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On undervoltage load shedding in power systems

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This paper presents an analytical basis and criterion for undervoltage load shedding in power systems. If the voltage is unstable, the proposed undervoltage load shedding criterion can be used to calculate the amount of load which must be shed to stabilise the voltage magnitude. By using a dynamic load model it has been possible to form the criterion which gives the amount of load which has to be shed to stabilise the system in each point in time. The criterion is general and can be applied to any power system. Copyright © 1996 Elsevier Science Ltd

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I. Introduction

There is a tendency for power transmission systems of today to operate closer and closer to their physical limits. It is not uncommon that the limiting factor for power transfers in systems today is the risk of voltage instability. As a consequence, at least some 15 incidents of voltage collapses occurred world-wide during the 1970s and 1980s [1]. In the event of an approaching blackout, the disconnection of loads under controlled conditions and/or blocking of the tap changers on transformers can minimise the damage sustained [2–4]. There is an obvious need to get the most benefit from the least disruption of the loads, raising questions of where, how much and when to shed. Some static sensitivity ideas have been studied [5]. A methodology which analytically accounts for the dynamic nature of voltage collapse is required, such as has already been reported for induction motor loads [6] and capacitor switching [7]. The present paper makes a step in this direction by developing a systematic procedure for showing how much and when to shed a single load. Emphasis is given to the role of load recovery dynamics.

The paper is organised in the following way. In Section II the special topic of voltage stability is introduced, including the systems and models used, and Section III gives a short theory background. Section IV presents the Load Shedding Criterion and Section V contains the conclusions that can be drawn from the present work.

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II. System and models

The test system first studied has one generator bus and one load bus, see Figure 1. At the load bus there is also a shunt capacitor for load compensation. There are two identical lines in parallel between the buses. This simple system is used to develop and illustrate the basic ideas, but it should be emphasised that these ideas can also be applied to larger systems, see Subsection IV.5.

To force the system into a voltage unstable state, one of the identical parallel lines is disconnected. The system is always exposed to the same initial disturbance.

There are no transformers in the system. This means that Load Tap Changer (LTC) dynamics are not explicitly modelled. But the dynamic load model which is used, with exponential recovery, can be used to model the aggregate effect of several LTCs seen from a high voltage level [8].

The generator has been modelled as a synchronous machine with one field winding, one damper winding in the d-axis and one damper winding in the q-axis. The exciter model used describes an alternator with controlled rectifier. A Power System Stabiliser (PSS) is also included

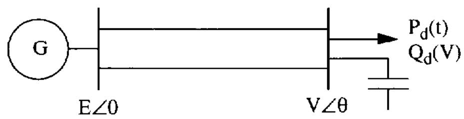

Figure 1. System for load shedding

in the Automatic Voltage Regulator (AVR) loop. The turbine is modelled as a hydro turbine with a speed-governor. Delayed rotor and stator current limiters in the generator and a dynamic load model have been implemented and used in the following simulations.

In looking for a simple dynamic load model based on the response to a voltage step, it is a useful approximation that the recovery is exponential [8]. A differential equation which captures that behaviour is

$$T_p \dot{P}_d + P_d = P_s(V) + k_p(V)\dot{V} \tag{1}$$

The behaviour of the load power, P_d, is determined by two non-linear functions $P_s(V)$, $k_p(V)$ and a time constant T_p. $P_s(V)$ is called the static load function and $k_p(V)$ is the dynamic load function. The equations and notation are analogous for the reactive power demand of the load.

The static load function expresses the amount of power the load needs at a certain voltage in the steady state. The used static load functions for active and reactive power are shown in Figure 2, and they are taken from [9].

The dynamic load function expresses the load power during a certain step change in voltage. The transient behaviour is here assumed to be

$$k_p(V) = k_p V^{\alpha - 1} \tag{2}$$

(Other functions could of course be used.) It is convenient to introduce $P_t(V)$ as

$$P_t(V) := \frac{1}{T_p}\int_0^V k_p(V)\,dV + P_t(0) \tag{3}$$

The transient change in power, denoted ΔP_0, after a transient change in voltage (from V_0 to V_1) can be expressed as

$$\Delta P_0 := P_d(t_{0-}) - P_d(t_{0+}) = P_t(V_0) - P_t(V_1) \tag{4}$$

Field measurements have shown that $\alpha = 2$ is often a reasonable approximation [10]. In this case we have that

$$\Delta P_0 = \frac{k_p(V_0^2 - V_1^2)}{2T_p} \tag{5}$$

The implementation of the load model is done by converting the differential equation to first-order normal form by introducing one state variable for the active and reactive power, x_p and x_q, respectively [8, 11]. It is convenient to define the state variable x_p and its derivative by

$$x_p = T_p(P_d - P_t(V)) \tag{6}$$

$$\dot{x}_p = -\frac{x_p}{T_p} + P_s(V) - P_t(V) \tag{7}$$

The load power, P_d, is given by

$$P_d = \frac{x_p}{T_p} + P_t(V) \tag{8}$$

One important relationship is when the load is in a steady state, i.e. $\dot{x}_p = 0$. In the following, the derivative of the state variable will be used to interpret the simulation results. It can be noted that the state variable is continuous but the derivative could be discontinuous. Corresponding equations apply for reactive load power.

III. Theory background

Consider the situation shown in Figure 1 where two lines feed a load. Assume initially that the load is modelled with full dynamic characteristics in active power and with only static characteristics in reactive power to reduce the

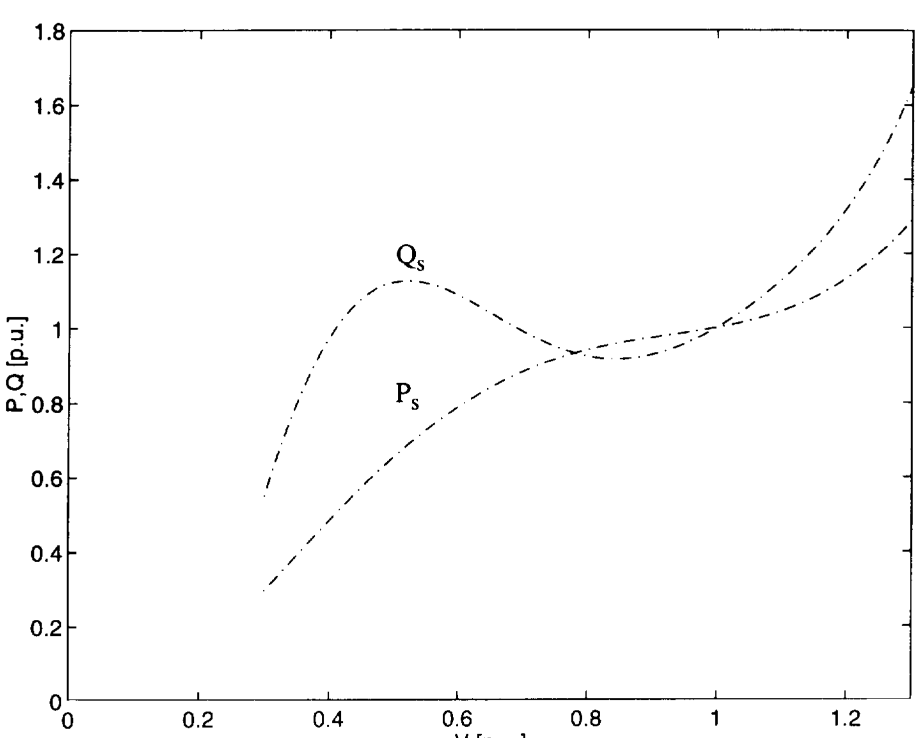

Figure 2. Static load characteristic

complexity of the different system dynamics. This is a close approximation to some loads observed in field tests where reactive power recovery was very small [10].

Combining the load model equations (7) and (8) with the power flow equations gives the simplified system model.

$$\dot{x}_p = P_s(V) - P_d \tag{9}$$

$$P_d = P_l(\theta, V) = \frac{x_p}{T_p} + P_t(V) \tag{10}$$

$$Q_d(V) = Q_l(\theta, V) \tag{11}$$

This ignores generator dynamics to allow a simple explanation of basic ideas. The procedure and examples in later sections use more complex models for the generator and loads.

In principle θ can be eliminated from equation (11), i.e. if $\partial Q_l/\partial\theta \neq 0$, the Implicit Function Theorem gives a function ψ such that $\theta = \psi(V, Q_d)$. Then the model becomes equation (9) with

$$P_d = P_l^*(V) := P_l(\psi(V, Q_d(V)), V) \tag{12}$$

If it is assumed that only active power is shed, then P_l^* is independent of shedding. In general, the (static) reactive power is modified and P_l^* changes also. Combining equation (6) with (12) gives

$$x_p = T_p(P_l^*(V) - P_t(V)) \tag{13}$$

The curve of x_p as a function of V according to equation (13) corresponds to the usual V–P 'nose curve' which gives a constraint for the dynamics (9). The analysis of the load shedding can be illustrated as shown in Figure 3.

The system capability curve P_l^* will be changed by a line disconnection (but not by load shedding). The load characteristics are changed by load shedding (but not the line disconnection). Thus, there are five curves of interest corresponding to pre- and post-fault line capability P_l^* (mapped through equation (13), giving pre-shed and post-shed versions) and the pre- and post-shed steady-state curves. The latter are given by $\dot{x}_p = 0$ in equation (7), i.e.

$$x_p = T_p(P_s(V) - P_t(V)) \tag{14}$$

Note that on the left-hand side of this curve, $\dot{x}_p > 0$. Conversely, on the right-hand side $\dot{x}_p < 0$.

The system is initially in equilibrium at point 1, determined by the intersection of the pre-fault curves. The fault reduces the line capability and the system moves to point 2; the state x_p is constant during the transient. At this point (on the left hand side of the $\dot{x}_p = 0$ curve) $\dot{x}_p > 0$ and x_p increases. From equation (13) we get the constraint

$$\dot{x}_p = C_d(V)\dot{V} \tag{15}$$

where the dynamic coefficient is

$$C_d(V) := T_p \frac{d(P_l^*(V) - P_t(V))}{dV} \tag{16}$$

At point 2, $C_d(V) > 0$, so voltage V increases initially as the trajectory moves towards point 3.

To prevent collapse, load could be shed such that:

(1) a new equilibrium point is created, point a;
(2) the system trajectory just after shedding is contained in the region of attraction of the equilibrium.

We refer to this strategy as the firm strategy. It is not hard to see that the region of attraction for this situation is all $V > V_b$ (within the region of validity of the model). A general stability criterion must also allow for points where the x_p–V curve has zero slope, i.e. $C_d(V) = 0$. Thus, more precisely, the load shedding action should achieve the following:

(1) point 2 should be on the right hand side of the post-shed $\dot{x}_p = 0$ curve;
(2) the post-shed voltage V_2 must lie in the stable region identified above.

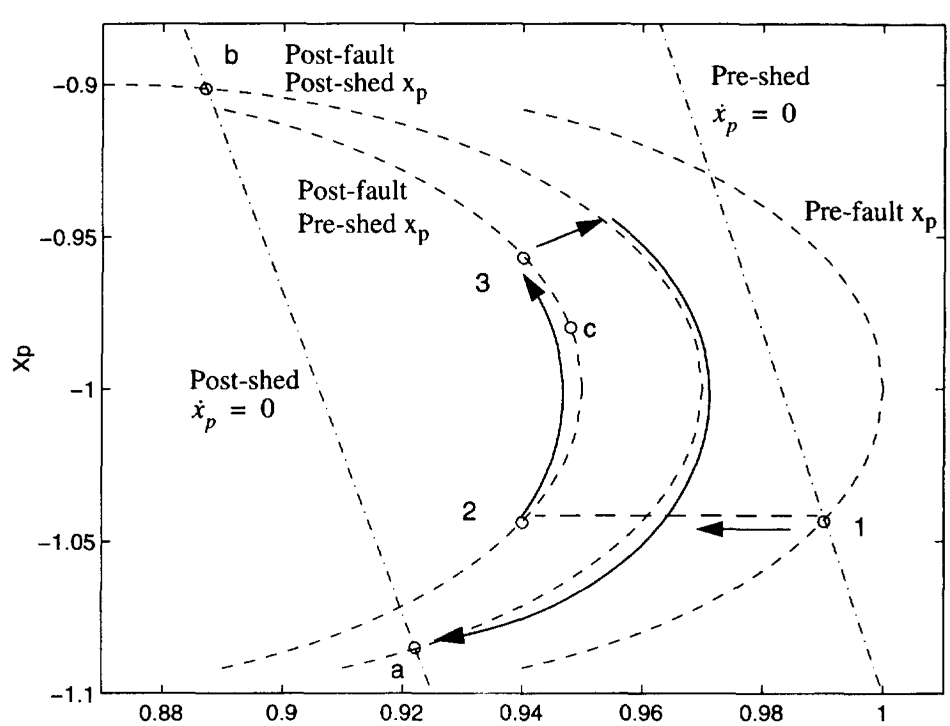

Figure 3. Analysis of load shedding

Within the above scenario, note that there is another possibility; there is no need to ensure point 2 is on the right hand side of the $\dot{x}_p = 0$ curve if the trajectory will be moving towards the new equilibrium. This can be illustrated using Figure 3. In that figure, point c is the point where the $\dot{x}_p = 0$ curve becomes tangential to the post-fault capability curve, as the amount of load shed is varied. (It is the bifurcation point where the post-fault equilibria coalesce.) Consider the case where the amount of load shed is such that the post-fault and post-shed curves intersect at an equilibrium point between points 2 and c. Then, provided the load is shed before the trajectory reaches that equilibrium point, the system will settle to that point. We will refer to this strategy as the soft strategy. If the load is shed after the system has already passed that point however, then the shedding strategy will fail. This highlights the trade-off between the amount and timing of load shedding. The firm and the soft strategies are further described below.

When using the more complete model described in Section II, the dynamics will be more complicated but will preserve the essential features of this discussion if the load dynamics are dominant.

IV. Load shedding experience

Only loads with a larger time constant than the time constant of the AVR will be studied here. This ensures that the load dynamics are dominating the other system dynamics in the long term [12]. The analysis could be carried out in the P–V plane but we choose to use the V–x_p plane since the load dynamics are dominant. Firstly, we consider the simple case of dynamic active power (as in Section III). Dynamics in both the active and reactive load are dealt with in Subsection IV.3. The load representation is assumed to be an aggregation of loads which all have the same characteristics.

IV.1 *Voltage collapse, simple case*

We shall consider first the situation when there is no load shedding. In that case the test system undergoes a voltage collapse approximately 4 minutes after the system is disturbed. The voltage at the generator has a rather flat profile because of the AVR. The load voltage is sagging all the time after line disconnection except for the very last seconds, where the generator dynamics are dominating and these few seconds will not be of interest in the following analysis.

IV.2 *Undervoltage load shedding, simple case*

In Figure 4 the important feature referred to at the end of Section III, should be noticed; the distance between the system trajectory and the static load characteristic. The minimum distance between the system trajectory and the static load characteristic is in this case situated at about 0.7 p.u. in voltage.

As long as the system trajectory is approaching the static load characteristic, both the soft and the firm strategy can be applied. By using the soft strategy we 'move' the system trajectory so close to the static load characteristic that an intersection will be found later on. If we 'move' the system trajectory to a position on the opposite side of the static load characteristic, the firm strategy has been applied. The three possible scenarios are shown in Figure 5. The minimum amount of load to shed to stabilise the system is related to the minimum distance.

When the system trajectory and the static load characteristic are diverging, only the firm strategy can be applied to stabilise the system. How to calculate the amount of load to shed will be discussed in Subsection IV.4.

Another condition for successful load shedding is that the system is short term viable, i.e. the system strives towards the static load characteristic without changing the voltage outside acceptable limits. The question of short term viability will be discussed later.

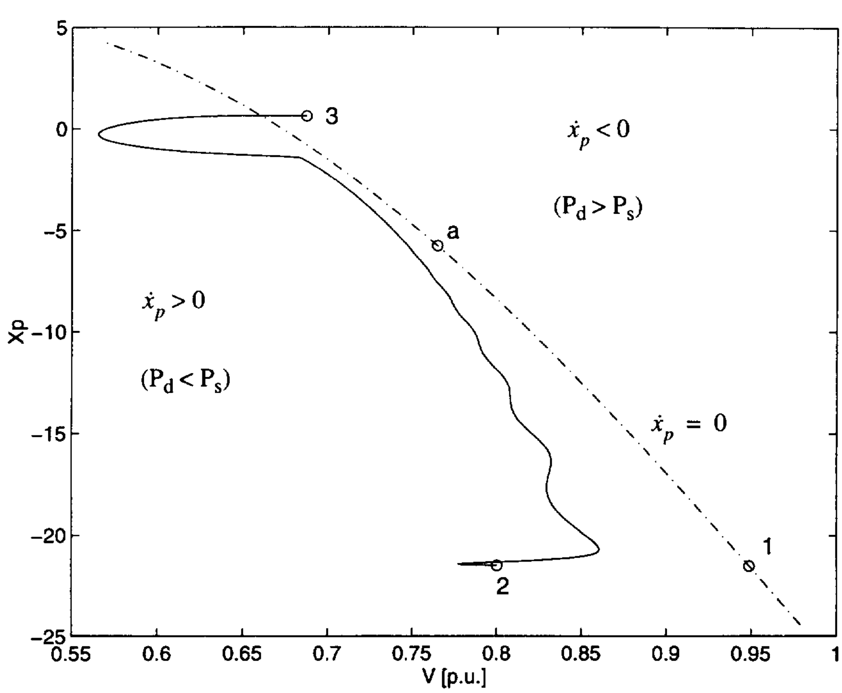

Figure 4. Collapse shown in V–x_p plane. (1) Pre-disturbance steady state operating point, (2) immediately after disturbance, (3) point for collapse and (a) post-disturbance steady state operating point

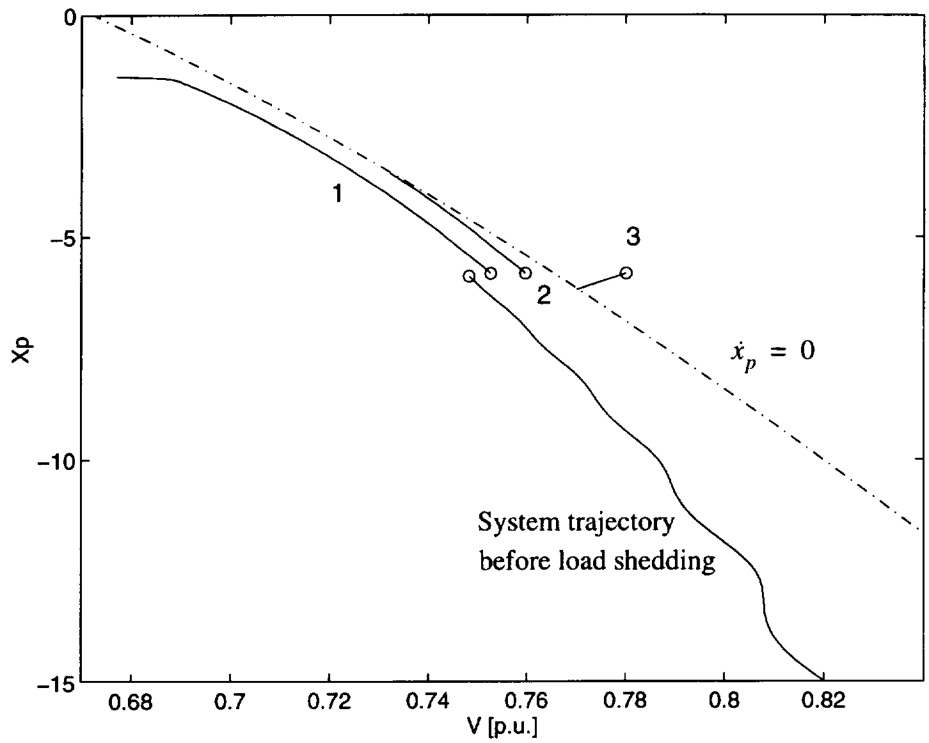

Figure 5. Levels of load shedding. (1) Small amount of load is shed, no intersection is found, gives collapse, (2) an amount of load corresponding to the soft strategy is shed and (3) an amount of load corresponding to the firm strategy is shed

In the following, two simulations will be presented. In these two cases the first 100 seconds simulated are exactly the same as in the voltage collapse case, Figure 4. At $t = 100$ undervoltage load shedding will be performed. The first case will be an example of the soft strategy.

The amount of load shed is such that the nominal active load power is decreased from 140 to 135 MW, reactive load power is unchanged. At the time of load shedding, the voltage at the load bus increases momentarily but continues to decrease for a long period of time until a steady state is reached. That equilibrium point is located at a voltage level lower than for the time of load shedding, see Figure 6 where the simulation result is plotted in the V–x_p plane. The initial fault forces the trajectory to jump from position number 1 to number 2. The load-shedding results in a jump from position number 3 to 4. Number 5 is the point for the final steady state and point a is the same point a as in Figure 4.

The second case of undervoltage load shedding will be an example of the firm strategy. At 100 seconds, some load is shed so that the nominal active load power is reduced from 140 to 125 MW. The reactive load power is unchanged. Also in this case the remaining load after load shedding was short term viable. The simulation result in the V–x_p plane is shown in Figure 7.

Immediately after the load shedding, the voltage at the generator is above the voltage set point. The AVR will, within a few seconds, bring the voltage back to normal at the generator terminals. The voltage at the load will also decrease because of the AVR reduction in voltage. The AVR dynamics are much faster than the load dynamics. This gives the result that, immediately after load shedding, the trajectory in the V–x_p plane moves with almost constant x_p.

The curve shown for the system capability is valid for the post-shedding steady state. Since that curve has an intersection with the static load characteristic for decreasing x_p, within reasonable voltage limits, a short term viable operating point can be expected to be found. If the load after load shedding is very capacitive, the system capability may not have an intersection within acceptable voltage limits.

For load shedding according to the firm strategy, the minimum possible voltage after shedding can be determined if the static load characteristic in equation (14) has a negative derivative ($\mathrm{d}x_p/\mathrm{d}V < 0$). This is possible since the trajectory jumps to a position where $\dot{x}_p < 0$. So, directly after the jump, the state variable x_p reaches a maximum value. That maximum value can then be used in equation (14) to determine the voltage when the load is in the steady state.

IV.3 *Load shedding criterion, general case*

In the previous subsection the analysis was based on a load with dynamics in only the active power. In the following the criterion will be extended to loads with dynamics in both active and reactive power. Two strategies for undervoltage load shedding have been discussed, but the recommended strategy is the firm one. There are several advantages to performing load shedding in that way instead of using the soft strategy. Some of these reasons are:

- The system will normally reach a steady state in a shorter period of time.
- The influence of unmodelled dynamics from other system equipment is minimised once the load shedding process starts to unload the system (the state variables of the load are decreasing).

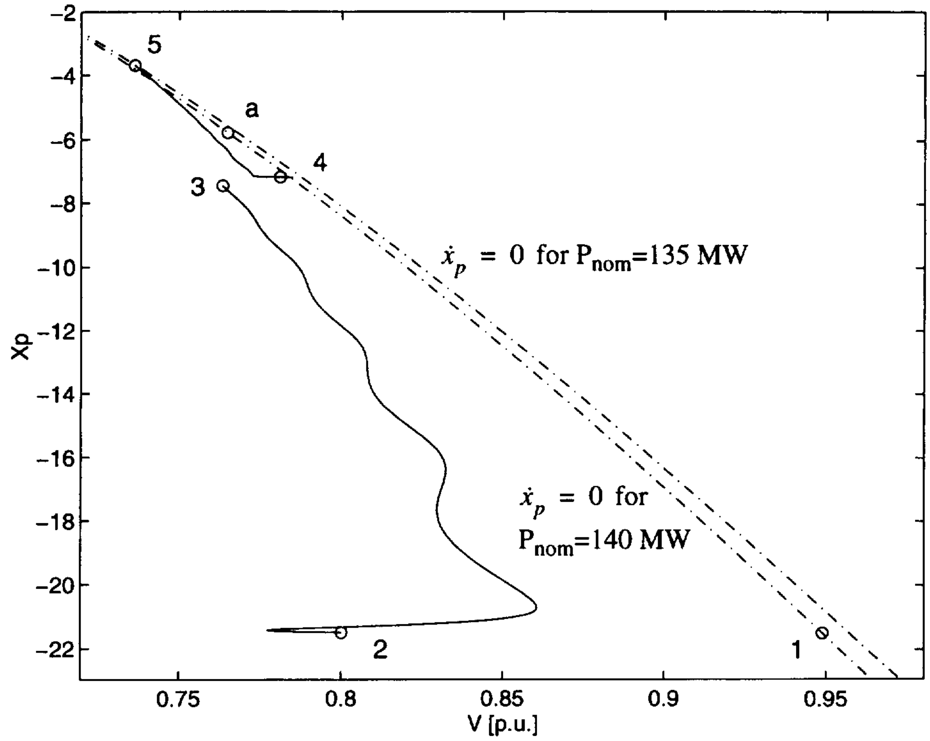

Figure 6. Soft load shedding

- The strategy is applicable whenever the load dynamics are dominating.

An undervoltage load shedding performed so that the trajectory moves to the region of decreasing state variable and the load which is short term viable is called successful in the short term in the following. Short term viability is the normal case for inductive loads.

The amount of load that needs to be shed is related to how large the 'disturbance' must be to bring the system back to a steady state. So by looking in the V–x_p, (V–x_q) plane the distance between the trajectory and the steady state characteristics can be taken as an indication of the amount of load that has to be shed. The closer the trajectory is to the static load characteristic the smaller is the amount of load that needs to be shed. Immediately after the fault and just before a possible collapse, a relatively large amount of load has to be shed to get a bad shedding successful in the short term.

The distance between the trajectory and the static load characteristic in the V–x_p plane can be different from the distance in the V–x_q plane. This means that, for a certain

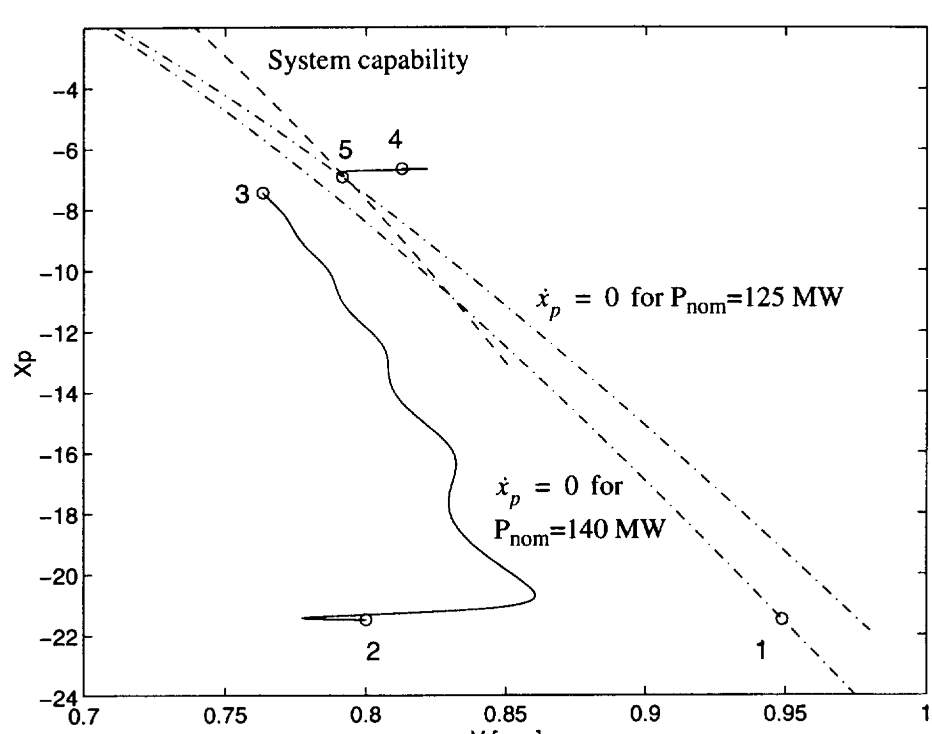

Figure 7. Firm load shedding

load shedding, the analysis result will be different if it is carried out in the V–x_p or in the V–x_q plane. If the time constants of the load recovery for active and reactive power are different, the results of the analyses will quite often be different. If the load shedding takes the trajectory to the area of decreasing state variable in both the V–x_p and V–x_q planes then the shedding is regarded as successful. If that area is not reached in either plane then the shedding is regarded as unsuccessful in the short term but it is not certain that a collapse will occur since the load shedding may fulfil the requirements according to the soft strategy.

For physical loads the active and reactive load powers are related, but in order to investigate the relative effectiveness they are chosen independently in this theoretical analysis. However, it is not obvious that by shedding reactive load the effectiveness is higher than for shedding active load.

In the analysis presented here, the load characteristic is assumed to be known. In practice, there will be some uncertainty in the load characteristic and this may influence the accuracy of the result. However, the method is independent of the load model used, and whether the load is modelled dynamically or statically. For instance, if tap changing transformers are explicitly modelled together with static loads, the dead band in the regulated voltage will introduce a 'dead band' in the analysis. When confronted by significant load parameter uncertainty, it is not hard to modify the procedure to ensure adequate control for all possible load parameters.

This technique of analysis is not limited to undervoltage load shedding. If there is a reactive power source available during a voltage unstable situation, this criterion for successful load shedding can be converted to a criterion for 'successful reactive power injection'.

IV.4 *Amount of load to shed, general case*

To calculate the amount of load that it is necessary to shed, some data must be available. Those are:

- Load flow data for the system for the point in time when the load shedding is going to be performed.
- Static load characteristics, $P_s(V)$ and $Q_s(V)$.
- Data for the transient behaviour of the load i.e. T_p, T_q, $k_p(V)$ and $k_q(V)$.

An important step in the calculations of the amount of load to shed, is the calculation of the load response to a voltage step. The load power after the voltage step, $P_{d,\text{new}}$, is equal to the initial load power, P_d, minus the transient response, ΔP_0, of the load (4), i.e.

$$P_{d,\text{new}} = P_d - \Delta P_0 \tag{17}$$

If the voltage step originates from load shedding, then the remaining proportion of the load after load shedding, z_p, must fulfil the just mentioned relationship,

$$P_{d,\text{new}} = z_p(P_d - \Delta P_0) \tag{18}$$

Equations (17) and (18) apply also for reactive load power. For the calculations of the amount of load to shed, the main structure of the iterative routine is:

(1) Using the monitored load powers at the instant when load shedding is going to be performed, solve a load flow for the current transient state.
(2) Choose new values of the load powers, $P_{d,\text{new}}$ and $Q_{d,\text{new}}$, which are the load powers after load shedding.
(3) Calculate the new voltage at the load, V_{new}. This new voltage must be higher than the current voltage, otherwise go back to step 2 and choose new load values.
(4) Calculate the transient change of the load, ΔP_0 (5) and ΔQ_0.
(5) Calculate how much load needs to be shed so that the voltages and load powers after load shedding correspond to the result with chosen load values. The factors of the load powers to shed, f_p and f_q, are calculated as $1 - z_p$ and $1 - z_q$ respectively. The values of f_p and f_q are between zero and unity.
(6) Scale the nominal powers (static load characteristic) of the load using the just calculated factors.
(7) Calculate the new static load values, $P_{s,\text{new}}$ and $Q_{s,\text{new}}$, of the load at the voltage V_{new}.
(8) If $P_{d,\text{new}} > P_{s,\text{new}}$ and $Q_{d,\text{new}} > Q_{s,\text{new}}$ then the load shedding is classified as successful in the short term. If these relationships are not fulfilled, go back to step 2 and choose new load values.

IV.5 *Nine bus test system*

To verify the analysis technique and load shedding criterion presented in the previous subsections, a larger test system, based on the WSCC 9 bus test system, was studied [12]. Some modifications had to be made to the WSCC 9 bus test system to make it suitable for long term dynamic studies. The static data, i.e. load flow data, were kept unchanged. The following modifications to the dynamic data were made:

- The three original loads were modelled by use of the dynamic load model described in Section II.
- Models for turbines and turbine regulators, power system stabilisers, stator and rotor current limiters, as described in Section II, were added to all three generators.

A voltage collapse is initiated by connecting an extra load to BUS-A. This load is modelled with impedance characteristics. At 30 seconds, this load with nominal power of $140 + j100$ MVA is connected. The 9 bus test system undergoes a voltage collapse after 400 seconds if no emergency control actions are performed. To prevent the system from this collapse, an undervoltage load shedding is performed after 200 seconds.

For simplicity we concentrate all load shedding on to one bus, BUS-B. A load flow solution at the time of load shedding must be available. The load power, P_d and Q_d, at BUS-B is at the time for load shedding $88.6 + j28.3$ MVA. Now, the load is reduced so that the criterion for successful load shedding is fulfilled in all three load buses. If the load power in BUS-B is lowered so that the new load powers, $P_{d,\text{new}}$ and $Q_{d,\text{new}}$ are $70 + j15$ MVA, a load flow solution shows that the voltage rise in the system makes the criterion for successful load shedding fulfilled in each bus. This reduction in load power at BUS-B, corresponds to a load shedding of 27.4% in active power and 58.4% in reactive power, i.e. $f_p = 0.274$ and $f_q = 0.584$. The load voltages in the system for this case of load shedding are shown in Figure 8. The influence of different load shedding sites on the system response of the 9 bus test system can be seen in [13].

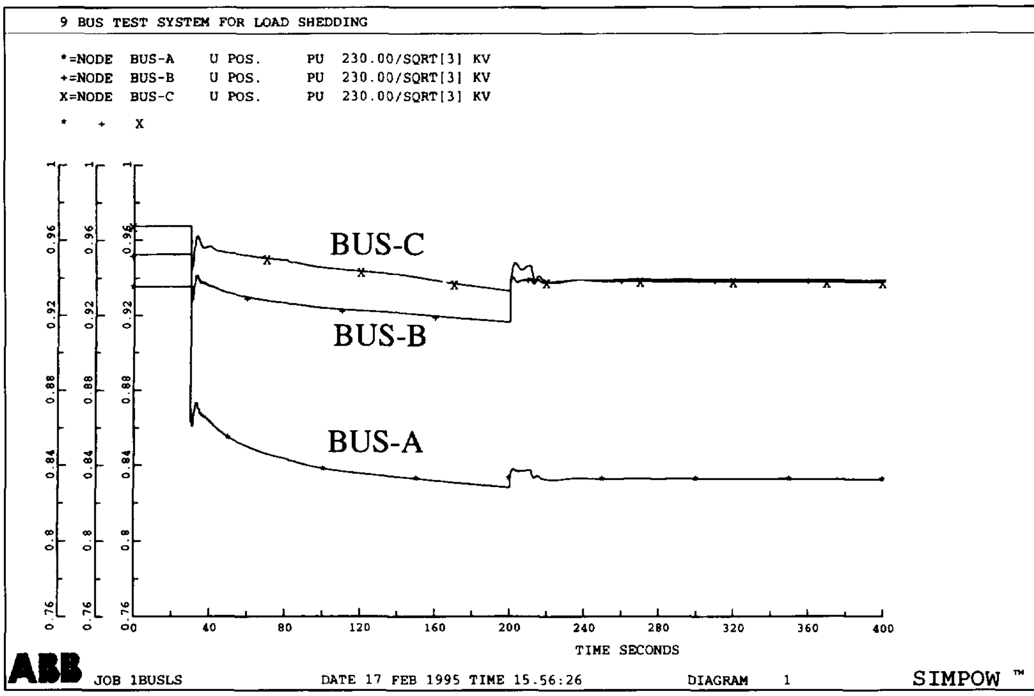

Figure 8. Successful load shedding

Shedding load at only one bus, as in this case, causes the conditions for successful load shedding to be fulfilled with an unnecessarily large safety margin (for this bus). But it is necessary to shed so much load to fulfil the criterion at all the other load buses. Thus, it is more effective to shed small amounts of load at several different buses. The amount should be such that the criterion is just fulfilled at each bus. To develop a strategy for multi-bus load shedding will be the subject of future research.

V. Conclusions

This paper has presented an analytically based criterion for undervoltage load shedding to avoid voltage collapse. The analysis technique can be applied to other kinds of emergency control actions apart from undervoltage load shedding. Capacitive switching has been studied in [7].

The course of events, in the 2-bus system, is very dependent upon the load characteristic. It is found that the system is more likely to experience a voltage collapse if the load has a pronounced transient characteristic, i.e. ΔP_0 in equation (4) is large.

Further work is needed to develop the methodology for large systems. The choice of load shedding sites can be approached by sensitivity analysis [5]. In general the sensitivity of a margin of stability must be studied. This requires further development of a general technique to estimate the regions of stability around equilibria. This was seen earlier in the case considered here (see Section III). In general, this assessment will need to allow carefully for generator/AVR dynamics although in many cases these are faster than the load dynamics and can be ignored.

Two strategies for undervoltage load shedding have been discussed. While each has advantages, at least where load change does not cause other problems, the firm strategy has been recommended. These strategies are dependent on knowledge of the dynamic load characteristics. While this is not widespread now, we can expect increasing attention to be paid to the collection of data [7, 10, 11].

VI. Acknowledgements

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Appendix

A.1 *Static data for 2-bus test system*

$S_{\text{base}} = 100\,\text{MVA}$, $V_{\text{base}} = 220\,\text{kV}$, $R_{\text{line}} = 0.044$ p.u., $X_{\text{line}} = 0.176$ p.u., $B_{\text{line}} = 0.30855$ p.u., Q_{nom} for shunt impedance $= -95$ Mvar, P_{nom} for load $= 140$ MW, Q_{nom} for load $= 130$ Mvar and V_{set} for generator $=$ 221 kV.

A.2 *Dynamic data for 2-bus test system*

Load: Active load power–dynamic representation, reactive power–static representation, $T_{\text{p}} = 20$ s, $k_{\text{p}} = 110$, $\alpha = 2$.

Synchronous machine: $S_{\text{rated}} = 280$ MVA, saturation excluded, $V_{\text{rated}} = 220$ kV, $H = 4.0$ MWs/MVA, $R_{\text{t}} = 0$ p.u., $X_{\text{t}} = 0.10$ p.u., $R_{\text{a}} = 0$ p.u., $X_{\text{a}} = 0.18$ p.u., $X_{\text{d}} = 1.0$ p.u., $X'_{\text{d}} = 0.4$ p.u., $X''_{\text{d}} = 0.249$ p.u., $T'_{\text{d0}} = 5.0$ s, $T''_{\text{d0}} = 0.06$ s, $X_{\text{q}} = 0.65$ p.u., $X''_{\text{q}} = 0.249$ p.u., $T''_{\text{q0}} = 0.125$ s.