Analysis Tools for Power Systems—Contending with Nonlinearities

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As systems become more heavily loaded, nonlinearities play an increasingly important role in power system behavior. Modeling must accurately reflect component and system response. Analysis tools should continue to work reliably, even under extreme system conditions, providing accurate predictions of system behavior. The paper considers some power system modeling issues and presents an overview of the source of nonlinearities in power systems. It then considers industry accepted analysis techniques, as well as those techniques that are more research orientated. The influence of nonlinearities on these tools is explored.

I. Introduction

Power system analysts need to accurately predict the behavior of their power systems. System planners, for example, must estimate future asset requirements, based on long range predictions of system performance. System operators must determine the robustness of the current operating point to parameter changes and disturbances. In both cases, the consequences of being misled by poor predictions can be quite significant. If the planner recommends plant that is not required, a large cost penalty is incurred. But if he underestimates plant requirements, consumers may be adversely affected by load restrictions and/or a reduction in reliability. Likewise, if operators are not guided accurately, they may miss an opportunity for a profitable power transaction with a neighboring utility, or they may be unaware that their system is at risk from certain contingencies.

Historically, power systems were designed and operated conservatively. It was comparatively easy to match load growth with new generation and transmission equipment. So systems normally operated in a region where behavior was fairly linear. Only occasionally would systems be forced to extremes where nonlinearities could begin to have some significant effect. However the recent trend is for power systems to be operated closer to limits. Also, as the electricity industry moves toward an open access market, operating strategies will become much less predictable. Hence the reliance on nearly linear behavior which was adequate in the past must give way to an acceptance that nonlinearities are going to play an increasingly important role in power system operation. It is therefore vital that analysis tools perform accurately and reliably in the presence of nonlinearities.

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Other papers within this special section are devoted to exploring the types of nonlinear phenomena that can occur in power systems. This paper focuses on commonly used analysis tools, and considers their ability to cope with those nonlinearities.

The types of analysis tools normally used by power system analysts are as follows:

- *Power flow* programs solve the algebraic equations which describe power system steady state conditions, i.e., a power flow solves for the system operating point (equilibrium point).
- *Small disturbance* stability analysis is concerned with the eigenstructure of the system linearized about an operating point of interest.
- *Large disturbance* simulation programs perform numerical integration of the differential-algebraic-discrete equations that describe the power system.

These tools will be considered further in later sections. For now we shall use a power flow example to motivate the idea that power systems can exhibit interesting nonlinear behavior. The algebraic equations solved by the power flow are in general multivalued. Fig. 1 provides a graphical illustration of the complex nature of solutions. This figure comes from [75], and describes solutions of a 23 bus section of the Queensland (Australia) power system. Each curve corresponds to a different (constant) value of voltage at the Cairns bus, and describes the relationship between the real and reactive power generation at Barron Gorge required to maintain that voltage. The curves together describe a complicated surface. We shall return to power flows, and this particular figure, later.

The paper is structured as follows. Section II describes the power system model, and addresses 'normal' modeling assumptions. Power system analysis tools which have been adopted by industry are described in Section III. Analysis tools which have more of a research orientation are considered in Section IV. Section V makes some predictions about future trends in analysis tools.

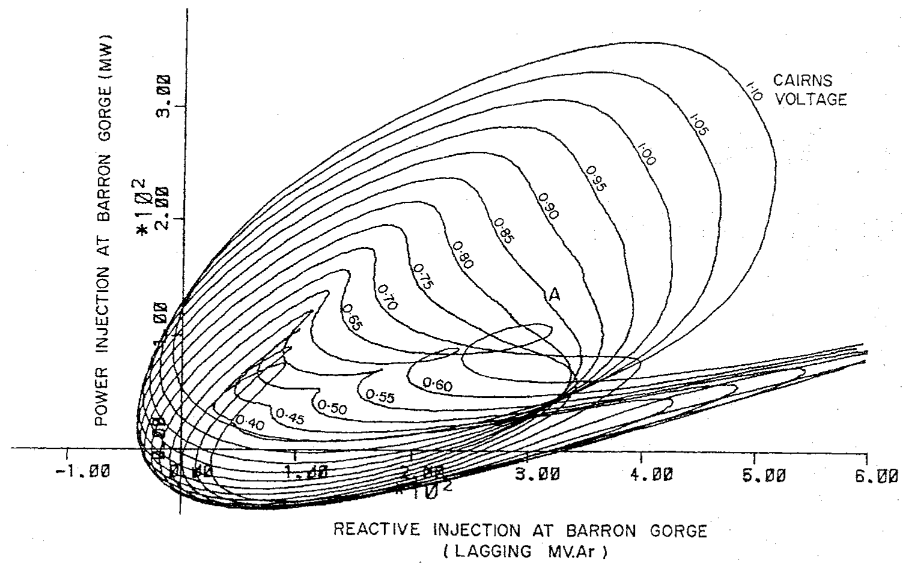

Fig. 1. Power flow solutions for various MW and MVAr parameters [75].

II. System Modeling Overview

A. Component Models

A major issue in the analysis of power systems is the modeling of the multitude of components that make up such complex interconnected systems. Analysis tools and techniques can only provide useful information if the models accurately reflect true component behavior over the range of interest. (Note that this range can vary depending on the nature of investigations.) In general, component models have been developed to a high degree of accuracy. One notable exception, which will be discussed later, is the modeling of loads.

Even if the model structure is well understood, often parameters will not be known exactly. Therefore parameter sensitivity analysis is an important aspect of modeling. This applies not only to the component models, but also to the complete interconnected system model. Generally industry has been limited to rather ad hoc approaches to sensitivity analysis. Fortunately newer techniques, such as those presented in [55], [66], offer a systematic way of analyzing parameter uncertainty. Validation is also a vitally important aspect of modeling. Unfortunately, for power systems this is often not easy [14], [31], but is worth persisting with.

Component modeling has been encountered in earlier papers in this issue [55], [66], [99]. Rather than reiterating modeling details, we shall highlight some areas where analysis tools, and the users of those tools, can be particularly vulnerable to nonlinear behavior. In this discussion, we shall focus on the inherent nonlinearities of components, modeling uncertainty and assumptions, and also on the influence of limits and switching.

1) Component Nonlinearities and Modeling Uncertainty: All power system components can exhibit some degree of nonlinearity under certain circumstances. However system dynamic behavior tends to be dominated by generation equipment and loads. Also power electronic devices are becoming increasingly significant in modern power systems. In some situations, such as voltage collapse, transformers play a dominant role. We shall therefore focus on these various components.

a) Generation equipment [5], [7], [54]: The modeling of synchronous machines is quite well established, with the Park–Blondel transformation forming the basis of the model development [5]. Even so, subtle differences can occur between machine models. For example, Canay's mutual inductance [9] is not always modeled. Also, there are different ways of handling saturation of the magnetic circuits [81]. Generally these modeling differences result in quite small discrepancies in simulated behavior. However occasionally the discrepancies become significant. This can be extremely frustrating, particularly when various utilities are undertaking parallel studies using different analysis packages. Benchmarking of the tools is necessary under those circumstances. Synchronous machine models are high order nonlinear differential-algebraic sets of equations. Therefore the structural consistency (in a structural stability sense [66]) of the models cannot even be guaranteed.

No matter how good the models, the accuracy of simulations is dependent on the quality of data. Standard measurement procedures generally provide accurate machine parameters. However these procedures rely on taking machines out of service, which is expensive due to lost production. Techniques for estimating parameters while machines remain in service are therefore required. The nonlinear nature of the models makes this a difficult identification problem though.

Because of the high order of synchronous machine models, it is common for simplifying assumptions to be made. This leads to some uncertainty in the model. Care must be taken to ensure the behavior of the simplified models is consistent with that of the more complete model. This cannot be guaranteed in general. References [61], [66] provide examples where this is not the case. Model reduction techniques must address these structural stability issues.

Historically the modeling of machine control loops was quite crude. Analysis packages were supplied with 'standard' models. It was necessary to massage actual controller structures and parameters into those standard forms. However discrepancies between predicted and measured system behavior have (in many cases) forced the development of more accurate models [53]. It is common now for controller model structures to be deduced directly from circuit diagrams, and parameters identified from tests. With the move toward control based expansion of power systems [33], increasing importance will be placed on controller modeling. Also, as more sophisticated control techniques gain favor, e.g., adaptive, robust and/or nonlinear control, controller structures and modeling will increase in complexity and importance.

Adaptation in controllers introduces an interesting challenge. Control parameters at any point in time will depend, through adaptation rules, on system conditions leading up to that point. The study of system dynamic behavior will therefore require a knowledge of prior system conditions. This challenge has not yet been faced in practical power system analysis.

Even though machines have the most direct influence on power system behavior, their prime movers are also vitally important. For a thermal plant, the dynamics of the steam cycle (boiler, turbine, governor) can affect the power system [18]. For a hydro unit, water column dynamics may be important. These phenomena tend to be slower than electromechanical transients, and will often only become influential after the initial transients have died away, maybe around 10–20 s. However they can be quite nonlinear, e.g., boiler-governor interaction of a thermal plant, or surging in the water column of a hydro unit. The effects can be crucial though in studies where the system undergoes a significant frequency excursion, for example following the tripping of a major unit. In such a case, incorrect analysis due to poor modeling could result in widespread underfrequency load shedding.

b) Loads [44], [45]: As a result of the time varying nature and complex composition of loads, it is often impractical (and perhaps impossible) for them to be accurately modeled. Certainly large individual predictable loads such as aluminium smelters, or some motor loads, can be accurately modeled. But in general, generic aggregate load models must be used. Such load models attempt to capture the dependence of a group of loads on variables such as voltage, frequency, and time. Generally load models should be nonlinear for large disturbance analysis.

Consider the voltage dependence of loads. The generic response of load to a step change in voltage is shown in Fig. 2 [32], [36]. (Real power load is shown, but reactive power generally exhibits a similar form of response.) The load undergoes an initial step change in response to the voltage step. A period of load recovery follows, with the load finally

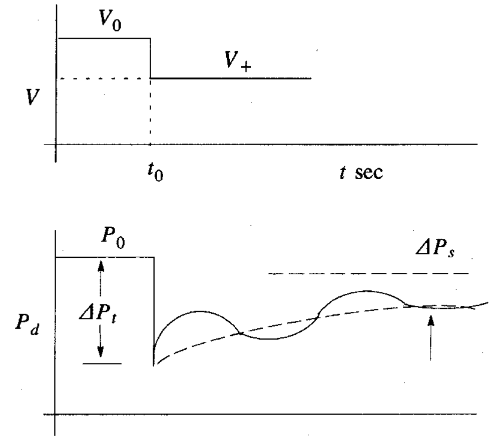

Fig. 2. Generic load response.

settling to a new steady state value. The load recovery may be monotonic, or display oscillatory behavior. This form of behavior can be observed in many different types of load [32], including induction motors, loads supplied by tap changing transformers, thermostatically controlled heating loads and aluminium smelters. However because of the aggregate nature of most loads, it is usually not appropriate to formulate a model based on any particular type of load. Rather the models of these different load types provide the motivation for generic load models.

A general (state) form of load response is given by [36]

$$\dot{x} = Fx + GN_1(V)$$
$$\begin{bmatrix} P_d \\ Q_d \end{bmatrix} = H^t x + N_2(V)$$

where x is an n-dimensional vector of load states, F, G, H are appropriately dimensioned matrices, and N_1, N_2 are nonlinear functions of voltage which are directly connected to the steady state and transient response of the load, respectively. This model is given in input-output form in [32]. These models are however too general to be of any practical use. More specific models are considered in [35], [36]. For example, monotonic recovery of real power load is given by

$$\dot{x}_p = -\frac{1}{T_p}x_p + N_p(V)$$
$$P_d = \frac{1}{T_p}x_p + P_t(V).$$

The modeling of loads can have quite a significant influence on power system dynamics, particularly slower behavior such as voltage collapse [89]. Interesting bifurcation phenomena have been observed, see for example [72], [98]. Load dynamics can also affect the damping of electromechanical oscillations. This is explored in [42] using generic recovery load models. Other studies have established a close connection between load modeling and the solvability of the power balance equations [38], [41], [66], [79]. (These solvability issues are considered further in Section II-C). Unfortunately though, load behavior is never exactly known. Therefore power system analysts must always check to ensure their results and conclusions are robust to load parameter uncertainty. The examples given in [66] further highlight the importance of this robustness analysis.

c) Power electronic devices [50], [62]: Power electronics can be used to quickly and reliably control power. Because of that, they are finding applications across the whole power system. At the load end, for example, they are fundamental to variable speed drives. They also form the basis for connecting renewable energy sources, e.g., photovoltaics and wind power, to power grids. Energy storage systems such as batteries and flywheels use similar technology.

Power electronic devices have found applications in transmission and distribution systems, where they can have a very significant influence on system behavior. High voltage direct current (HVDC) schemes have been in commercial operation for many years [64]. Recently many newer forms of power electronic devices have been proposed. They have been generically referred to as flexible ac transmission system (FACTS) devices [22], and include static var compensators (SVC's), thyristor controlled series compensators (TCSC's), static converters (STATCON's) and thyristor switched phase shifting transformers. The development of this equipment has been driven by the desire to use the controllability of the power electronics, together with modern control techniques, to increase the operational flexibility of power systems and to improve system stability margins [33].

Generally, in the modeling of such devices for studies of power system behavior, the fast switching action inherent in power electronics is ignored. Instead the devices are represented by approximate models which exhibit continuous behavior. The aim is to ensure that the exact and approximate representations have a similar 'average' effect on the system, i.e., phasor dynamics remain consistent. Of course any physical limitations in the actual device must be accurately reflected in the approximate model.

The SVC provides a good example. An actual SVC generally consists of a fixed shunt capacitor in parallel with a shunt connected thyristor controlled reactor (TCR). The thyristors regulate the current flowing through the inductor. Therefore the thyristor firing angle determines the effective susceptance of the TCR branch, and hence of the SVC. Control of the susceptance allows regulation of the bus voltage, with associated stability improvement. The SVC can be modeled approximately, but sufficiently accurately (for most studies), as a susceptance which can vary continuously between an inductive limit and a capacitive limit. However it is important to remember that the model is only approximate, and does have its limitations [76], [77]. It is possible, under certain circumstances, for unmodeled nonlinear dynamics to have a significant effect.

The TCSC provides another interesting example. It has basically the same configuration as an SVC. But instead of being connected as a shunt device, the TCSC is connected in series with a transmission line. Therefore, by varying the effective impedance of the TCSC, the impedance of the overall transmission connection can be altered. This can be used to control power flows and improve system damping. However the variable susceptance model developed for the SVC is no longer appropriate. Instead the TCR of the TCSC must be modeled as a current source which is dependent on the thyristor firing angle [47]. Ignoring these nonlinear effects can lead to quite erroneous results.

These examples highlight the fact that care is required in developing phasor-based models for fast switching devices.

d) Transformers [7], [26]: Many transformers have the capability of performing tap changing on-line. A voltage regulator compares the measured voltage with a setpoint, and orders a tap change if the difference has been greater than some deadband for a preset time. Note that the transformer ratio changes in a finite number of discrete steps. Transformers therefore introduce a number of significant nonlinear effects, viz., deadbands, time delays, discrete switching, and limits.

To overcome the difficulties introduced by these nonlinearities, various simplifying assumptions are often made. In particular, for long time frame studies, the tap position is frequently treated as a continuous variable τ, driven (between limits) by the simple differential equation

$$\dot{\tau} = \frac{1}{T}(V_0 - V_{\text{meas}}) \tag{1}$$

where V_0 is the setpoint voltage, V_{meas} is the measured bus voltage and T is a time constant representing tapping time delays. A relationship between T and the actual transformer time delays is established in [78]. However, the model (1) is fairly crude, ignoring significant nonlinear behavior. Care must be taken to ensure it is used appropriately.

2) Limits and Switching: All power system components need to be protected against damage which could result from abnormal operating conditions. In particular, faults need to be cleared quickly and reliably. However protection devices have an inherently nonlinear effect on systems. When a limit is encountered, the resulting system behavior tends to be non-smooth, though continuous. Switching action results in discontinuous behavior. The effects of these nonlinearities on systems can be quite significant. However rigorous analysis is usually nontrivial.

Protection of power generation equipment and subsystems includes over/under excitation, over/under frequency, and volts per hertz limits. The process of voltage collapse, for example, often involves over-excitation limitations [89]. Inaccurate modeling could lead to erroneous conclusions about power system security.

Lines, cables, and transformers (network connections) are normally protected against faults or excessive currents [8], [68]. If the protection detects an abnormality, the connection will be tripped. This results in a change in network topology, and a discontinuity in the mathematical description of the system. Protection operation often has a significant influence on system stability. For example, voltage collapse is frequently associated with cascaded line tripping. Careful tuning of protection settings can therefore lead to improved system security [90]. (Alternatively, poor tuning can result in a reduction in system security.) Further, in order to maximize system security over a wide range of operating scenarios, adaptive relaying concepts are being proposed and implemented [70]. The influence of network protection on system dynamic behavior implies that these devices should be accurately modeled. However modeling has traditionally been rather crude; the exception being in postmortem analysis [14].

Loads are also often subject to switching action. If the frequency or voltage of a controlled load falls below some threshold, the load will be disconnected after a preset time delay. The location and timing of this protection action can influence system stability [6].

B. System Modeling

The complete power system model is formed from all the component models. Interconnection is achieved by ensuring that Kirchoff's current law is satisfied at all nodes, i.e., currents add to zero. The model has the generic structure

$$\dot{x} = f(x, y, z; p) \tag{2}$$
$$0 = g(x, y, z; p) \tag{3}$$
$$z_{k+1} = h(x, y, z_k; p) \tag{4}$$

where x are dynamic states, y are algebraic states, z are discrete states, and p are parameters.

Various simplifying assumptions are commonly made, depending upon the nature of the study. If the focus is on fast transient processes, such as switching behavior of power electronic circuits, or resonance between transformers and cables (for example), then slower behavior could be neglected. These studies would typically have a time window of less then a second, so it would not be necessary to model effects such as tap changing, or boiler/turbine/governor dynamics.

A significant proportion of power system studies focus however on (slower) electromechanical phenomena. In such cases it is normal to assume that system quantities can be adequately represented by phasors (rather than instantaneous values). Network modeling detail is then significantly reduced, but more precise modeling of the slower devices is required. Also, the approximations discussed earlier can (generally) be made for power electronic devices. In using a phasor representation however, care must be taken to ensure that system behavior is slow relative to the system frequency. Otherwise the phasor-based models may become invalid [16], [100]. This can become significant if a system trajectory approaches conditions where the model is not solvable. These conditions are considered in Section II-C.

Even after making simplifying assumptions, the system model may contain a wide range of time constants. Some components, such as AVR's for power electronic devices, have small time constants in the order of milliseconds, whereas time constants for other components, e.g., boilers, are in the order of minutes. Such stiff systems must be handled carefully by numerical integration techniques so that numerical stability is ensured [82]. (Numerical instability can lead to meaningless simulation results. An even more troublesome possibility though is that numerical instability may be mistaken for system instability.)

The power system model wili often include time delays. Implementation in variable step size numerical integration techniques can be difficult. Also, switching action typically does not coincide with time steps. Interpolation techniques are frequently used to reflect actual switching times.

C. Generic Properties of the System Model

Significant analysis of the differential-algebraic-discrete model (2)–(4) has been presented in the literature. This special issue provides some insights into such investigations. In this paper we wish to explore features of this model which can potentially cause difficulties for analysis tools.

The first thing to note is that the model generally contains algebraic equations; see (3). These equations relate to constraints such as power balance at network buses. Ignoring discontinuities caused by discrete states for now, the model simplifies to a differential-algebraic (DA) structure

$$\dot{x} = f(x, y; p) \tag{5}$$
$$0 = g(x, y; p). \tag{6}$$

The algebraic equations define the *constraint manifold,* a manifold in the space of dynamic and algebraic state variables [37], [55], [94], [97]. Satisfying the algebraic equations ensures that system trajectories always remain on that manifold. The differential equations drive the system over the constraint manifold.

It should be recognized that there are generally underlying dynamics associated with the algebraic constraints (6). However these dynamics tend to be neglected as part of the model reduction process. They may be discarded, for example, through the use of the phasor representation, or the simplified modeling of power electronic equipment. The more complete differential equation model can often be viewed as a singular perturbation [52] of the DA model. These issues are explored further in [55], [97].

Referring to the DA model (5), (6), while $D_y g$ (the matrix of partial derivatives of algebraic equations with respect to algebraic variables) remains nonsingular, the Implicit Function Theorem [24] ensures the existence of *local* functions $y = \psi(x; p)$. The algebraic variables y can then be replaced in (5), yielding the local differential equation form

$$\dot{x} = f(x, \psi(x); p) := f'(x; p). \tag{7}$$

Under certain conditions, this local result can be extended to a differential equation form over simply connected regions [34], [74]. Numerical techniques do not rely explicitly on the functions ψ. However, they are greatly influenced by the existence of such functions.

In general, the local functions ψ are not unique. For a given value of dynamic state x, (6) may be satisfied by a number of different values of the algebraic state y,

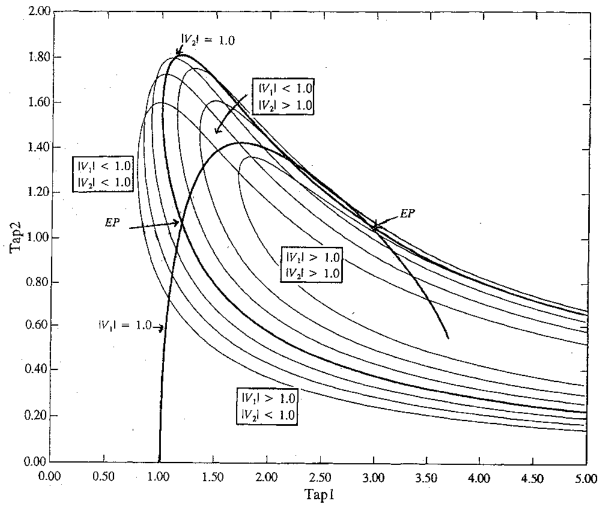

Fig. 3. Power system constraint manifold [40].

with a different ψ_i corresponding to each solution. As x varies in response to system dynamics (5), a point may be encountered where different solutions of (6) coalesce. (This is a form of saddle node bifurcation, where x is the varying parameter.) At such a point, $D_y g$ becomes singular and the algebraic equations become unsolvable. The model breaks down, so the system trajectory cannot be continued. Extensive investigations of this phenomenon are reported in [94], [97]. As mentioned earlier, a close connection exists between this algebraic singularity (model breakdown) and load modeling [38], [66].

To illustrate these concepts, Fig. 3 provides a simple example taken from [40]. The power system in this case has a radial structure, with two load buses being supplied from an infinite bus. A tapping transformer regulates the voltage magnitude of each load bus. The dynamic states are therefore tap positions τ_1, τ_2, and the algebraic variables are bus voltage magnitudes and angles. In [40], the tap positions are modeled as continuous variables driven by differential equations of the form (1). However the shape of the constraint manifold is independent of the dynamics driving τ_1, τ_2, or whether they are discrete or continuous variables. Fig. 3 shows a projection of the constraint manifold. Notice that no solutions of the algebraic equations exist beyond the boundary of this manifold. System dynamics will cause τ_1, τ_2 to vary, producing a trajectory over this manifold. But if the trajectory encounters the boundary, it will not be able to continue. The model will break down. Other (power system) examples which illustrate this type of behavior are given in [38], [55], [74], [95].

Typically power system models can exhibit such behavior. Real systems of course do not reach points from which they cannot continue. The model breakdown is due to modeling assumptions, for example the use of phasors where they are not appropriate [16], [100] or oversimplification (or inappropriate choice) of load models [41]. In reality, the trajectory would continue under the action of unmodeled dynamics. However the assumptions which underlie this problem are common in the analysis of power systems. Therefore analytical tools must be able to cope with these model breakdown conditions.

In Section II-A.2 we saw that many power system devices involve limits. Examples include generator overexcitation limits, controller saturation, and physical limitations of devices. Included in this latter category are transformer tap limits, SVC susceptance limits and induction motor slip. Both windup and non-windup limits can be found in power systems. Limits can have a significant influence on system behavior. An important aspect of this, namely the structure of the stability boundary, is explored in [96], [97].

Limits often lead to a shrinking of the stability region. For example, limits on the field forcing capability of generator AVR's can result in a generator being unable to maintain synchronism following a major system disturbance. However some limits can be beneficial, e.g., limits on generator stabilizer output signals. Another classic illustration of the influence of limits can be found in [71], [99] where it is shown that generator overexcitation limits play a major role in the voltage collapse phenomenon. That same illustration can however also be used to show that some limits may lead to an improvement in stability. Transformer tapping plays a major role in voltage collapse. But when transformers encounter tap limits, the destabilizing effect of tapping is removed.

Limits can have other interesting effects also. A case is reported in [98] where limits led to sustained oscillations. Without limits, oscillations grew and the system lost stability. The limits forced the system trajectory to remain bounded.

Discontinuities, such as produced by protection switching action, can cause further complications. For example, with no discontinuities the constraint manifold is continuous, and so trajectories are continuous. (If there are no limits, the constraint manifold and trajectories are smooth.) But in the presence of discontinuities, the constraints describe a 'surface' which is composed of disconnected sections. Algebraic variables undergo step changes when discontinuities are encountered. This corresponds to jumping from one section of the constraint surface to another. As mentioned earlier, generally the constraints will be multivalued, i.e., for a given x, the constraints may be satisfied by a number of different y. Therefore, when switching actions occur, analysis tools must ensure that the algebraic variables jump to the correct solution. Otherwise the trajectory will continue on the wrong component of the constraint surface, giving meaningless results.

D. System Equilibria

The starting point for the analysis of the power system model (2)–(4) is the establishment of system equilibria. For a given set of parameters p, the equilibria are given by solutions of the (generally) nonlinear set of equations

$$0 = f(x, y, z; p) \tag{8}$$

$$0 = g(x, y, z; p) \tag{9}$$

$$z = h(x, y, z; p). \tag{10}$$

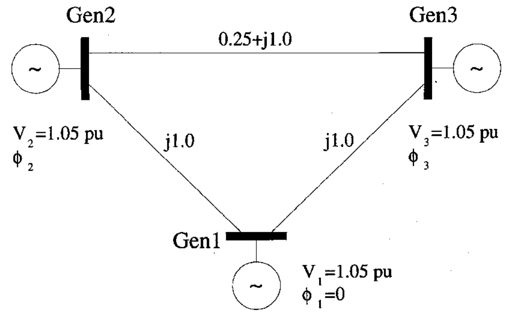

Fig. 4. Simple 3 bus power system.

The primary concern in solving (8)–(10) is ensuring that power balance is achieved at all buses in the network. Because boundary conditions at buses are usually specified in terms of power, i.e., generation or load, the equations are nonlinear.

Having satisfied the network constraints, it is then generally possible to determine steady state conditions for system components such as generators and controllers. Usually this is quite a straightforward procedure. Therefore power flow analysis and steady state analysis can be considered as synonymous. Equilibria are commonly referred to as power flow solutions.

As parameters vary, so do the equilibria. Note that due to the nonlinear nature of (8)–(10), if a power flow solution exists, then generally there will be more than one solution. Also, the power flow solution space is commonly bounded, so parameters can be found for which there are no solutions. Fig. 1 provides an indication of these properties. They are more clearly illustrated by the very simple system of Fig. 4. (This is a lossy version of a system analyzed by Tavora and Smith [87].) A view of the power flow solution space is shown in Fig. 5. Each curve which makes up this figure corresponds to a different (constant) value of ϕ_2, the voltage angle at Gen2. The complete surface shown in Fig. 5 results when ϕ_2 is varied from -180° to 180° in steps of 10°. This surface shows all possible solutions for parameters P_2 and P_3. Multiple solutions exist within the boundary of this surface. There are no solutions outside the boundary. This simple example provides a clear illustration of the nonlinear nature of the power flow equations.

Iterative techniques are used to solve for power flow solutions. Therefore, because of the existence of multiple solutions, the iterative process must be carefully initialized to ensure the appropriate solution is obtained. Poor initialization can result in nonconvergence, or convergence to the wrong solution.

At the solution space boundary, generically two solutions coalesce then disappear in a saddle node bifurcation [55]. Similar bifurcation behavior occurs at other points in parameter space which lie within the solution space. (But at those internal points, there always exist at least two solutions, even though two other solutions may be coalescing.) Fig. 5 illustrates this. It follows from the Implicit Function Theorem that at bifurcation points, the Jacobian of (8)–(10) must be singular. Normal power flow solution techniques rely (at least implicitly) on inverting this Jacobian. Therefore solving the power flow equations at bifurcation points is not possible using traditional techniques. This is discussed by Kwatny *et al.* [55], and is considered further in later sections of this paper. Solving for points near bifurcations can also be difficult due to Jacobian ill-conditioning.

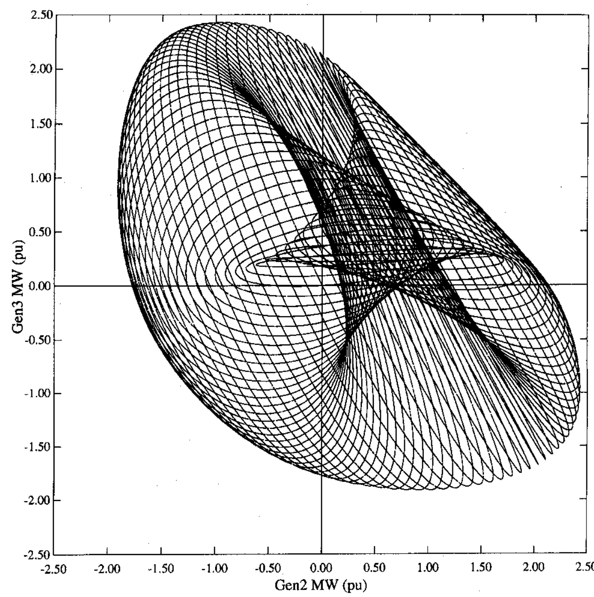

Fig. 5. Power flow solutions for the 3 bus system.

Some power system devices, such as tap-changing transformers, have controllers which regulate to within a deadband, rather than to a particular setpoint. In the transformer case, no tap changing is initiated if the voltage lies within a specified range. Therefore, rather than the steady state equations (8)–(10) defining equilibrium points, equilibria cover regions of state space. This is difficult to handle computationally. Traditionally this difficulty was overcome by assuming that any solution point which lay within the solution regions was acceptable. However that introduced a problem with repeatability of results. (Starting the solution process from different initial points would generally result in slightly different final solutions.) A more reliable way of overcoming this problem is to approximate deadbands by continuous curves which force the system to point solutions. Results are then repeatable. Returning to the tap-changing transformer illustration, solutions will usually be forced to the midpoint of the deadband range.

III. Industry Analysis Techniques

A. Power Flow Analysis

The power flow equations (8)–(10) can be formulated in either polar or rectangular form. Polar form is more intuitive because the state variables are voltage magnitudes and angles, and so have physical meaning. Also, there is a close connection between the inertia of the power flow Jacobian matrix and the small disturbance stability properties of the equilibrium point. However, when formulated in rectangular form, the power flow equations are quadratic. Some numerical advantages flow from that form. Also, the properties of quadratic equations provide some clearer insights into questions of multiple equilibria [57], [63].

Traditionally the Newton–Raphson algorithm has been used for power flow algorithms. Let the power flow equations have the general form

$$F(\mathrm{x}; p) = 0. \tag{11}$$

(Note that the 'x' in this equation is not the same as the x of previous equations.) The ith iteration of the power flow solution process solves the equations,

$$\Delta \mathrm{x}_i = -J(\mathrm{x}_{i-1})^{-1} F(\mathrm{x}_{i-1}; p) \tag{12}$$

$$\mathrm{x}_i = \mathrm{x}_{i-1} + \lambda_i \Delta \mathrm{x}_i \tag{13}$$

where J is the power flow Jacobian, i.e., $J = \partial F/\partial \mathrm{x}$. The Jacobian J is usually large and sparse. The inversion of J in (12) is normally achieved through LU factorization. Special care must be taken in ordering nodes so that fill-in is minimized during the factorization process.

The λ_i in (13) is a scalar multiplier used to control the updating of variables at each iteration. In the traditional Newton–Raphson method, $\lambda_i = 1$ at each iteration. Other choices have been proposed. The rectangular form of the power flow problem leads naturally to the idea of an 'optimal multiplier' [46], [57]. This multiplier ensures that the updates of variables at each iteration converge in a (locally) optimal way to the solution point. It is obtained as a solution of

$$\min_{\lambda_i} \|F(\mathrm{x}_{i-1} + \lambda_i \Delta \mathrm{x}_i; p)\|^2 \tag{14}$$

which can be manipulated to yield a cubic function.

One of the features of the optimal multiplier is that it reduces if the Jacobian becomes ill-conditioned. This results in superior convergence properties when the solution point lies near the solution space boundary, e.g., when the power system is heavily stressed. Another interesting property is that the optimal multiplier can be used to find multiple power flow solutions [86]. The cubic that results from (14) usually has a single real solution (and a complex pair of solutions.) However it can have three real solutions. In that case, the largest multiplier will force the solution process toward a second solution point. A number of voltage collapse proximity indicators rely on this second solution [63], [86], [93].

Power systems tend to exhibit a reasonably strong coupling between real power and voltage angle $(P - \phi)$, and between reactive power and voltage magnitude $(Q - V)$. This has been exploited in the *Fast Decoupled* power flow formulation [84], where the $(P - V)$ and $(Q - \phi)$ cross coupling terms are ignored. That assumption, together with the assumptions that voltage magnitudes are all close to 1.0 pu, and that voltage angles are all nearly zero, allows a $(2n \times 2n)$ Jacobian to be replaced by two constant $(n \times n)$ matrices. Therefore, rather than inverting the full Jacobian at each iteration, it is only necessary to invert the $(n \times n)$ matrices once, before the start of the iterative process. So an iteration of the Fast Decoupled method is faster than a Newton–Raphson iteration. However these simplifications destroy the quadratic convergence properties of the Newton–Raphson algorithm, so the Fast Decoupled power flow requires more iterations.

Iterative solution procedures such as Newton–Raphson and its derivative the Fast Decoupled power flow, can be viewed as nonlinear dynamical systems [67]. The region of convergence of the solution procedure equates to the stability region in the dynamical systems view. Power flow techniques can display an interesting range of nonlinear behavior. Examples include:

- The boundary of the convergence region has a fractal form [67], [91].
- Fast Decoupled power flows can occasionally exhibit sustained oscillations as iterations proceed around, but never converge to, a solution point.
- When a power flow diverges, iterations display a chaotic form of behavior.
- The optimal multiplier, given by (14), acts as an adaptive gain which seeks to maximize the rate of convergence at each iteration.
- It is shown in [57] that when the Newton–Raphson power flow is formulated in rectangular coordinates, and the optimal multiplier is used, then a line through any pair of solutions is an invariant manifold. If iterations encounter that line, then they never again leave it.

As systems become more heavily stressed, the $(P - V)$ and $(Q - \phi)$ cross coupling terms become significant. Also, the assumptions of 1.0 pu voltage magnitudes and angles close to zero become less acceptable. For that reason, the region of convergence of the Newton–Raphson power flow is generally larger than that of Fast Decoupled techniques. However the fractal nature of the boundary ensures that convergence behavior is not totally predictable. Further, for both techniques, the convergence region shrinks as solution points approach the solution space boundary, e.g., as systems become more heavily loaded.

B. Power Flow Curves

In the usual formulation of the power flow problem (11), the dimension of F and x are the same. So, for a given value of parameters p, solution points are specified. However it can be useful to introduce an extra variable, by allowing a parameter to vary. The power flow equations are then underconstrained because there is one more variable than constraint. The set of equations defines a 1-manifold, or curve. Such curves can be useful for:

- investigating the sensitivity of the system state to parameter variation
- finding the point of maximum loadability (saddle node bifurcation point) in a particular direction
- finding multiple solution points.

Figs. 1 and 4 show examples of power flow curves.

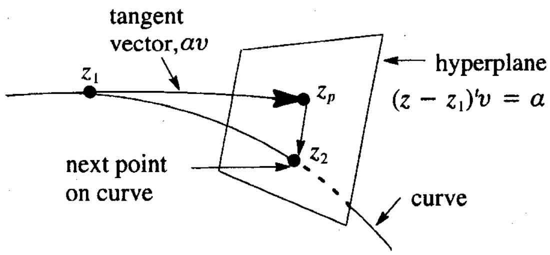

Fig. 6. Predictor-corrector process.

A number of *continuation* methods have been proposed for numerically determining power flow curves; see for example [2], [10], [11], [55], [75]. (Figs. 1 and 4 were produced using the algorithm described in [75].) Typically some form of predictor/corrector technique is employed [28], [80]. The Euler homotopy approach [28] shall be used to illustrate this process.

Referring to Fig. 6, the first step in moving from a point z_1 on the curve to the next point z_2 on the curve is to obtain an estimate of z_2, i.e., to predict ahead to z_p. To do that, the vector v which is tangent to the curve at z_1 is obtained. (Once two points on the curve have been found, the tangent vector v can be approximated by the unit vector which passes through those two points.) The predicted point z_p is obtained by moving along v a predefined distance α.

The next step is to correct to the point z_2 on the curve. In the Euler method, this is achieved by solving for the point of intersection of the curve and the hyperplane that is perpendicular to v and which passes through z_p. The intersection point is therefore defined by the original under-determined set of equations (11) which describe the curve, together with an extra equation describing the hyperplane. The corrector problem therefore involves the same number of equations as unknowns, and so can be solved using a standard technique such as Newton–Raphson.

Industry exposure to and acceptance of power flow curves is still rather limited. It is quite common for power system analysts to manually vary parameters to obtain sensitivity information around operating points. This procedure of manual parameter variation is also used to determine points of maximum loadability. The system is progressively loaded until the power flow ceases to converge. This process assumes that power flow divergence implies Jacobian singularity and hence maximum loadability. This may be misleading though. Power flow divergence is certainly related to Jacobian singularity, but may well be due to a poor initial guess, or the particular implementation of the power flow algorithm.

The use of power flow curves is beginning to gain industry acceptance however. Concern over the possibility of system failure via some form of voltage collapse mechanism has prompted greater interest in methods for determining points of maximum loadability. Continuation methods are a simple way of obtaining such points [10].

Note that as a curve is traversed, points will generally be found where the network constraints change. For example, a generator may encounter its reactive power limit. It would then cease regulating voltage, but may instead maintain a constant reactive power output. At such points the tangent to the curve will undergo a step change. This can be easily handled in most continuation methods, but slows them down. In large systems, with many machines and tap changing transformers encountering limits, the effect on the performance of the continuation method can be quite significant.

C. Small Disturbance Analysis

Small disturbance analysis is the analysis of system behavior in response to small perturbations about an operating point [88]. This analysis is undertaken by linearizing the system description (2)–(4) at an operating point of interest. The eigenstructure of the linearized system is then determined. Small disturbance stability corresponds to all eigenvalues lying in the open left half of the complex plane.

The focus of this publication is on nonlinear behavior of power systems. Therefore we shall not dwell on small disturbance (linear) behavior. However small disturbance analysis can provide quite valuable qualitative information about the large disturbance behavior of a system. Certain 'slower' eigenvalues correspond to electromechanical modes of oscillation of the power system. Information can be obtained from the associated eigenvectors which provides an indication of the way in which groups of machines participate in oscillatory behavior [69]. This participation information is also very useful for understanding the effects of controls on oscillatory behavior, and for tuning controllers.

Unstable equilibrium points (UEP's) are often classified by the number of unstable eigenvalues of the system when it is linearized at the UEP. A type-n UEP has n unstable eigenvalues. The stable manifolds of type-1 UEP's that lie on the stability boundary form important sections of the stability boundary [12]. Therefore (small disturbance) classification of UEP's can be important in the large disturbance analysis of systems. Eigenvectors of type-1 UEP's on the stability boundary can also provide valuable information about modes of instability.

Power system analysts are often interested in the sensitivity of system behavior to parameter variation. In particular, sensitivity information is useful for determining the robustness of an operating point to parameter uncertainty. Eigenvalue sensitivity is helpful for such investigations. For example, the sensitivity of modes to load model parameters is explored in [42]. Sensitivity information is frequently used in the tuning of control loops [60]. In such studies the objective is to maximize damping by moving relevant eigenvalues away from the imaginary axis.

Small disturbance studies indicate the existence or otherwise of Hopf bifurcations [30], [55]. In the power system context, Hopf bifurcations have traditionally been associated with electromechanical modes, and in particular with poor tuning of generator control loops [1], [3], [17], [66]. However recent studies have shown that interactions between load-end devices can lead to oscillatory instability through Hopf bifurcations [72], [98]. This is considered further in Section IV-D.

D. Large Disturbance Analysis

Large disturbance analysis of power systems is the study of the response of the (nonlinear) system over time following some (large) disturbance. Two important features of large disturbance analysis are, 1) nonlinearities must be modeled, and 2) the time domain response is required. Numerical simulation techniques therefore form the basis for large disturbance investigations.

Sections II-A through II-C provide an overview of the modeling of power systems. From that overview it can be seen that there are a number of modeling issues which simulation tools must contend with. They can be summarized as follows:

- nonlinearities of components cannot be ignored,
- the model has a differential-algebraic-discrete structure, so algebraic singularity may occur,
- the power system model is generally stiff, i.e., a large range of time constants are often present,
- relative to the smallest time constants, long simulation times are generally required,
- discontinuities are a major feature of large disturbance behavior, and
- events are likely to occur midway through a time step.

Different analysis packages handle these modeling requirements in different ways. Consider the algebraic singularity problem. Algebraic singularity manifests as nonconvergence of the algebraic constraints. To overcome this problem, a number of commercial packages arbitrarily alter load characteristics at low voltages. The loads are modified to behave like constant admittances, independent of the user specified load models. The network constraints become linear and solvable. Whilst this generally solves a symptom of the problem, viz., nonconvergence of the algebraic constraints, it does nothing to address the real problem, namely incorrect modeling. Further, it hides from the analyst the fact that modeling is not accurately reflecting true system behavior. Unfortunately such packages generally do not even inform the user when this alteration has occurred during a simulation. Repeatability of results using a different package can become extremely difficult. Also, care must be taken in postmortem analysis of disturbances where the aim is to match simulated and observed behavior.

Until comparatively recently, power system simulation programs used Euler-based numerical integration techniques. However these explicit techniques are prone to numerical instability, no matter how small the integration step size is made [82]. Therefore, considering the wide range of time constants and long simulation times which are typical of power system studies, these techniques are inappropriate for power system applications.

An alternative approach is based on implicit techniques such as trapezoidal integration [82]. Trapezoidal techniques are numerically stable for large time steps, but they are not necessarily very accurate when the system has small time constants and a large time step is used. However they have been adapted to handle this situation [27]. Initially an appropriately small time step is chosen. Activity of the states is monitored as the simulation progresses. When there is comparatively little change between time intervals, the time step is increased. Subsequent disturbances may again excite fast states though. Care must be taken to ensure that the time step is reduced when necessary, and that the fast states are reinitialized correctly.

The time step variation idea described above duplicates, in a rather ad hoc way, the behavior of variable step size algorithms such as Gear's method [82]. Some power system simulation packages have adopted such variable order, variable step size algorithms [23], [85]. The implementation is much more difficult, but there are benefits in being able to discard the heuristic rules that are built into the other techniques.

Variable step size algorithms work well for long term simulations when there are only a few discontinuities. Each time a discontinuity occurs though, it subjects the system to a step change, which excites fast states. Therefore at each discontinuity, the time step must shrink to a value small enough to accurately simulate the behavior of the fastest states. In some studies however, discontinuities such as protection operations and/or transformer tap steps can dominate behavior. The study of voltage collapse is one such case.

Voltage collapse often involves many limits, such as generator overexcitation limits and transformer tap limits, being encountered. Also, typically many switching operations occur, for example cascaded tripping of feeders, connection of capacitors and disconnection of loads and reactors. An approach which is specifically aimed at simulating those types of scenarios has therefore been developed. It is based on time scale decoupling (singular perturbation) ideas [92]. The fast states are ignored, and instead effectively replaced by equilibrium equations. The simulation consists of a number of 'snapshots' of the system as the collapse process proceeds. This approach certainly introduces some approximations. However it appears to fairly accurately capture the slowly evolving phenomena of interest.

IV. Research Analysis Tools

In Section III we discussed analysis tools which had generally been adopted by industry. There exist other tools, and variations of those in Section III, which to date remain largely within the research community. This section provides an overview of some of those tools.

A. Continuation Methods

It can be seen from Figs. 1, 3, and 5 that considerable information is available from continuation methods. Such techniques can provide valuable insights into the geometry of the power flow solution space, and questions of multiple equilibria. Industry is not directly interested in such questions, but it remains a fairly active research area. Multiple equilibria are important for voltage collapse proximity

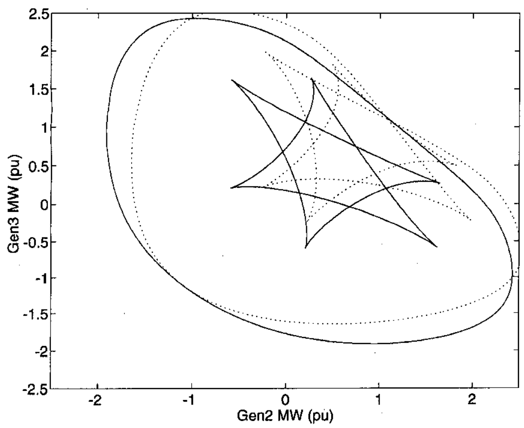

Fig. 7. Saddle node bifurcation curves for the 3 bus system.

indicators (VCPI's) [63], [86], [93]. An understanding of the solution space geometry may lead to better techniques for finding multiple equilibria, and ultimately to better VCPI's for industry.

Continuation methods can also be used to produce bifurcation curves, e.g., curves composed of points where saddle node or Hopf bifurcations occur. Fig. 7 provides an example of saddle node bifurcation curves. It was produced using the predictor/corrector technique described in [43]. The solid lines show the saddle node bifurcation curves that correspond to Fig. 5. The dotted curve shows the distortion which occurs as the resistance in the line between buses Gen2 and Gen3 of Fig. 4 is increased from 0.25–0.5 pu. Note that the system is structurally unstable [66] for this parameter change, with the inner and outer curves becoming connected. These types of curves are helpful in addressing questions, such as posed in [48], relating to the geometry of the solution space boundary.

The continuation method used to produce bifurcation curves such as shown in Fig. 7 is effectively the same as that used to obtain power flow curves. The only difference is in the formulation of the problem. We saw in Section III-B that power flow curves were obtained by releasing a single parameter of the power flow problem (11). To produce bifurcation curves, the equations describing a bifurcation point must first be established. For saddle node bifurcations, the equations can be written as [4]

$$F(\mathrm{x};p) = 0 \tag{15}$$
$$D_{\mathrm{x}}F(\mathrm{x};p)v = 0 \tag{16}$$
$$v^t v = 1. \tag{17}$$

Equation (15) ensures that solutions satisfy the power flow equations. Equations (16) and (17) force the power flow Jacobian $D_{\mathrm{x}}F$ to be singular, with v being the right eigenvector corresponding to a zero eigenvalue. (Singularity of $D_{\mathrm{x}}F$ is a necessary condition for a saddle node bifurcation.) When there is one free parameter $p \in \mathbb{R}^1$, (15)–(17) describe saddle node bifurcation points. When there are two free parameters though, curves of bifurcation points are described. In producing the curves of Fig. 7, the two free parameters were P_2 and P_3, the real powers at Gen2 and Gen3, respectively.

Continuation methods are also useful for exploring the constraint manifold of differential-algebraic systems. Fig. 3 is an example. A clearer picture of the geometry of the constraint manifold can assist in interpreting DA system behavior.

B. Closest Bifurcation Points

In Section III-B, it was mentioned that continuation methods could be used to find the saddle node bifurcation point in a particular loading direction. The distance to that bifurcation point provides a measure of system vulnerability to voltage collapse. However in determining the robustness of an operating point, it is appealing to have a measure of the absolute minimum distance from the operating point to a bifurcation, rather than a distance in some arbitrary direction. Various algorithms have been proposed [19], [20], [49], [58] for determining such critical points, i.e., closest bifurcation points. The ideas are applicable for both saddle node and Hopf bifurcations [20], [59]. However we shall focus on saddle node bifurcations.

As we saw from (15)–(17), if one parameter is free to vary, bifurcations occur at points. With two free parameters, curves of bifurcations are formed. Examples are given in Fig. 7. If more than two parameters are free, then bifurcations exist as surfaces or hypersurfaces. In establishing procedures for finding critical points, i.e., points on bifurcation surfaces which are (locally) closest to the operating point, we use the fact that the vector from a critical point to the operating point is normal to the bifurcation surface. (This criterion is also satisfied by points which are (locally) furthest from the operating point. However those points can be subsequently eliminated.)

To illustrate these ideas, we will use the approach given in [58]. That approach is based on reformulating the power flow (11) as

$$F(\mathrm{x};p_0) = \begin{bmatrix} F_1(\mathrm{x}) + p_{0,1} \\ F_2(\mathrm{x}) + p_{0,2} \end{bmatrix} = 0 \tag{18}$$

where p_0 gives the values of the parameters at the operating point. F is partitioned such that the first equations correspond to parameters that are free to vary at the critical point, while the parameters of the second equations remain fixed at their operating point values $p_{0,2}$.

The critical points are given by nontrivial solutions of

$$-s + p_{0,1} + F_1(\mathrm{x}) = 0 \tag{19}$$
$$p_{0,2} + F_2(\mathrm{x}) = 0 \tag{20}$$
$$D_{\mathrm{x}}F_1^t s + D_{\mathrm{x}}F_2^t \Lambda = 0 \tag{21}$$

where s is the change in parameters between the operating point and the critical point. Note that (21) ensures that vector s is normal to the saddle node bifurcation surface by forcing $[s^t \Lambda^t]^t$ to be the left eigenvector of $D_{\mathrm{x}}F$ corresponding to a zero eigenvalue. The magnitude of s is a measure of system robustness. The orientation of s in parameter space provides information on the most sensitive parameters.

The procedure for finding critical points initially involves taking a guess as to the direction in parameter space of the critical point. This is required because the numerical techniques only find local minima. However it is generally not restrictive, because power system analysts usually have a good feel for reasonable loading directions. The saddle node bifurcation point in that specific direction is obtained. A continuation method is then used to move from that initial bifurcation point to the critical point. The procedure described in [58] ensures that the distance to the operating point always reduces along the continuation path. This guarantees that a local minima is obtained.

The analysis and numerical techniques that have been applied to this problem rely on smoothness of the bifurcation surface. If that smoothness property is lost due to limits and/or discontinuities, the problem becomes immensely more difficult. This is an area for future research.

C. Energy Function Techniques

Energy (Lyapunov) functions provide a way of quickly, though approximately, determining the large disturbance stability of power systems [25], [65]. As discussed in [13], during the faulted period, the system acquires "energy." At the end of the faulted period, the energy acquired by the system is compared with a critical value of energy to determine whether or not stability will be maintained. If the system has obtained less energy than the critical value, then it will be stable. If not, stability may be lost. Note that it is possible to obtain a *measure* of how stable a system is, rather than just determining whether it is stable or unstable.

There are two main challenges with energy function techniques [13]: 1) development of energy functions that (at least approximately) satisfy Lyapunov criteria, and 2) calculation of the critical value of energy. They are now considered.

Energy function techniques are motivated by Lyapunov stability concepts, and so are reliant on the development of functions of the appropriate form. Strict Lyapunov functions are positive definite, and have a nonpositive time derivative along trajectories [51]. Unfortunately, functions which satisfy those conditions have only been found for rather restrictive power system models. For example, no Lyapunov function has been developed for systems which include sixth order machine models, though functions do exist for third order models. Transfer conductance and voltage dependent real power loads present some fundamental problems, as they introduce path dependent integral terms into the Lyapunov function. A Lyapunov function of nonstandard form which incorporates transfer conductances was proposed in [73]. Its usefulness for practical power systems has not been reported though. To overcome modeling limitations, often the troublesome terms are approximated in some way [25], [65]. Rigorous stability results are then not possible, but the resulting energy functions generally appear to provide reliable stability assessment for practical power systems.

The second main challenge of energy function techniques is the calculation of the critical value of energy. The critical energy is the minimum amount of energy required by the system to exit the stability region following a particular disturbance. A number of techniques have been proposed for estimating that energy value. Among those techniques, the controlling unstable equilibrium point (UEP) approach appears to give the most reliable results [13]. Using this approach, the critical energy is given by the potential energy of the UEP which the system would pass close to if the disturbance was critically cleared. Practical systems have many UEP's. Therefore the task of determining which UEP is the appropriate one is nontrivial. The most promising procedure is the BCU method [12], [13], but even it is not completely reliable [56].

Energy function techniques have traditionally been applied to generator (angle) stability assessment. By including the effects of (static) load models, some conclusions have been drawn regarding transient voltage collapse [37], [74]. However it is appealing to considered the inclusion of load dynamics, a fundamental aspect of longer-term voltage collapse. With this aim, Lyapunov functions for systems of dynamic loads were proposed in [36]. Also, dynamic loads were incorporated into 'standard' multimachine Lyapunov functions in [15]. By taking account of reactive power limits, as explained in [39], energy function analysis can therefore be extended to voltage collapse scenarios. Further, the energy function of [15] provides a framework for investigating dynamic interaction between generators and loads.

As well as being useful for estimating the stability region about an operating point, energy functions can provide a geometric interpretation of system behavior. There are some interesting connections between modes of oscillation (as discussed in Section III-C), unstable equilibrium points and the geometry of the potential energy surface. The energy flows can also provide a picture of the stabilizing and destabilizing forces within a system.

Recently energy functions have been used to motivate control strategies for FACTS devices [29]. The proposed control strategies ensure that energy dissipation is a maximum along the system trajectory.

D. Hopf Bifurcation Computations

Hopf bifurcations correspond to a transition from stable oscillatory to unstable oscillatory behavior or vice versa [55]. Small disturbance (linear) analysis can be used to detect a Hopf bifurcation. However the information available from linear analysis is incomplete. Limit cycles, which are a nonlinear phenomenon, are generally associated with Hopf bifurcations. A subcritical Hopf bifurcation corresponds to the coalescing of a stable oscillatory equilibrium point and an unstable limit cycle. A supercritical Hopf bifurcation occurs when an unstable equilibrium point and a stable limit cycle coalesce [30]. Because of the nonlinear nature of limit cycles, they cannot be analyzed using small disturbance analysis.

Information about limit cycles can be very important though. When a stable equilibrium point is surrounded by an unstable limit cycle, the limit cycle determines the boundary of the stability region. If a disturbance bumps the system to a point outside the limit cycle, stability will be lost. As system parameters vary, the unstable limit cycle can shrink around the equilibrium point, resulting is a reduction of the stability region, and hence the security of the power system.

Further, it is important to be able to distinguish between subcritical and supercritical Hopf bifurcations. When stability of the equilibrium point is lost at a subcritical bifurcation, the system loses stability. However when stability of the equilibrium point is lost via a supercritical bifurcation, the system remains stable, though with sustained oscillations. In the operation of a power system, this distinction would be crucial. It is very difficult though, using traditional analysis techniques, to determine limit cycle details.

AUTO [21] is a general bifurcation analysis tool which is proving useful for analyzing power system behavior [3], [72]. It uses a continuation method to identify bifurcations and track equilibria and limit cycles as a parameter of the system is varied. It also provides information regarding the nature of equilibria and limit cycles. Unfortunately AUTO cannot be used directly with large systems. It does not employ sparsity techniques, and so suffers from dimensioning limitations. Also, it is restricted to systems of ordinary differential equations. Though if algebraic singularity is not an issue, DA systems can be transformed to the required form.

The computations associated with identifying and classifying Hopf bifurcations and limit cycles are presented in [55]. Those numerical techniques provide a basis for the analysis of large power systems. Development of these tools is progressing.

V. Future Directions

As power system operating paradigms move toward more heavily stressed, less secure systems, the influence of nonlinearities on system behavior will increase. Power system analysts will need to be equipped with tools which are capable of providing insights into quite complicated behavior. The trend will be for research tools to be increasingly accepted and adopted by industry.

As systems become more heavily loaded, limits and protection devices will come into effect more frequently. Such discontinuities can introduce complicated nonlinear behavior such as limit induced bifurcations and sustained oscillations [97], [98]. Also, they cause considerable difficulties for tools that rely on smoothness properties, such as the techniques for finding closest bifurcation points given in Section IV-B. Those difficulties must be addressed.

The role of sophisticated controls, for example adaptive, robust and/or nonlinear techniques, will increase as power systems become more heavily stressed and more flexibly operated [33]. Investigating and tuning such controllers will require advanced analysis tools, generally beyond the capability of those currently used for power systems. Such tools will grow from techniques used in dynamical systems theory [30], [83].

VI. Conclusions

Power system components are inherently nonlinear. Further, the interconnection of components results in a nonlinear differential-algebraic-discrete model for the complete system. With traditional system operation, nonlinear effects were largely insignificant. However, as systems become more heavily, and less predictably loaded, nonlinearities will begin to have a noticeable influence on system behavior.

A major issue in the analysis of power systems is the modeling of the multitude of components that make up such complex systems. Analysis tools and techniques can only provide useful information if the models accurately reflect true component behavior over the range of interest. The paper highlights areas where analysis tools, and the users of those tools, can be particularly vulnerable to nonlinear behavior. These areas include the inherent nonlinearities of components, modeling uncertainty and assumptions, and the influence of limits and switching.

Analysis tools must continue to work reliably and accurately in the presence of nonlinearities. The paper provides an overview of factors which can adversely affect existing analysis tools. These include Jacobian ill-conditioning near the power flow solution space boundary, algebraic singularity, stiffness of the system time constants, and nonsmooth behavior.

The paper identifies a number of analysis tools which are generally accepted in industry, and other tools which are presently more orientated toward research applications. It is likely that as nonlinearities become more dominant, existing industry tools will be unable to effectively probe some of the more complex forms of observed behavior. Industry analysts will need to supplement their present tools with newer, more systematic techniques which currently reside largely within the research community.

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