefits [11] . However FACTS controls must be coordinated with each other, and with AGC. Distributed MPC offers an effective means of achieving such coordination, whilst alleviating the organizational and computational burden associated with centralized control.

II. Models

Distributed MPC relies on decomposing the overall system model into appropriate subsystem models. A system comprised of M interconnected subsystems will be used to establish these concepts.

A. Centralized model

The overall system model is represented as a discrete, linear time-invariant (LTI) model of the form

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) \end{aligned} \tag{1}$$

in which k denotes discrete time (A, B) stabilizable, (A, C) detectable and

$$A = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1M} \\ \vdots & \vdots & \ddots & \vdots \\ A_{i1} & A_{i2} & \dots & A_{iM} \\ \vdots & \vdots & \ddots & \vdots \\ A_{M1} & A_{M2} & \dots & A_{MM} \end{bmatrix} \quad B = \begin{bmatrix} B_{11} & B_{12} & \dots & B_{1M} \\ \vdots & \vdots & \ddots & \vdots \\ B_{i1} & B_{i2} & \dots & B_{iM} \\ \vdots & \vdots & \ddots & \vdots \\ B_{M1} & B_{M2} & \dots & B_{MM} \end{bmatrix}$$

$$C = \begin{bmatrix} C_{11} & 0 & \dots & 0 \\ 0 & C_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & C_{MM} \end{bmatrix} \qquad u = \begin{bmatrix} u_1{}' & u_2{}' & \dots & u_M{}' \end{bmatrix}'$$

$$x = \begin{bmatrix} x_1{}' & x_2{}' & \dots & x_M{}' \end{bmatrix}' \qquad y = \begin{bmatrix} y_1{}' & y_2{}' & \dots & y_M{}' \end{bmatrix}'.$$

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We use the notation $\{1, M\}$ to denote the sequence of integers $1, 2, \ldots M$. For each subsystem $i \in \{1, M\}$, the triplet (u_i, x_i, y_i) represents the subsystem input, state and output vector respectively with $u_i \in \mathbb{R}^{m_i}, x_i \in \mathbb{R}^{n_i}$ and $y_i \in \mathbb{R}^{z_i}$. Define $n = \sum_i n_i, m = \sum_i m_i$ and $z = \sum_i z_i$.

B. Decentralized model

In the decentralized modeling framework, the effect of the external subsystems on the local subsystem is assumed to be negligible. The decentralized model for subsystem $i,\ i \in \{1, M\}$ is written as

$$\begin{aligned} x_i(k+1) &= A_{ii}x_i(k) + B_{ii}u_i(k) \\ y_i(k) &= C_{ii}x_i(k) \end{aligned} \tag{2}$$

C. Compound models (CM)

The CM for each subsystem i combines the effect of the local subsystem variables as well as the effect of the states and inputs of the interconnected subsystems. The CM for subsystem i follows directly from (1) and can be written as

$$\begin{aligned} x_i(k+1) &= A_{ii}x_i(k) + B_{ii}u_i(k) + \sum_{j \neq i}(A_{ij}x_j(k) + B_{ij}u_j(k)) \\ y_i(k) &= C_{ii}x_i(k) \end{aligned} \tag{3}$$

III. Distributed MPC

A. Preliminaries

The compound models for each subsystem $i \in \{1, M\}$ are assumed to be available. For the class of distributed MPC methods considered in this work, an iteration and exchange of variables between subsystems is performed during a sample time. We may choose not to iterate to convergence. The iteration number is denoted by p.

The set of admissible controls for subsystem $i, \Omega_i \subseteq \mathbb{R}^{m_i}$ is assumed to be a nonempty, compact, convex set containing the origin in its interior. For convenience, we define

$$\Omega_i = \{u_i \in \mathbb{R}^{m_i} | D_i u_i \leq d_i, d_i > 0\} \tag{4}$$

The set of admissible controls for the whole plant Ω is defined to be the Cartesian product of the admissible control sets of each of the subsystems. For subsystem i at time k, the predicted state vector at time $t > k$ is denoted by $x_i(t|k)$. By definition $x_i(k|k) \equiv x_i(k)$.

The *cost function* for subsystem i is defined over an infinite horizon and written as

$$\begin{aligned} &\phi_i\left(\boldsymbol{x}_i, \boldsymbol{u}_i; x_i(k)\right) \\ &\quad = \frac{1}{2}\sum_{t=k}^{\infty} x_i(t|k)' Q_i x_i(t|k) + u_i(t|k)' R_i u_i(t|k) \end{aligned} \tag{5}$$

in which $Q_i > 0,\ R_i > 0$ are symmetric weighting matrices and $\boldsymbol{x}_i(k) = [x_i(k+1|k)', x_i(k+2|k), \ldots\ldots]'$ and $\boldsymbol{u}_i(k) = [u_i(k|k)', u_i(k+1|k)', \ldots\ldots]'$.

Previous papers [12], [1], discuss existing distributed MPC strategies and their drawbacks. In particular, the unreliability of the class of communication-based strategies [1], in which each subsystem's MPC has no information about the objectives of the interconnected subsystems' MPCs, is demonstrated.

[1]Similar strategies have been proposed by [4], [5]

B. Feasible cooperation-based MPC (FC-MPC)

To arrive at a reliable, distributed, systemwide MPC framework, we modify the objectives of the subsystems' MPCs to provide a means for cooperative behavior among the controllers. Each local controller objective ϕ_i is replaced by one that measures the systemwide impact of local control actions. Here, we choose the simplest such measure, the overall plant objective which is a strong convex combination of the individual subsystems' objectives $i.e.$, $\phi = \sum w_i \phi_i,\ w_i > 0,\ \sum w_i = 1$ [2].

For notational simplicity, we drop the time dependence of $(\boldsymbol{x}_i^p(k), \boldsymbol{u}_i^p(k))$ and represent it as $(\boldsymbol{x}_i^p, \boldsymbol{u}_i^p)$. The control horizon is denoted by N. For each subsystem i at iteration p, only the subsystem input sequence $\boldsymbol{u}_i^p$ is optimized and updated. The other subsystems' inputs are not altered during this optimization; subsystem i holds their values at $\boldsymbol{u}_j^{p-1}, j \neq i$.

In large-scale implementations, the system sampling interval may be insufficient for convergence of the iterative, cooperation-based algorithm. In such cases, the cooperation-based algorithm has to be terminated prior to convergence of the exchanged input trajectories. The last calculated input trajectories are used to define a suitable control law. To facilitate intermediate termination, it is imperative that all iterates generated by the cooperation-based algorithm are systemwide feasible (*i.e.*, satisfy all model and inequality constraints) and the resulting distributed control law is closed-loop stable. In the following section, a distributed MPC algorithm (Algorithm 1) that maintains strict feasibility of all intermediate iterates is described. Algorithm 1 also allows the definition of a distributed control law (for both state and output feedback) that assures nominal closed-loop stability for all values of the iteration number.

Define the finite sequences $\overline{\boldsymbol{x}}_i(k)' = [x_i(k+1|k)', \ldots, x_i(k+N|k)']$ and $\overline{\boldsymbol{u}}_i(k)' = [u_i(k|k)', u_i(k+1|k)', \ldots, u_i(k+N-1|k)']$. For convenience, we drop the k dependence of $\overline{\boldsymbol{x}}_i$ and $\overline{\boldsymbol{u}}_i$. It is shown in [1] that for each $i \in \{1, M\}$, $\overline{\boldsymbol{x}}_i$ can be expressed as

$$\overline{\boldsymbol{x}}_i = E_{ii}\overline{\boldsymbol{u}}_i + f_{ii}x_i(k) + \sum_{j \neq i}[E_{ij}\overline{\boldsymbol{u}}_j + f_{ij}x_j(k)]. \tag{6}$$

in which E_{ij}, f_{ij} are functions of the subsystem model matrices A_{ij}, B_{ij}, $\forall\ i, j \in \{1, M\}$.

The input trajectory $\boldsymbol{u}_i$ is obtained by augmenting $\overline{\boldsymbol{u}}_i$ with the input sequence $u_i(t|k) = 0, k+N \leq t$. The state trajectory $\boldsymbol{x}_i$ is derived from $\overline{\boldsymbol{x}}_i$ by propagating the terminal state $x_i(k+N|k)$ using (3) and $u_i(t|k) = 0, k+N \leq t, \forall\ i \in \{1, M\}$. For subsystem i, the FC-MPC optimization problem is

$$\min_{\boldsymbol{u}_i} \quad \sum_{r=1}^{M} w_r \Phi_r\left(\boldsymbol{u}_1^{p-1}, \ldots, \boldsymbol{u}_{i-1}^{p-1}, \boldsymbol{u}_i, \boldsymbol{u}_{i+1}^{p-1}, \ldots, \boldsymbol{u}_M^{p-1}; x_r(k)\right) \tag{7a}$$

subject to

$$u_i(j|k) \in \Omega_i, \quad k \leq j \leq k+N-1 \tag{7b}$$

$$u_i(j|k) = 0, \quad k+N \leq j \tag{7c}$$

[2]In contrast, each communication-based MPC optimizes over its local objective. Convergence of this formulation is assumed and is therefore a drawback. At convergence of communication-based MPC, the NE solution (suboptimal) is obtained.

The cost function $\Phi_i(\cdot)$ is obtained by eliminating the state trajectory $\boldsymbol{x}_i$ from (5). The solution to the above optimization problem is denoted by $\boldsymbol{u}_i^{p(*)}$. By definition,

$$\overline{\boldsymbol{u}}_i^{p(*)\prime} = [u_i^{p(*)}(k|k)', u_i^{p(*)}(k+1|k)', \ldots, u_i^{p(*)}(k+N-1|k)']$$

Details of the state feedback distributed MPC framework for stable power systems is available in [1]. A brief description is included in this paper for the sake of completeness.

IV. Implementable FC-MPC algorithm

For $\phi_i(\cdot)$ convex, quadratic (5), the FC-MPC optimization problem for each subsystem $i \in \{1, M\}$ can be written explicitly. Details of the optimization problem are given in Appendix A.

A. FC-MPC algorithm and properties

The state sequence generated by the input sequence $\boldsymbol{u}$ and initial state z is represented as $\boldsymbol{x}^{(\boldsymbol{u};z)}$. The following algorithm is employed for cooperation-based distributed MPC.

Algorithm 1:

Given $\left(\boldsymbol{u}_i^0, x_i(k)\right), \mathbb{Q}_i \geq 0, \mathbb{R}_i \geq 0, i \in \{1, M\}$
$p_{\max}(k) \geq 0$ and $\epsilon > 0$
$p \leftarrow 1, \rho_i \leftarrow \Gamma\epsilon, \Gamma \gg 1$
while $\rho_i > \epsilon$ for some $i \in \{1, M\}$ and $p \leq p_{\max}$
do $\forall\ i \in \{1, M\}$
$\boldsymbol{u}_i^{p(*)} \in \arg\ \min$ FC-MPC$_i$, (see (7), (24))
end (do)
for each $i \in \{1, M\}$
$\boldsymbol{u}_i^p = w_i \boldsymbol{u}_i^{p(*)} + (1 - w_i)\, \boldsymbol{u}_i^{p-1}$
$\rho_i = \|\boldsymbol{u}_i^p - \boldsymbol{u}_i^{p-1}\|$
end (for)
Transmit $\boldsymbol{u}_i^p,\ \forall\ i \in \{1, M\}$ among interconnected subsystems.
$\boldsymbol{x}_i^p \leftarrow \boldsymbol{x}_i^{[\boldsymbol{u}_1^p, \boldsymbol{u}_2^p, \ldots, \boldsymbol{u}_M^p]; x(k))},\ \forall\ i \in \{1, M\}$
$p \leftarrow p + 1$
end (while)

Denote the cooperation-based cost function after p iterates by $\Phi([\boldsymbol{u}_1^p, \boldsymbol{u}_2^p, \ldots, \boldsymbol{u}_M^p]; x(k))$. Therefore,

$$\Phi([\boldsymbol{u}_1^p, \boldsymbol{u}_2^p, \ldots, \boldsymbol{u}_M^p]; x(k)) = \sum_{r=1}^{M} w_r \Phi_r\left([\boldsymbol{u}_1^p, \boldsymbol{u}_2^p, \ldots, \boldsymbol{u}_M^p]; x_r(k)\right)$$

The following properties can be established for the FC-MPC formulation (7), (24) employing Algorithm 1.

Lemma 1: Given the distributed MPC formulation FC-MPC$_i$ defined in (7) and (24), $\forall\ i \in \{1, M\}$, the sequence of cost functions $\{\Phi([\boldsymbol{u}_1^p, \boldsymbol{u}_2^p, \ldots, \boldsymbol{u}_M^p]; x(k))\}$ generated by Algorithm 1 is a nonincreasing function of the iteration number p.

Using Lemma 1 and the fact that $\Phi(\cdot)$ is bounded below assures convergence.

Lemma 2: All limit points of Algorithm 1 are optimal.

Lemma 2 implies that the solution obtained at convergence of Algorithm 1 is within a pre-specified tolerance of the centralized MPC solution.

B. Distributed MPC control law under state feedback

Let $\mathcal{X}$ represent the constrained stabilizable set for the system under the set of input constraints $\Omega_1 \times \Omega_2 \times \ldots \times \Omega_M$. At time k, let the FC-MPC algorithm (Algorithm 1) be terminated after $p(k) = q$ iterates, with

$$\boldsymbol{u}_i^q(x(k)) = \left[u_i^q(x(k), k)', u_i^q(x(k), k+1)', \ldots, \right]', \quad \forall\ i \in \{1, M\} \tag{8}$$

representing the solution to Algorithm 1 after q cooperation-based iterates. The distributed MPC control law is obtained through a receding horizon implementation of optimal control whereby the input applied to subsystem i is

$$u_i(k) = u_i^q(k|k) \equiv u_i^q(x(k), k). \tag{9}$$

For open-loop stable systems, nominal exponential closed-loop stability under the state feedback distributed MPC control law can be established for all $x(k) \in \mathcal{X}$ and all $p(k) > 0$ (see [1] for details).

V. Distributed MPC for unstable/integrating systems

For A containing unstable or integrating modes, a terminal state constraint that forces the unstable modes to the origin at the end of the control horizon is necessary to ensure closed-loop stability in the distributed MPC framework. Let $\mathcal{X}_N$ denote the N-step constrained stabilizable set for the system. It is assumed that $x(k) \in \mathcal{X}_N$. The real Schur decomposition of A is defined as

$$A = [U_s \quad U_e] \begin{bmatrix} A_s & \mathbb{A} \\ & A_e \end{bmatrix} \begin{bmatrix} U_s' \\ U_e' \end{bmatrix}$$

in which A_s and A_e represent the stable and unstable eigenvalue blocks of A respectively.

For unstable or integrating systems, the terminal state constraint $U_e'x(k+N|k) = \sum_i U_{e_i}'x_i(k+N|k) = \sum_i \beta_i'\overline{\boldsymbol{x}}_i = 0$ is necessary to ensure closed-loop stability. Using (6) and (29), we define $\forall\ i \in \{1, M\}$

$$S_i = \sum_{j=1}^{M} \beta_j' E_{ji}\ ,\ s \equiv s(x(k)) = \sum_{j=1}^{M} \beta_j' \boldsymbol{g}_j \tag{10}$$

The terminal state constraint $U_e'x(k+N|k) = 0$ can therefore be re-written as

$$\sum_{j=1}^{M} S_j \overline{\boldsymbol{u}}_j + s = 0 \tag{11}$$

An exact penalty approach is employed to enforce the coupled input constraint (11). The FC-MPC optimization problem for each subsystem $i \in \{1, M\}$ is written as

$$\min_{\overline{\boldsymbol{u}}_i, v} \quad \frac{1}{2}\overline{\boldsymbol{u}}_i'\mathcal{R}_i^u\overline{\boldsymbol{u}}_i + \gamma'v + \left(\sum_{j=1}^{M} E_{ji}' \sum_{s \neq j} \mathbb{M}_{js} \sum_{l \neq i}(E_{sl}\overline{\boldsymbol{u}}_l^{p-1} + \boldsymbol{g}_s) + \sum_{j=1}^{M} E_{ji}'\mathbb{Q}_j \sum_{l \neq i}(E_{jl}\overline{\boldsymbol{u}}_l^{p-1} + \boldsymbol{g}_j) + S_i'\Gamma \sum_{j \neq i}\left(S_j\overline{\boldsymbol{u}}_j^{p-1} + s\right)\right)'\overline{\boldsymbol{u}}_i$$

subject to

$$u_i(j|k) \in \Omega_i, \quad k \leq j \leq k+N-1 \tag{12a}$$

$$-v \leq S_i\overline{\boldsymbol{u}}_i + \sum_{j \neq i} S_j \overline{\boldsymbol{u}}_j^{p-1} + s \leq v \tag{12b}$$

in which

$$\mathcal{R}_i^u = \mathcal{R}_i + S_i'\Gamma S_i, \qquad \Gamma = \delta I,$$
$$\gamma = \sigma[1, 1, \ldots, 1]', \quad \sigma, \delta \gg 1$$

and the terminal penalty $\overline{\mathbb{Q}}$ (see (30)) is given by $U_s\Sigma U_s'$, and is obtained as the solution to the centralized Lyapunov equation

$$A_s'\Sigma\ A - \Sigma = -U_s'\mathbb{Q}U_s \tag{13}$$

At time $k = 0$, a feasible input trajectory for each subsystem is generated by solving a linear program.

Closed-loop stability for the nominal system can be established for all $x(k) \in \mathcal{X}_N$ and all $p(k) > 0$. However, as a consequence of the coupled constraint (11), the solution obtained at convergence of Algorithm 1 can no longer be guaranteed to be optimal.

VI. Distributed state estimation for FC-MPC

All the states of a system are seldom measured. Estimating the states of the system from available measurements constitutes a key component of any realistic MPC formulation. Theory for centralized estimation is well understood. For many large, networked systems, organizational and geographic constraints may preclude the use of centralized estimation strategies. A plant decomposition algorithm for parallel state estimation was proposed in [13]. A decentralized state estimation strategy for large-scale state estimation was described in [14].

The conditional density of the subsystem state x_i, given the set of measurements $y_i,\ \forall\ i \in \{1, M\}$ is assumed to be normally distributed. For each subsystem i, the vectors $w_{x_i} \sim N(0, Q_{x_i}) \in \mathbb{R}^{n_{w_i}}$ and $\nu_i \sim N(0, R_{v_i}) \in \mathbb{R}^{z_i}$ denote the zero-mean disturbances on the subsystem model state equation and output equation respectively. $G_i \in \mathbb{R}^{n_i \times n_{w_i}}$ denotes the shaping matrix for the state disturbance w_{x_i}. The state and output equation for each subsystem $i \in \{1, M\}$ is written as

$$x_i(k+1) = A_{ii}x_i(k) + B_{ii}u_i(k) + \sum_{j\neq i}[A_{ij}x_j(k) + B_{ij}u_j(k)] + G_i w_{x_i} \tag{14a}$$

$$y_i(k) = C_{ii}x_i(k) + \nu_i \tag{14b}$$

At time k, let $\widehat{x}_i(k|k-1)$ represent an estimate of the states of subsystem i given measurements up to and including time $k-1$, obtained using a distributed estimation strategy. The observer predictor equation for subsystem i is written as

$$\begin{aligned}\widehat{x}_i(k+1|k) &= A_{ii}\widehat{x}_i(k|k-1) + B_{ii}u_i(k) + \sum_{j\neq i}\Big[A_{ij}\widehat{x}_j(k|k-1) \\ &+ B_{ij}u_j(k)\Big] + L_{ii}\Big[y_i(k) - C_i\widehat{x}_i(k|k-1)\Big] \\ &+ \sum_{j\neq i} L_{ij}\Big[y_j(k) - C_j\widehat{x}_j(k|k-1)\Big]\end{aligned} \tag{15}$$

Define

$$L = \begin{bmatrix} L_{11} & L_{12} & \dots & L_{1M} \\ L_{21} & L_{22} & \dots & L_{2M} \\ \vdots & \ddots & \ddots & \vdots \\ L_{M1} & L_{M2} & \dots & L_{MM}\end{bmatrix} \quad \mathcal{A}_i = \begin{bmatrix} 0 & A_{12} & \dots & A_{1M} \\ A_{21} & 0 & \dots & A_{2M} \\ \vdots & \ddots & \ddots & \vdots \\ A_{M1} & A_{M2} & \dots & 0\end{bmatrix}$$

$$\mathcal{A}_d = \text{diag}(A_{11}, A_{22}, \dots, A_{MM}) \qquad A = \mathcal{A}_d + \mathcal{A}_i$$

$$\mathcal{B}_d = \text{diag}(B_{11}, B_{22}, \dots, B_{MM}) \qquad \mathcal{C}_d = C$$

Since (A, C) is detectable, a gain matrix L may be selected to ensure the desired degree of estimator convergence. Let $\widehat{x}_i^*, i \in \{1, M\}$ denote the optimal state estimate (centralized Kalman filter) and let L^* represent the corresponding steady-state centralized Kalman predictor gain (optimal).

A. Distributed estimation with local measurements

Let (A_{ii}, C_{ii}) detectable for each $i \in \{1, M\}$. Assume that the subsystems are completely decoupled *i.e.*, $A_{ij} = B_{ij} = 0$. For each decoupled subsystem $i \in \{1, M\}$, it is possible to construct local, steady-state observers of the form

$$\begin{aligned}\widehat{x}_i^d(k+1|k) &= A_{ii}\widehat{x}_i^d(k|k-1) + B_{ii}u_i(k) \\ &+ L_{ii}^d\Big[y_i(k) - C_{ii}\widehat{x}_i^d(k|k-1)\Big]\end{aligned} \tag{16a}$$

$$\text{in which } L_{ii}^d = A_{ii}P_{ii}C_{ii}'\left(R_{v_i} + C_{ii}P_{ii}C_{ii}'\right)^{-1}, \tag{16b}$$

$$P_{ii} = G_iQ_{x_i}G_i' + A_{ii}P_{ii}A_{ii}' - L_{ii}^dC_{ii}P_{ii}A_{ii}' \tag{16c}$$

and $\widehat{x}_i^d$ represents an estimate of the subsystem states using the decentralized estimation strategy. Since (A_{ii}, C_{ii}) detectable, $(A_{ii} - L_{ii}^dC_{ii})$ is stable.

Let $\mathcal{L}_d = \text{diag}(L_{11}^d, L_{22}^d, \dots, L_{MM}^d)$, $e_i^d(k) = x_i(k) - \widehat{x}_i^d(k|k-1)$ and $e(k) = [e_1(k)', e_2(k)', \dots, e_M(k)]'$. For the decoupled system $(\mathcal{A}_d, \mathcal{B}_d, \mathcal{C}_d)$, the set of decentralized steady-state estimators are stable and optimal. However, in general, $A_{ij}, B_{ij} \neq 0$. Stability of the set of decentralized steady-state estimators is assured iff $|\lambda_{\max}(\mathcal{H}(\mathcal{L}_d))| < 1$ in which $\lambda_{\max}$ denotes the maximum eigenvalue and

$$\mathcal{H}(\mathcal{L}_d) = (\mathcal{A}_d - \mathcal{L}_d\mathcal{C}_d) + \mathcal{A}_i \tag{17}$$

The observer predictor equation for subsystem i is

$$\begin{aligned}\widehat{x}_i(k+1|k) &= A_{ii}\widehat{x}_i(k|k-1) + B_{ii}u_i(k) + \sum_{j\neq i}[A_{ij}\widehat{x}_j(k|k-1) \\ &+ B_{ij}u_j(k)] + L_{ii}^d\left[y_i(k) - C_{ii}\widehat{x}_i(k|k-1)\right]\end{aligned} \tag{18}$$

B. Distributed estimation with measurement exchange

The advantage of the approach in Section VI-A is that it requires only local measurements for subsystem state estimation. In many situations, however, it may not be possible to find a $\mathcal{L}_d$ that satisfies $|\lambda_{\max}(\mathcal{H}(\mathcal{L}_d))| < 1$ and gives an acceptable degree of estimator convergence. The following lemma establishes a design procedure for distributed estimation. A similar design procedure for distributed state estimation for continuous time systems is described in [15].

Lemma 3: Let (A, C) be detectable and let (A_{ii}, C_{ii}) detectable for each $i \in \{1, M\}$. The set of steady-state subsystem-based distributed observers given by (15) with

- $L_{ii} = L_{ii}^d$ (from (16))
- P_{ii} obtained as the solution to the Riccati equation (16)
- $L_{ij} = A_{ij}C_{jj}'(C_{jj}C_{jj}')^{-1}$

for all $i, j \in \{1, M\}, j \neq i$, is stable. Further, the estimator error $e(k)$ decays to zero at the same rate as that for the set of decentralized steady-state estimators (16) designed for the system $(\mathcal{A}_d, \mathcal{B}_d, \mathcal{C}_d)$ in which the interconnections are identically zero.

C. Suboptimality and convergence

Lemma 4: Given (A, C) detectable and $\tau_i(k) = \widehat{x}_i(k|k-1) - \widehat{x}_i^*(k|k-1)$, in which $\widehat{x}_i(k|k-1)$ is the subsystem state estimate obtained using the distributed estimation strategy described in either Section VI-A or Section VI-B, then $\tau_i(k) \to 0, \forall\ i \in \{1, M\}$ exponentially.

Remark 1: The distributed estimation strategies of Sections VI-A, VI-B are both suboptimal estimation strategies. In general, it is not possible to a priori establish which suboptimal distributed estimation strategy will yield superior estimates. However, using Lemma 4, we know that the subsystem state estimates obtained using either distributed estimation procedure (Section VI-A or VI-B) converge to the optimal subsystem state estimates (obtained using a centralized Kalman filter, for example) exponentially.

VII. Distributed MPC under output feedback

Let the states of each subsystem $i \in \{1, M\}$ be estimated using distributed observers designed using the approach described in either Section VI-A or Section VI-B. At time k, let the FC-MPC algorithm (Algorithm 1) be terminated after $p(k) = q$ iterates. For notational convenience, we write $\widehat{x}_i(k) \equiv \widehat{x}_i(k|k)$. The solution to Algorithm 1 after q cooperation-based iterates is represented as

$$\boldsymbol{u}_i^q(\widehat{x}(k)) = \left[u_i^q(\widehat{x}(k), k)', u_i^q(\widehat{x}(k), k+1)', \ldots, \right]', \qquad \forall\, i \in \{1, M\} \tag{19}$$

The input injected into each subsystem i,under the output feedback distributed MPC control law, is

$$u_i(k) = u_i^l(k|k) \equiv u_i^l(\widehat{x}(k), k). \tag{20}$$

Exponential stability of the closed-loop system under the output feedback distributed MPC control law is assured by the following theorem, which requires that the local observers are exponentially stable but makes no assumptions on the optimality of the obtained state estimates.

Theorem 1: Given Algorithm 1 and the distributed MPC formulation (7), (24) with $N \geq 1$. Let the subsystem states be estimated using a set of distributed steady-state observers designed using the approach described in either Section VI-A or VI-B. If A is stable, $\overline{\mathbb{Q}}$ is obtained from (31), the distributed control law defined in (20) and

$$\begin{aligned} Q_i(0) = Q_i(1) = \ldots = Q_i(N-1) = Q_i > 0 \\ R_i(0) = R_i(1) = \cdots = R_i(N-1) = R_i > 0 \\ \forall\, i \in \{1, M\} \end{aligned}$$

then the origin is an exponentially stable equilibrium for the closed-loop system

$$x(k+1) = Ax(k) + Bu(\widehat{x}(k))$$

in which

$$u(\widehat{x}(k)) = \left[u_1^{p(k)}(\widehat{x}(k), k)', \ldots, u_M^{p(k)}(\widehat{x}(k), k)'\right]'$$

for all $\widehat{x}(k) \in \mathcal{X}$ and all $p(k) = 1, 2, \ldots$

A similar result can be established for systems with unstable/integrating modes employing the FC-MPC framework under output feedback. The details are omitted due to space constraints.

VIII. Disturbance modeling for FC-MPC

Disturbance models are employed to eliminate steady-state offset in the presence of nonzero mean, constant disturbances. Inclusion of disturbance models is a prerequisite in any practical MPC implementation. Presently, the constant output disturbance model is the most widely used disturbance model to achieve zero steady-state offset [16], [17]. However, inspite of its simplicity, the output disturbance model may lead to poor closed-loop performance. Output disturbance models are also unsuitable for use in plants with integrating modes as the effects of the augmented disturbance and the plant integrating mode cannot be distinguished. The idea of using a input disturbance model to eliminate steady-state offset was first proposed by [18] for the linear quadratic regulator (LQR).

For single MPCs [19], [20] derive conditions that permit zero off-set control, using suitable disturbance models, in the presence of unmodelled effects and/or nonzero mean disturbances. In a distributed MPC framework, many choices of disturbance models are possible. From a practitioner's standpoint, it is usually convenient to use local integrating disturbances.

For each subsystem $i \in \{1, M\}$, the subsystem state vector $\widehat{x}_i$ is augmented with the integrating disturbance vector $\widehat{d}_i$. The augmented subsystem model for subsystem i is

$$\begin{bmatrix}\widehat{x}_i \\ \widehat{d}_i\end{bmatrix}(k+1) = \widetilde{A}_{ii}\begin{bmatrix}\widehat{x}_i \\ \widehat{d}_i\end{bmatrix}(k) + \widetilde{B}_{ii}u_i(k) + \sum_{j \neq i}\left[\widetilde{A}_{ij}\begin{bmatrix}\widehat{x}_j \\ \widehat{d}_j\end{bmatrix}(k) + \widetilde{B}_{ij}u_j(k)\right] \tag{21a}$$

$$y_i(k) = \widetilde{C}_{ii}\begin{bmatrix}\widehat{x}_i \\ \widehat{d}_i\end{bmatrix}(k) \tag{21b}$$

in which

$$\widetilde{A}_{ii} = \begin{pmatrix} A_{ii} & B_{ii}^d \\ 0 & I \end{pmatrix} \qquad \widetilde{A}_{ij} = \begin{pmatrix} A_{ij} & 0 \\ 0 & 0 \end{pmatrix}$$
$$\widetilde{B}_{ii} = \begin{pmatrix} B_{ii} \\ 0 \end{pmatrix} \qquad \widetilde{B}_{ij} = \begin{pmatrix} B_{ij} \\ 0 \end{pmatrix}$$
$$\widetilde{C}_{ii} = \begin{pmatrix} C_{ii} & P_{ii}^d \end{pmatrix} \qquad \forall\, i, j \in \{1, M\},\ j \neq i$$

in which $\widehat{d}_i \in \mathbb{R}^{n_{d_i}}$, $B_{ii}^d \in \mathbb{R}^{n_i \times n_{d_i}}$, $P_{ii}^d \in \mathbb{R}^{z_i \times n_{d_i}}$. The pair (B_{ii}^d, P_{ii}^d) represent the input–output disturbance models for subsystem i.

Lemma 5: For each subsystem $i \in \{1, M\}$, let (A_{ii}, C_{ii}) be detectable. The augmented model $(\widetilde{A}_{ii}, \widetilde{C}_{ii})$ is detectable if

$$\operatorname{rank}\begin{bmatrix} I - A_{ii} & -B_{ii}^d \\ C_{ii} & P_{ii}^d \end{bmatrix} = n_i + n_{d_i} \tag{22}$$

and $n_{d_i} \leq z_i$.

Zero off-set steady-state tracking performance can be established in the FC-MPC framework using the following lemma.

Lemma 6: Given (A, B) stabilizable and let $\forall\ i \in \{1, M\}$

- (A_{ii}, C_{ii}), $(\widetilde{A}_{ii}, \widetilde{C}_{ii})$ detectable
- $n_{d_i} = z_i$

If the closed-loop system under FC-MPC is stable and none of the input constraints are active at steady state, then the FC-MPCs with distributed steady-state observers, designed using the approach described in either Section VI-A or VI-B, and local disturbance models track their respective output targets with zero steady-state offset.

IX. Examples

A. Performance comparison

The examples use the cumulative stage cost as an index for comparing the performance of different controller paradigms. Accordingly, define

$$\Lambda = \frac{1}{t}\sum_{k=0}^{t-1}\sum_{i=1}^{M}\frac{1}{2}\left[x_i(k)'Q_ix_i(k) + u_i(k)'R_iu_i(k)\right]. \tag{23}$$

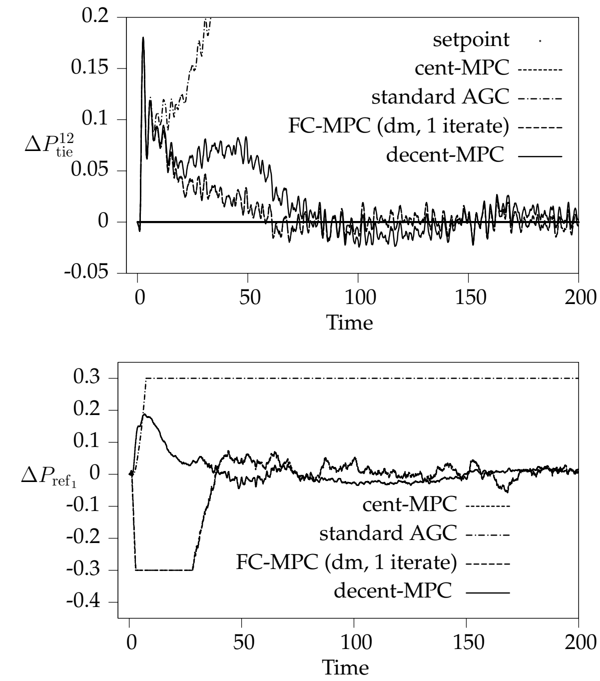

Fig. 1. Change in tie line power flow ($\Delta P^{12}_{\text{tie}}$) and load reference setpoint for area 1 (ΔP_{ref_1}). FC-MPC (dm) employs the distributed estimation strategy described in Section VI-B.

TABLE I
PERFORMANCE OF DIFFERENT CONTROL FORMULATIONS W.R.T. CENT-MPC, $\Delta\Lambda\% = \frac{\Lambda_{\text{config}} - \Lambda_{\text{cent}}}{\Lambda_{\text{cent}}} \times 100$.

	Λ	$\Delta\Lambda\%$
cent-MPC	0.64	−
standard AGC	∞	
decent-MPC	0.97	51.6
FC-MPC (1 iterate) (centralized estimation)	0.643	0.49
FC-MPC (dm, 1 iterate) (distributed estimation (VI-B))	0.644	0.52

B. Two area power system network

A power system AGC example with two control areas interconnected through a tie line is considered. In the distributed MPC framework, each control area employs a MPC to reject disturbances due to load fluctuations. The MPCs drive the frequency and tie line power flow deviations $(\Delta\omega_i, \Delta P^{ij}_{\text{tie}})$ to zero by manipulating the load reference setpoint ΔP_{ref_i}. The performance of the different MPC frameworks is compared against standard AGC. In particular, the performance of standard AGC (with anti-reset windup), centralized MPC (cent-MPC), decentralized MPC (decent-MPC) and FC-MPC is assessed when a 25% load disturbance affects area 2.

Each MPC uses an observer to estimate the relevant states from noisy measurements and an input disturbance model to eliminate steady-state offset. In the FC-MPC framework, the distributed controller is defined by terminating Algorithm 1 after 1 iterate. The performance of the different control frameworks rejecting the tie line power flow transients and the corresponding load reference input profile for area 1 is shown in Fig. 1. The closed-loop performance of the different control formulations are compared in Table I. In this case, the stability condition $|\lambda_{\max}\mathcal{H}(\mathcal{L}_d)| < 1$ is satisfied and the distributed estimation strategy of Section VI-A may be used. However, for this example, we will use the distributed state estimation strategy of Section VI-B for estimating subsystem states in the FC-MPC framework (FC-MPC (dm)).

Under the influence of the load disturbance, the inputs under standard AGC saturate at their bound constraints and the resulting system exhibits closed-loop unstable behavior. The FC-MPC framework, terminated after 1 cooperation-based iterate, and employing a centralized Kalman filter achieves performance that is within 0.5% of the optimal, centralized MPC performance. If the subsystem states are estimated using the distributed estimation strategy of Section VI-B, the performance loss (relative to centralized MPC) incurred with the FC-MPC formulation terminated after 1 cooperation-based iterate is $\sim 0.52\%$. In fact, the performance of the FC-MPC framework with distributed estimation is almost indistinguishable from that of centralized MPC. For the sake of comparison, the set of distributed estimators designed using Section VI-A results in a performance loss of $\sim 0.6\%$ relative to centralized MPC.

C. Four area power system network

We consider an example with four interconnected control areas as shown in Fig. 2. Power flow through tie line connections $1-2$, $2-3$, $3-4$ are the sources of interactions between the control areas. The performance of cent-MPC, decent-MPC and FC-MPC are analyzed when there is a 25% load increase in area 2 and a simultaneous 25% load drop in area 3. For the distributed MPC framework, two cases are considered. In the first case, the states of the system are estimated using a centralized Kalman filter and in the second case, the states of each subsystem are estimated using the distributed estimation methodology described in Section VI-A (FC-MPC (lm)). The latter case is a feasible framework for distributed estimation since the stability condition $|\lambda_{\max}\mathcal{H}(\mathcal{L}_d)| < 1$ is satisfied. Another advantage of the estimation strategy of Section VI-A is that only local measurements are required to estimate subsystem states. In both cases, Algorithm 1 is terminated after 1 cooperation-based iterate. An input disturbance model is used in each MPC to eliminate steady-state offset. The load reference setpoint (P_{ref}) in each area is manipulated to reject the load disturbances and drive the deviation in frequencies and tie line power flows to zero.

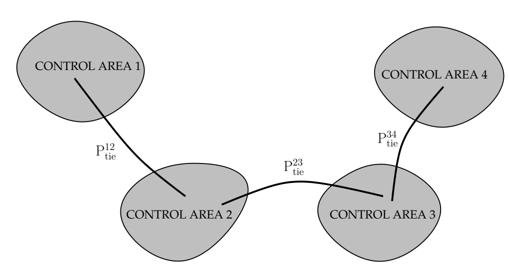

Fig. 2. Four area power network.

TABLE II

PERFORMANCE OF DIFFERENT CONTROL FORMULATIONS W.R.T. CENT-MPC, $\Delta\Lambda\% = \frac{\Lambda_{\text{config}} - \Lambda_{\text{cent}}}{\Lambda_{\text{cent}}} \times 100$.

	Λ	$\Delta\Lambda\%$
cent-MPC	0.24	−
decent-MPC	∞	
FC-MPC (1 iterate) (centralized estimation)	0.26	8
FC-MPC (lm, 1 iterate) (distributed estimation (VI-A)	0.28	15.5

TABLE III

PERFORMANCE OF DIFFERENT CONTROL FORMULATIONS W.R.T. CENT-MPC, $\Delta\Lambda\% = \frac{\Lambda_{\text{config}} - \Lambda_{\text{cent}}}{\Lambda_{\text{cent}}} \times 100$.

	Λ	$\Delta \times 10^2\Lambda\%$
cent-MPC	1.9	−
decent-MPC	2.6	37
FC-MPC (dm, 1 iterate) (distributed estimation (VI-B)	2.1	1.83
FC-MPC (lm, 1 iterate) (distributed estimation (VI-A)	1.93	0.84

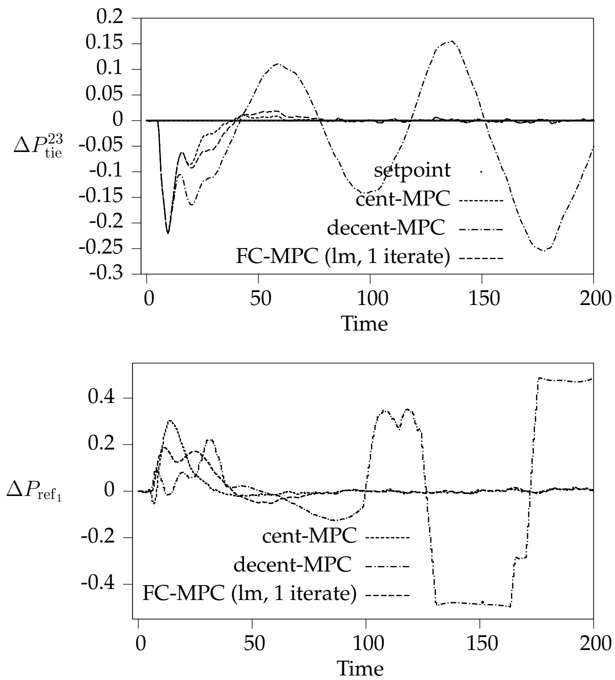

Fig. 3. Change in tie line flow ($\Delta P_{\text{tie}}^{23}$), and load reference setpoint for area 1 (ΔP_{ref_1}). FC-MPC (lm) employs the distributed estimation strategy described in Section VI-A.

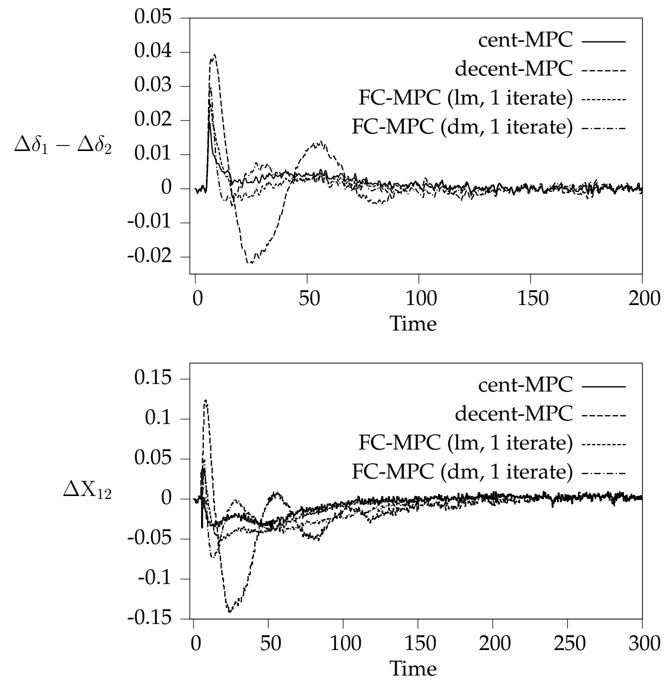

Fig. 4. Relative phase difference ($\Delta\delta_1 - \Delta\delta_2$), and change in FACTS impedance (ΔX_{12}). FC-MPC (lm) and FC-MPC (dm) employ the distributed estimation frameworks described in Section VI-A and Section VI-B respectively.

The performances of the different control frameworks rejecting the tie line power flow transients between areas 2 and 3 and the corresponding load reference input profile for area 1 are shown in Fig. 3. A closed-loop performance comparison of the different MPC frameworks is given in Table II. We observe from Fig. 3 that the system is unstable under decentralized MPC. The performance of the FC-MPC framework with centralized estimation is within 8% of centralized MPC performance. The performance loss, relative to centralized MPC, incurred by the FC-MPC framework employing the distributed estimation strategy described in Section VI-A is $\sim 16\%$.

D. Two area power system network with FACTS device

In this example, a two area network interconnected through a tie line is considered. A FACTS device is employed by area 1 to manipulate the effective impedance of the tie line and control power flow between the two control areas. The performance of the cent-MPC, decent-MPC and FC-MPC formulations rejecting a 25% increase in load of area 2 is investigated. The FC-MPC algorithm is terminated after 1 cooperation-based iterate and efficacy of the two distributed estimation strategies described in Section VI is evaluated. In all cases, an input disturbance model is employed.

The relative phase deviation in the two areas and the change in impedance due to the FACTS device under the different MPC frameworks is shown in Fig. 4. A closed-loop performance comparison of the different MPC frameworks rejecting the load disturbance is given in Table III. Under decentralized MPC, the incurred performance loss, relative to centralized MPC, is $\sim 37\%$. With the distributed estimation strategy of Section VI-B, the performance loss drops to $\sim 1.9\%$. For this system, $|\lambda_{\max}\mathcal{H}(\mathcal{L}_d)| < 1$ and hence the distributed estimation framework of Section VI-A can be employed. This distributed estimation framework results in performance that is within 1% of centralized MPC performance.

X. CONCLUSIONS

Centralized MPC is not well suited for control of large-scale, geographically expansive systems such as power systems. However, the performance benefits obtained with centralized MPC can be realized through distributed MPC strategies. Such strategies rely on decomposition of the overall system into interconnected subsystems, and iterative exchange of information between these subsystems. An MPC optimization problem is solved within each subsystem, using an estimate of the current subsystem state and the latest available external state estimate.

For consistency with the distributed control philosophy, the estimation process must also be distributed across the subsystems. If a certain (easily verifiable) condition is satisfied, local measurements are sufficient for observer stability. Otherwise, a stable observer can always be designed by exchanging measurements between adjacent subsystems. Both estimation strategies are suboptimal, but the estimates generated converge exponentially to the optimal estimates. Furthermore, use of either observer, in conjunction with the defined distributed MPC control law under output feedback, guarantees nominal closed-loop stability for all values of the iteration number. This feature allows the practitioner to terminate the distributed MPC algorithm at the end of the sampling interval, even if convergence is not achieved. A disturbance modeling framework for achieving zero-offset control in the presence of nonzero mean disturbances and modeling errors has been established.

In this paper, a number of power system examples have applied distributed output feedback MPC to automatic generation control (AGC). MPC outperforms standard AGC, due to its ability to account for process constraints. The distributed MPC framework also allows coordination of FACTS controls with AGC.

XI. ACKNOWLEDGMENT

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APPENDIX

A. FC-MPC optimization for stable power systems

$$\text{FC-MPC}_i \triangleq$$

$$\min_{\overline{\boldsymbol{u}}_i} \quad \frac{1}{2}\overline{\boldsymbol{u}}_i{}'\mathcal{R}_i\overline{\boldsymbol{u}}_i + \left(\sum_{j=1}^{M} E_{ji}{}' \sum_{s\neq j} \mathbb{M}_{js} \sum_{l\neq i}(E_{sl}\overline{\boldsymbol{u}}_l^{p-1} + \boldsymbol{g}_s) + \sum_{j=1}^{M} E_{ji}{}'\mathbb{Q}_j \sum_{l\neq i}(E_{jl}\overline{\boldsymbol{u}}_l^{p-1} + \boldsymbol{g}_j)\right)'\overline{\boldsymbol{u}}_i \tag{24a}$$

subject to

$$u_i(j|k) \in \Omega_i, \quad k \leq j \leq k+N-1 \tag{24b}$$

in which

$$\mathcal{R}_i = w_i\mathbb{R}_i + \sum_{j=1}^{M} w_j E_{ji}{}'\mathbb{Q}_j E_{ji} + \sum_{j=1}^{M} E_{ji}{}' \sum_{s\neq j} \mathbb{M}_{js} E_{si} \tag{25}$$

$$\mathbb{Q}_i = \text{diag}\left(Q_i(1), \ldots, Q_i(N-1), \overline{Q}_{ii}\right) \tag{26}$$

$$\mathbb{M}_{ij} = \text{diag}\left(0, \ldots, 0, \overline{Q}_{ij}\right) \tag{27}$$

$$\mathbb{R}_i = \text{diag}\left(R_i(0), R_i(1), \ldots, R_i(N-1)\right) \tag{28}$$

$$\boldsymbol{g}_i = \sum_{j=1}^{M} f_{ij} x_j(k) \tag{29}$$

and

$$\overline{\mathbb{Q}} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \cdots & \cdots & \overline{Q}_{1M} \\ \overline{Q}_{21} & \overline{Q}_{22} & \cdots & \cdots & \overline{Q}_{2M} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \overline{Q}_{M1} & \overline{Q}_{M2} & \cdots & \cdots & \overline{Q}_{MM} \end{bmatrix} \tag{30}$$

is a suitable terminal penalty matrix. Restricting attention to open-loop stable systems simplifies the choice of $\overline{\mathbb{Q}}$. For each $i \in \{1, M\}$, let $Q_i(0) = Q_i(1) = \ldots = Q_i(N-1) = Q_i$. The terminal penalty $\overline{\mathbb{Q}}$ can be obtained as the solution to the centralized Lyapunov equation

$$A' \, \overline{\mathbb{Q}} \, A - \overline{\mathbb{Q}} = -\mathbb{Q} \tag{31}$$

in which $\mathbb{Q} = \text{diag}(w_1 Q_1, w_2 Q_2, \ldots, w_M Q_M)$.

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