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Lyapunov Function Analysis of Power Systems with Dynamic Loads

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Abstract

A recently developed Lyapunov (energy) function incorporates both generator and load dynamics. This paper reviews that energy function. Its use in the assessment of the stability of power systems where generator and load dynamics are active is presented. Further, generator/load interaction is explored. This new energy function allows for direct assessment of (dynamic) voltage collapse scenarios. It provides an analytical basis for establishing critical capacitor and load switching times. These issues are considered.

1 Introduction

Application of energy function ideas to power systems was originally motivated by the desire for rapid assessment of intermachine (angle) stability [12]. This was a natural focus, as power system stability was typically concerned with ensuring that the angles between machines remained bounded. Instability occurred when angle differences increased to the point where there was pole slipping (and subsequent machine tripping). Recently however, other (network related) dynamic phenomena have also had a major influence on system planning and operation. These phenomena are generally referred to as voltage instability or voltage collapse [4].

Voltage collapse can occur over different time frames. A transient form of voltage collapse occurs when network voltages decline rapidly in response to an increase in intermachine angles [8, 14]. This form of voltage collapse is closely related to singularity of the algebraic equations of the power system model. Energy function analysis of this situation is presented in [7, 14]. In that analysis, loads are modelled as statically dependent on voltage.

However in the more traditional view of voltage collapse, the dynamic behaviour of loads plays a major role [6]. In response to a voltage step, loads often exhibit dynamic recovery of the form shown in Figure 1 [5, 10]. In a weakened system, this restoration of load at reduced voltage can lead to a continual decline in voltage. Ultimately the process ends in cascaded protection operation and/or machine (angle) instability.

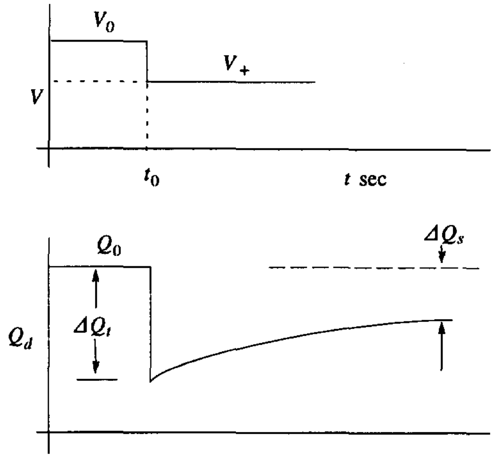

Figure 1: Generic load response

It is appealing to consider the extension of energy function techniques to situations where both generator and load dynamics are active. This has been made possible by a recent extension [3] of the 'structure preserving' energy function to incorporate dynamic load behaviour of the form shown in Figure 1. Of particular interest is the direct assessment of (load driven) voltage instability. In that case, limits on reactive power sources must be taken into account. That extension of energy functions is described in [9].

Traditionally angle stability and voltage stability issues have been treated separately. However angles and voltages are all states of the one system, and so must interact. Certainly there can be a time scale separation between angle and voltage effects. However that is not necessarily always the case. For example, in the voltage collapse scenario, frequently machine separation is the ultimately mode of failure [8]. Further, dynamic loads which have a comparatively fast response time

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can interact with intermachine oscillations to affect the damping of those oscillations [11]. This generator (angle) – load (voltage) interaction will be considered in this paper from an energy function perspective.

Energy functions offer a number of benefits in the analysis of system stability [13]. These include direct (fast) assessment, and the provision of a 'measure' of system stability. These benefits have previously been recognized for angle stability assessment, but can now be extended to situations where load dynamics are important, such as voltage instability phenomena. Also, critical clearing time ideas can be extended to applications such as determining critical capacitor or load switching times for alleviating voltage collapse.

The paper is structured as follows. In Section 2 we provide an outline of the load and system modelling details that are necessary for establishing the energy function. Section 3 then gives an energy function that incorporates generator and load dynamics. Energy function analysis of generator/load systems is presented in Section 4. Conclusions are given in Section 5.

2 Modelling

2.1 System model

In extending the structure preserving model to incorporate load dynamics, the usual assumptions relating to system modelling shall be made. Therefore synchronous machines are represented by the classical machine model, with dynamics given by the usual swing equations,

$$M_i \dot{\omega}_{g_i} + D_{g_i}\omega_{g_i} + P_{ELEC_i} = P_{M_i} \tag{1}$$

Also, the network is assumed to be lossless. Full details can be found in [7].

Let the complex voltage at the ith bus be the (time varying) phasor $V_i \angle \delta_i$ where δ_i is the bus phase angle with respect to a synchronously rotating reference frame. The bus frequency deviation is given by $\omega_i = \dot{\delta}_i$. We shall use the machine reference model, so that all angles are referenced to the nth bus angle, i.e., $\alpha_i = \delta_i - \delta_n$. Define,

$$\underline{\alpha} = [\alpha_1, ..., \alpha_{n-1}]^t$$

$$\underline{\omega}_g = [\omega_{n_0+1}, ..., \omega_n]^t$$

$$\underline{V} = [V_1, ..., V_n]^t$$

2.2 Load model

In the development of strict Lyapunov functions of the form commonly used in power system analysis, it is necessary to assume real power demand is given by [7],

$$P_{d_i}(\omega_i) = P^o_{d_i} + \omega_i D_{l_i} \tag{2}$$

We wish to allow reactive power load to have a dynamic response of the form shown in Figure 1. A model which captures that behaviour is given by [5],

$$\begin{aligned} T_{q_i}\dot{x}_{q_i} &= -x_{q_i} + Q^o_{s_i} - Q_{t_i}(V_i) && (3) \\ Q_{l_i}(\underline{V}, \underline{\alpha}) &= x_{q_i} + Q_{t_i}(V_i) && (4) \end{aligned}$$

where x_{q_i} is an internal state of the load. To ensure that a strict Lyapunov function is obtained, it is necessary to restrict the reactive power transient response to,

$$Q_{t_i}(V_i) = Q^o_{t_i} \ln\left(\frac{V_i}{\mu_i}\right) \tag{5}$$

This form for $Q_{t_i}(V_i)$ is rather unusual. However the free parameters in the model, i.e., $Q^o_{t_i}$ and μ_i, can be varied to provide a good (local) approximation to the more usual exponential form for $Q_{t_i}(V_i)$. This is illustrated in [3].

In the model (3), the steady state load characteristic $Q^o_{s_i}$ is a constant. This restriction is not necessary though. It is shown in [3] that the Lyapunov function can be easily adapted to allow for a voltage dependent characteristic $Q_{s_i}(V_i)$ of the form given in (5).

3 Lyapunov (Energy) Function

The complete model for the system with dynamic reactive power loads is assembled in [3]. A Popov criterion analysis is undertaken to obtain the corresponding Lyapunov (energy) function,

$$\begin{aligned} &\mathcal{V}(\underline{\omega}_g, \underline{\alpha}, \underline{x}_q, \underline{V}, \underline{\alpha}^s, \underline{x}^s_q, \underline{V}^s) = \\ &\quad \frac{1}{2}\underline{\omega}^t_g \underline{M}_g \underline{\omega}_g + \frac{1}{2}(\underline{x}_q - \underline{x}^s_q)^t [Q^o_t]^{-1} (\underline{x}_q - \underline{x}^s_q) \\ &\quad -\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}(V_iV_j\cos\alpha_{ij} - V^s_iV^s_j\cos\alpha^s_{ij}) \\ &\quad -\sum_{i=1}^{n-1} P_i(\alpha_i - \alpha^s_i) + \sum_{i=1}^{n_o-m} x_{q_i}\ln\left(\frac{V_i}{V^s_i}\right) \\ &\quad +\sum_{i=1}^{n_o-m}\frac{Q^o_{t_i}}{2}\left(\ln^2\left(\frac{V_i}{\mu_i}\right) - \ln^2\left(\frac{V^s_i}{\mu_i}\right)\right) \end{aligned} \tag{6}$$

It is interesting to consider the terms of this energy function which are introduced by the dynamic behaviour of reactive power loads. Comparisons with the static load energy function reveal that the second term of (6) results from the load dynamics. The fifth and sixth terms also appear to be quite different. However it is shown in [3] that these latter terms can be replaced by

$$\sum_{i=1}^{n_o-m}\int_{V^s_i}^{V_i}\frac{Q_{d_i}(x_{q_i}, z_i)}{z_i}dz_i$$

which is very similar to the corresponding term of the static load energy function. Interestingly, both functions give exactly the same value of energy at equilibrium points (EPs).

For a power system with no generator or load damping, $\dot{\mathcal{V}}$ along trajectories is given by,

$$\dot{\mathcal{V}} = -\sum_{i=1}^{n_0-m} \frac{T_{q_i}(\dot{x}_{q_i})^2}{Q^o_{t_i}} \qquad (7)$$

i.e., the dynamics of the reactive power loads provide damping in the system.

As mentioned earlier, this new Lyapunov function relies on the usual assumptions that the system is lossless, and that real power loads are constant. This of course does not reflect realistic system conditions. A number of numerical approximations have been developed for the static load case to overcome these restrictions [13]. These approximations are just as applicable for the dynamic load energy function. The resulting energy functions do not (generally) satisfy strict Lyapunov function properties. However they have proven to be useful for stability analysis of real systems [1].

4 Energy Function Analysis

4.1 Stability analysis

The use of energy function techniques for analysing the stability of power systems is well documented [13]. The first step is to find a critical value of energy $\mathcal{V}_c$ such that the region of state space defined by $\{\underline{x} : \mathcal{V}(\underline{x}) < \mathcal{V}_c\}$ provides a good estimate of the region of attraction. A number of different methods have been proposed for finding $\mathcal{V}_c$. The 'controlling UEP' method is a common choice [2, 13]. In that case, the critical energy is given by the potential energy of the unstable equilibrium point (UEP) associated with a particular (controlling) mode of instability. There are a number of issues relating to finding the controlling UEP. However they are beyond the scope of this paper. Details can be found in [2].

The second step is to determine the system energy at the beginning of the post-disturbance period, i.e., the energy acquired during the disturbance. If that energy is less than $\mathcal{V}_c$, the system will be stable. Otherwise it may be unstable. These ideas will now be illustrated.

The aim of the examples is to explore the characteristics of the new Lyapunov function (6). It is therefore convenient to use simple illustrative power systems. However the ideas extend naturally to large systems. Also, the examples satisfy the modelling assumptions which underlie the (strict) Lyapunov function. Relaxation of these assumptions for practical analysis was discussed in Section 3.

The two machine, single load system of Figure 2 involves both generator and load dynamics. For a reasonably heavy load, the operating point, or stable equilibrium point (SEP), is accompanied by a nearby UEP. For this system there is only one UEP, so it is necessarily the controlling UEP. The power flow solutions for the SEP and UEP are given in Tables 1 and 2 respectively.

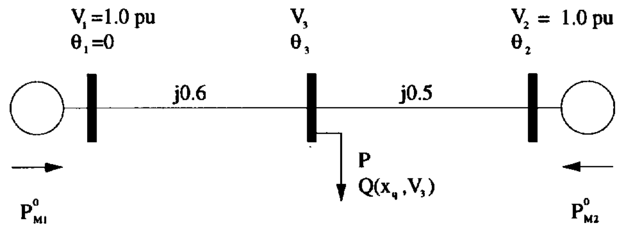

Figure 2: Two generator, single load system

Bus number	V	Angle (deg.)	P	Q
1	1.0000	0.00	-0.5	0.5087
2	1.0000	50.90	0.7	0.6581
3	0.7568	23.35	-0.2	-0.4000

Table 1: SEP power flow

Bus number	V	Angle (deg.)	P	Q
1	1.0000	0.00	-0.5	1.0881
2	1.0000	90.55	0.7	1.4067
3	0.4588	40.84	-0.2	-0.4000

Table 2: UEP power flow

The inertia of the generator at bus 1 was set to a large value, so that the bus acted as an infinite bus. The generator at bus 2 had an inertia constant of 0.1pu. Machine damping was set to zero to allow investigation of load damping effects. Load parameters were: T_q = 1.0sec, Q^o_s = 0.4pu, Q^o_t = 0.1pu, μ = 0.2pu.

The potential energy of the UEP was evaluated as $\mathcal{V}_c$ = 0.0404pu. As mentioned in Section 3, this energy is independent of load dynamics.

The system was faulted by opening the feeder between buses 1 and 3. The feeder was returned to service when system energy reached the critical value $\mathcal{V}_c$ = 0.0404pu. Figure 3 shows the behaviour of energy for this case. The damping provided by the dynamic load is indicated by the steady decline in energy following fault clearing. The system is clearly stable. The load damping leads to conservativeness in the stability estimate. It was found that the energy at clearing time could be as high as 0.047pu before instability occurred. This represents a margin of approximately 16%. The initial portion of the trajectory corresponding to the critically cleared case is shown in Figure 4. This three dimensional plot shows potential energy on the vertical axis. The potential energy well, a characteristic of energy functions, is clearly evident in this figure. Notice that the first oscillation passes close to the UEP, but the second swing shows that significant damping has occurred.

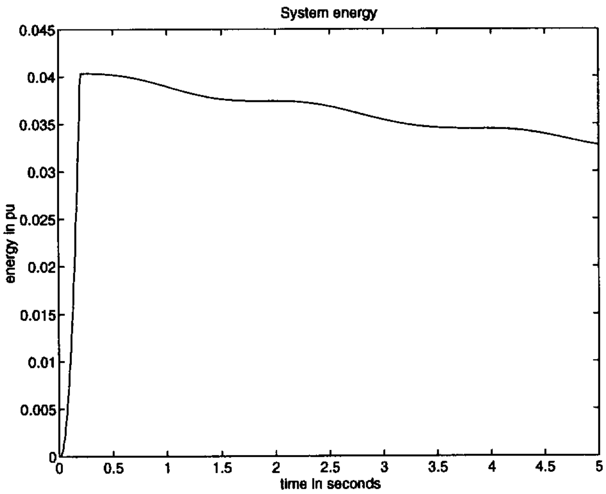

Figure 3: System cleared at UEP energy

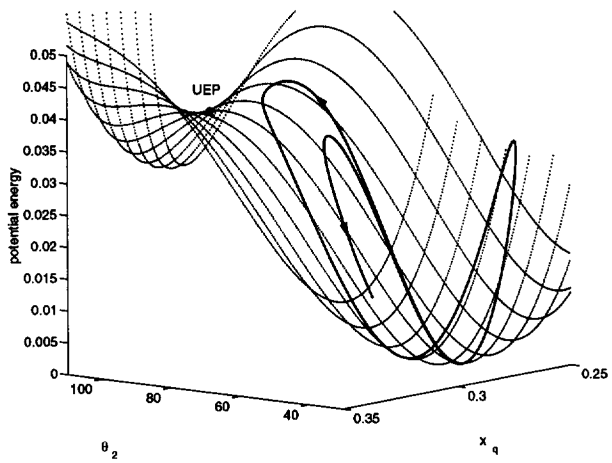

Figure 4: Stable trajectory in a potential energy well

The amount of load damping was reduced by reducing Q_t^0 by a factor of 10. The reactive power load and load state were then less sensitive to voltage variations. In this case the conservativeness of the stability estimate was less than 2.7%.

The controlling UEP method is based on the assumption that when a system goes unstable, it will pass close by the UEP of interest. However is systems with dynamic loads, that may not always be the case. Two factors have a significant influence, 1) the transient voltage dependence of the load, and 2) the load time constant.

At a UEP, the demand of a dynamic load is given by its steady state characteristic $Q_s(V)$. If the transient characteristic $Q_t(V)$ is significantly more sensitive to voltage, then the system may not (initially) pass near the UEP. The load time constant plays an important role in the transition from behaviour dominated by $Q_t(V)$ to behaviour given by $Q_s(V)$.

The system may not pass near the controlling UEP if machine and load behaviour can be decomposed into slow and fast subsystems. If a load time constant is short, the load dynamics behave as a singular perturbation to the machine dynamics. The energy associated with the load is then quite distinct from the energy due to the machines. After fault clearing the total energy may be higher than the energy of the UEP, since the energy associated with the load is quickly built up and dissipated. An extreme example of this is shown in Figure 5. The load time constant was decreased to 0.01 seconds. (Alternatively the machine inertia could have been increased). When the fault occurred, energy built up very quickly. However energy was quickly dissipated when the fault was cleared. Some energy due to machine motion remained. The phase portrait shows these two distinct modes of system behaviour.

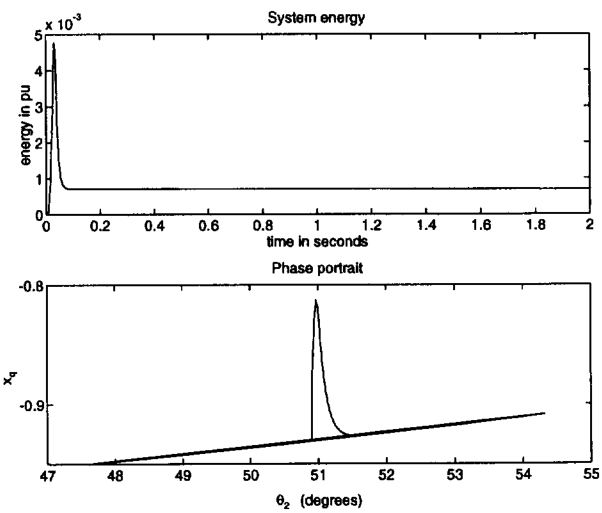

Figure 5: System behaviour with short time constant

If the load time constant is relatively long, then load behaviour (in the short term) will be given by the transient voltage characteristic. Therefore the disturbed system may not pass near the UEP. This is shown in Figure 6, where the load time constant was adjusted to 50 seconds. With almost no damping, system energy is sustained at a value well above that of the UEP. The phase portrait again shows a decomposition of system beahviour. The load state has moved very little, even though angles are deviating quite significantly. Ultimately, as the load does respond, the system may approach the UEP and go unstable. However this would be a slow process.

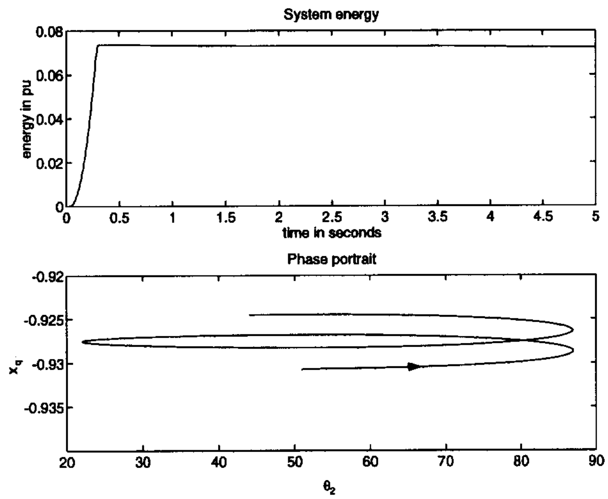

Figure 6: System behaviour with long time constant

In cases where the controlling UEP method is unreliable, for example due to time scale decoupling effects, the 'potential energy boundary surface' (PEBS) method [13] may be used to obtain a less-conservative estimate of the critical energy.

4.2 Load system analysis

The system shown in Figure 7 will be used to illustrate the stability analysis of load systems. Load parameters are given in Table 3. Bus 1 is an infinite bus, so there are no generator dynamics.

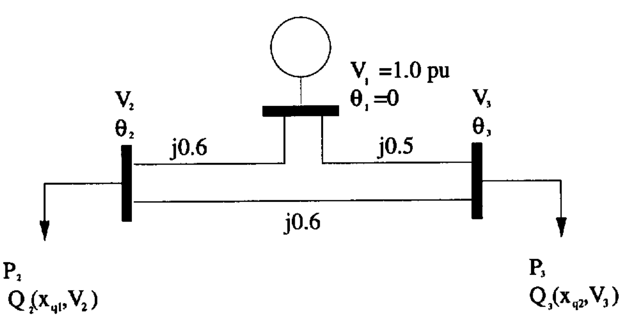

Figure 7: Dynamic load system

Bus number	T_q	Q_s^o	Q_t^o	μ
2	4.0	0.4	0.3	0.05
3	1.0	0.4	1.0	0.2

Table 3: Load parameters

In this case the system was disturbed by increasing the impedance between buses 1 and 2. With no corrective switching action, the system was unstable. The voltage at bus 2 steadily declined. This is shown in Figure 8. However stability could be maintained by switching a shunt capacitor (with a susceptance of 0.4pu) before the system acquired energy equal to the critical value $\mathcal{V}_c$ given by the controlling UEP. Time domain simulation of this stable case is also shown in Figure 8, along with a delayed switching case. A potential-energy-well view of critical capacitor switching is shown in Figure 9. This clearly illustrates the significance of the UEP energy in determining the critical switching time.

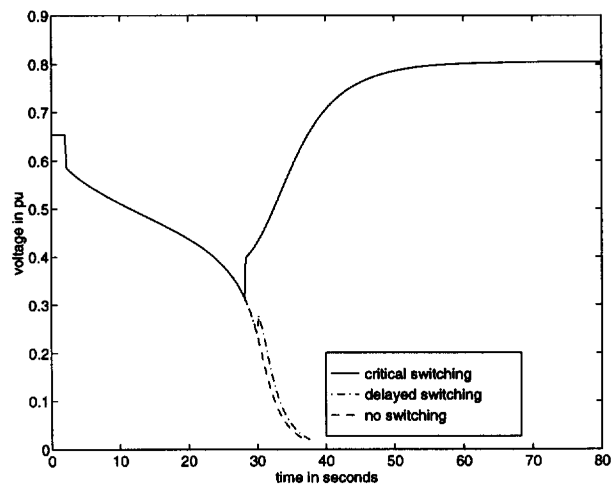

Figure 8: Critical capacitor switching

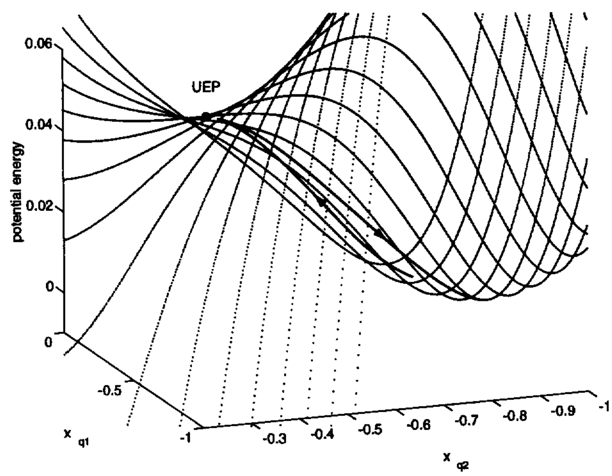

Figure 9: Energy view of critically switched system

An energy-type analysis of critical capacitor switching was undertaken in [15]. This example reinforces the value of such an approach. However the ideas can now be justified on the basis of rigorous Lyapunov stability arguments.

5 Conclusions

Energy function techniques have long been recognized as a useful way of analysing machine (angle) stability. By using an energy function which incorporates the effects of load dynamics, these techniques can be extended to systems where generators and loads interact dynamically. In fact, direct assessment of (dynamic) voltage collapse is possible. This is illustrated in the paper. The controlling UEP method is used to determine the critical switching time of a capacitor. Delayed switching results in voltage collapse.

In systems with dynamic loads, unstable trajectories may not always pass close by a UEP. Two factors have a significant influence, 1) the transient voltage dependence of the load, and 2) the load time constant. In particular, if machine and load behaviour can be decomposed into slow and fast subsystems, then the relevance of the UEP may be diminished. Potential energy boundary surface (PEBS) ideas then become important.

Acknowledgement

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