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# HYBRID SYSTEMS VIEW OF POWER SYSTEM MODELLING

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## ABSTRACT

The large disturbance behaviour of power systems often involves complex interactions between continuous dynamics and discrete events. The paper proposes a differential-algebraic-discrete (DAD) model structure which captures those interactions in a systematic way. It is shown that the model is a realization of a general hybrid system model. The DAD model opens up opportunities for the application to power systems of hybrid system results in stability analysis and control. The paper presents a practical approach to implementing the DAD model structure.

## 1. INTRODUCTION

The large disturbance behaviour of power systems is often characterised by complex interactions between continuous dynamics and discrete events. Components such as generators and loads drive the continuous dynamic behaviour. They obey physical laws, and are usually represented by coupled differential and algebraic equations [1]. Other components exhibit event-driven discrete behaviour though. Examples include tap-changing transformers, switched shunts and protection devices. The dynamics in this case are often governed by logic rules that depend on inputs from the continuous dynamics.

Systems which involve both continuous and discrete event dynamics, such as power systems, have become known as *hybrid systems*. Significant attention has been directed towards hybrid systems recently, with exciting progress being made in stability analysis and control [2, 3]. These developments open up new opportunities for power system security assessment and control design. However to fully exploit these opportunities, a systematic model of power systems must be established. This paper addresses that modelling issue.

Analysis of the large disturbance dynamic behaviour of power systems has historically relied on time domain simulation. Therefore *ad hoc* approaches to modelling discrete events have been adequate. Development of systematic models has received little attention. An exception is the work of [4], where precise descriptions of protection devices have been presented. Interestingly, even common devices such as tap-changing transformers exhibit dynamic behaviour that is difficult to describe analytically. The proposed model captures such complexities.

## 2. HYBRID SYSTEM MODEL

As indicated in Section 1, hybrid systems are characterized by:

- continuous and discrete states,
- continuous dynamics,
- discrete events, or triggers, and
- mappings that define the evolution of discrete states at events.

Conceptually such systems can be thought of as an indexed collection of continuous dynamical systems $\dot{x} = f_q(x)$, along with a mechanism for 'jumping' between those systems, i.e., for switching between the various $f_q$. The continuous and discrete states are $x$ and $q$ respectively. The jumping reflects the influence of the discrete event behaviour, and is dependent upon both the trigger condition and the discrete state evolution mapping. Overall system behaviour can be viewed as a sequential patching together of dynamical systems, with the final state of one dynamical system specifying the inital state for the next.

A formal presentation of these concepts is given in [5], where a general hybrid dynamical system is defined as

$$H = [Q, \Sigma, A, G] \tag{1}$$

and

- $Q$ is the set of discrete states;
- $\Sigma = \{\Sigma_q\}_{q\in Q}$ is the collection of dynamical systems $\Sigma_q = [X_q, \Gamma_q, f_q]$ where each $X_q$ is an arbitrary topological space forming the continuous state-space of $\Sigma_q$, $\Gamma_q$ is a semigroup over which the states evolve, and $f_q$ generates the continuous state dynamics;
- $A = \{A_q\}_{q\in Q}$, $A_q \subset X_q$ for each $q \in Q$, is the collection of autonomous jump sets, i.e., the conditions which trigger jumps;
- $G = \{G_q\}_{q\in Q}$, where $G_q : A_q \to S = \bigcup_{q\in Q} X_q \times \{q\}$ is the autonomous jump transition map.

The hybrid state-space of $H$ is given by $S$. In this paper we restrict attention to hybrid systems where $Q$ is countable, each $X_q \subset \mathbb{R}^{d_q}$, $d_q \in \mathbb{Z}_+$, and each $\Gamma_q = \mathbb{R}$.

The abstract model (1) is not immediately useful for power system analysis. Therefore a model which is more conducive to such analysis is presented in the following section. It is then shown that the proposed model is consistent with (1).

## 3. POWER SYSTEM MODEL

Many different types of systems, including power systems, can be generically described by a parameter dependent differential-

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algebraic-discrete (DAD) model of the form,

$$\dot{x} = f(x, y, z; \lambda) \quad (2)$$

$$0 = g^{(0)}(x, y, z; \lambda) \quad (3)$$

$$0 = \begin{cases} g^{(i-)}(x, y, z; \lambda) & y_{d,i} < 0 \\ g^{(i+)}(x, y, z; \lambda) & y_{d,i} > 0 \end{cases} \quad i = 1, ..., d \quad (4)$$

$$z^+ = h_j(x^-, y^-, z^-; \lambda) \quad y_{e,j} = 0 \quad j \in \{1, ..., e\} \quad (5)$$

$$\dot{z} = 0 \quad y_{e,j} \neq 0 \quad \forall j \in \{1, ..., e\} \quad (6)$$

where

$$x \in X \subseteq \mathbb{R}^n, y \in Y \subseteq \mathbb{R}^m, z \in Z \subseteq \mathbb{R}^l, \lambda \in L \subseteq \mathbb{R}^p$$

$$y_d = Dy$$

$$y_e = Ey$$

$$f : \mathbb{R}^{n+m+l+p} \to \mathbb{R}^n$$

$$g = \begin{bmatrix} g^{(0)} \\ g^{(1)} \\ \vdots \\ g^{(d)} \end{bmatrix} : \mathbb{R}^{n+m+l+p} \to \mathbb{R}^m$$

$$h_j : \mathbb{R}^{n+m+l+p} \to \mathbb{R}^l \qquad j = 1, ..., e$$

and $D \in \mathbb{R}^{d \times m}, E \in \mathbb{R}^{e \times m}$ are matrices of zeros, except that each row of each matrix has a single 1 in an appropriate location. The $y_d$ and $y_e$go vern the $d$ switching events and $e$ reset events respectively. There is no restriction on $y_d$ and $y_e$ sharing some common elements. In (5), $x^-$, $y^-$, $z^-$ refer to the values of $x$, $y$, and $z$ just prior to the reset condition, whilst $z^+$ denotes the value of $z$ just after the reset event.

In this model, which is similar to a model proposed in [6], $x$ are continuous dynamic state variables, $y$ are algebraic state variables, $z$ are discrete state variables, and $\lambda$ are parameters. For example, in the power system context $x$ would include machine dynamic states such as angles, velocities and fluxes, $y$ would include network variables such as load bus voltage magnitudes and angles, $z$ could represent transformer tap positions and/or relay internal states, and $\lambda$ could be chosen from a diverse range of parameters, from loads through to fault clearing time.

Note that the model does not allow discontinuities in the dynamic states, i.e., impulse effects. This is not a restriction forced by analysis. However the model adopts the philosophy that the dynamic states of *actual* systems cannot undergo step changes.

The proposed model (2)-(6) captures all the important aspects of hybrid system behaviour, namely the interaction between continuous and discrete states as they evolve over time. Between events, system behaviour is governed by the differential-algebraic (DA) dynamical system

$$\dot{x} = f(x, y, z; \lambda) \quad (7)$$

$$0 = g_q(x, y, z; \lambda) \quad (8)$$

where $g_q$ is composed of $g^{(0)}$, together with functions from (4) chosen depending on the signs of the elements of $y_d$. Each different composition of $g_q$ is indexed by a unique $q$. An event is triggered by an element of $y_d$ changing sign and/or an element of $y_e$ passing through zero. At an event, the composition of $g_q$ changes and/or elements of $z$ are reset. Therefore, in this hybrid system model, each DA dynamical system is effectively indexed by $q$ and $z$. At an event, this index changes and a jump is made to the new dynamical system.

Assuming the Jacobian $\partial g_q/\partial y$ is nonsingular, i.e., the system has not encountered an impasse surface [7], the implicit function theorem allows (8) to be solved (locally) giving $y = \varphi_{(q,z)}(x; \lambda)$. Substitution into (7) yields $\dot{x} = f^*_{(q,z)}(x; \lambda)$. This representation allows the DAD model to be related directly to the general hybrid dynamical system model (1). The discrete states are $(q, z) \in Q$. The dynamical systems $\Sigma_{(q,z)}$ are defined by (7),(8), with $f^*_{(q,z)}$ generating the continuous state dynamics. Each jump set $A_{(q,z)}$ is composed of conditions $y_{d,i} = 0$ and $y_{e,j} = 0$, where $y_d$, $y_e$ are given by $\varphi_{(q,z)}$. The general nature of $g_q$, and hence $\varphi_{(q,z)}$, allows arbitrarily complicated sets of event triggering conditions to be described for each $(q, z)$. The jump transition map $G_{(q,z)}$ is defined by the change in $q$ that corresponds to each $y_{d,i} = 0$, along with the reset map (5) corresponding to each $y_{e,j} = 0$.

The following example illustrates the DAD model structure (2)-(6).

## Example

In order to demonstrate the ability of the DAD structure (2)-(6) to model rule-based systems, this example considers a relatively detailed representation of the automatic voltage regulator (AVR) of a tap-changing transformer. The logic flow of the AVR for low voltages, i.e., for increasing tap ratio, is outlined in Figure 1. The corresponding DAD representation is,

$$\dot{x}_1 = y_1$$

$$0 = y_3 - y_4 + z_1$$

$$0 = y_6 - n + n_{\max} - n_{\text{step}}/2$$

$$0 = nV_1 - V_2$$

$$0 = y_1 - 1 \qquad y_2 < 0$$

$$\left.\begin{aligned} 0 &= y_1 \\ 0 &= y_4 - x_1 \end{aligned}\right\} \qquad y_2 > 0$$

$$0 = y_2 - V_2 + V_{\text{low}} \qquad y_6 < 0$$

$$0 = y_2 - 1 \qquad y_6 > 0$$

$$0 = y_5 - x_1 + z_1 + T_{\text{tap}} \qquad y_3 < 0$$

$$0 = y_5 - x_1 + y_4 + T_{\text{tap}} \qquad y_3 > 0$$

$$\left.\begin{aligned} z_1^+ &= x_1^- \\ n^+ &= n^- + n_{\text{step}} \end{aligned}\right\} \qquad \text{when } y_5 = 0.$$

To assist in connecting AVR logic with the model, Figure 1 indicates the variables that are related to particular functions.

The dynamics of this device are driven by a number of interacting events which govern the behaviour of the timer. If the tap is at the upper limit ($y_6 > 0$), or the voltage is within the deadband ($y_2 > 0$) then the timer is blocked. If the voltage is outside the deadband ($y_2 < 0$) then the timer will run. If the timer reaches $T_{\text{tap}}$ ($y_5 = 0$) then a tap change will occur and the timer will be reset, but not necessarily blocked. If the voltage returns to within the deadband, because of smooth system dynamics, or a tap change, or some other system event, then the timer is blocked and reset. □

It is clear that the notation of (2)-(6) can quickly become unwieldy. Therefore it is convenient to write the model more com-

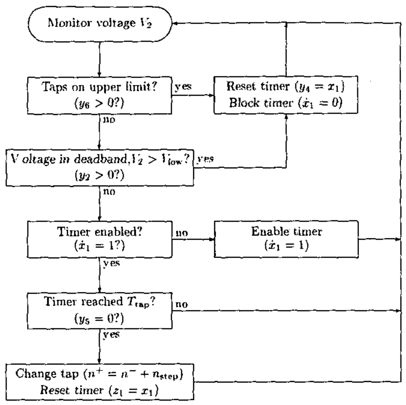


Figure 1: Transformer AVR logic for increasing tap.

pactly as

$$\underline{\dot{x}} = \underline{f}(\underline{x}, y) \tag{9}$$

$$0 = g^{(0)}(\underline{x}, y) \tag{10}$$

$$0 = \begin{cases} g^{(i-)}(\underline{x}, y) & y_{d,i} < 0 \\ g^{(i+)}(\underline{x}, y) & y_{d,i} > 0 \end{cases} \quad i = 1, \ldots, d \tag{11}$$

$$\underline{x}^{+} = \underline{h}_j(\underline{x}^{-}, y^{-}) \qquad y_{e,j} = 0 \quad j \in \{1, \ldots, e\} \tag{12}$$

where

$$\underline{x} = \begin{bmatrix} x \\ z \\ \lambda \end{bmatrix}, \quad \underline{f} = \begin{bmatrix} f \\ 0 \\ 0 \end{bmatrix}, \quad \underline{h}_j = \begin{bmatrix} x \\ h_j \\ \lambda \end{bmatrix}.$$

Let the times at which events occur be given by $\{\tau_k : t_0 < \tau_1 < \tau_2 < \ldots\}$.

Notice that the definition of $\underline{f}$ ensures that $z$ and $\lambda$ remain constant away from reset events (12). Further, $\underline{h}_j$ ensures that $x$ and $\lambda$ remain unchanged at a reset event. As with (7),(8), over each of the open time intervals $(\tau_k, \tau_{k+1})$ the system is described by a smooth differential-algebraic (DA) model

$$\underline{\dot{x}} = \underline{f}(\underline{x}, y) \tag{13}$$

$$0 = g(\underline{x}, y) \tag{14}$$

where $g$ is composed of (10) together with functions from (11) chosen depending on the signs of the elements of $y_d$. (Recall that the definition of the $\tau_k$ ensures that no elements of $y_d$ can change sign during the period $(\tau_k, \tau_{k+1})$.)

## 4. IMPLEMENTATION

Models of large systems are most effectively constructed using a hierarchical, object-oriented approach. With such an approach, components are grouped together as subsystems, and the subsystems are combined to form the full system. This allows component and subsystem models to be developed and tested independently. It also allows flexibility in interchanging models. For example, a dynamic load model may be swapped for a static load to explore the influence on voltage collapse. At a higher level, an economic dispatch subsystem model could be replaced by a market dispatch model to compare outcomes of the different scheduling strategies.

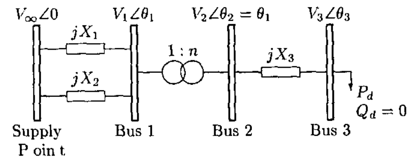


Figure 2: Power system example.

The interactions inherent in hybrid systems are counter to this decomposition into subsystems and components. However the algebraic equations of the DAD model can be exploited to achieve the desired modularity. Each component or subsystem should be modelled autonomously in the DAD structure, with all 'interface' quantities, i.e., inputs and outputs, established as algebraic variables. The components are then interconnected by introducing simple algebraic equations that 'link' the interface variables. For example, say the $n$-th algebraic state of component $j$, denoted $y_{j,n}$, is required as an input by component $k$. In the model of component $k$, that input would appear as an algebraic variable $y_{k,m}$. The connection is made via the simple algebraic equation $0 = y_{j,n} - y_{k,m}$. In general, all linking can be achieved by summations of the form

$$0 = \sum c_k y_{i,j} \tag{15}$$

where $c_k$i s $\pm 1$. Notice that all connections are external to the component models.

The process introduces extra algebraic variables and equations, so is a little inefficient. However the connections are extremely sparse, so careful use of sparsity techniques makes the approach tractable.

The proposed modular approach to constructing hybrid systems has been implemented in Matlab. The following example illustrates the concepts.

## Example (continued)

The simple power system of Figure 2 consists of a dynamic load supplied from an infinite bus via a tap-changing transformer. The continuous dynamics of the real power load are given by an exponential recovery model.

This system is described by the data file of Figure 3. Each component of the system is described by its corresponding model; the dynamic load and tap changer are represented by **load_dyn1** and **tap_changer** respectively, whilst the models for the infinite bus, network and switched line are appropriately named. Each model is associated with three data vectors. These specify initialization values for $\underline{x}_0$, $y_0$ and parameters respectively.

As indicated earlier, the models are interconnected through interface variables. Consider the connection of components such as the load to the network. The model **network** provides a nodal representation of the network constraints, and introduces four algebraic variables at each bus, viz., real and imaginary components of bus voltage $V_r$, $V_i$ and injected current $I_r$, $I_i$. The model **load_dyn1** describes load behaviour in terms of terminal bus algebraic variables $V_r$, $V_i$, $I_r$ and $I_i$. The link between the network and load variables is established via **connections**. Each vector in **connections** contains pairs of indices which set up an equation of the form (15). Referring to the data file of Figure 3, the first vector [1 2 3 −13], for example, introduces the equation $0 = y_{1,2} - y_{3,13}$ where $y_{i,j}$ refers to the $j$-th algebraic variable of the $i$-th model. This particular equation ensures the real part of the voltage seen by the load $(y_{1,2})$ is equal to the appropriate network voltage $(y_{3,13})$.

```
model_data = {'load_dyn1' [0 5 0.4 2.0] [0.4 1 0 -0.4 0] [0.4] ;
              'tap_changer' [0 -1 1.0375 20 0.0125] ...
                 [0 0 0 0 0 0 1 0 1 0 -0.4 0 0.4 0] ...
                 [1.04 1.1] ;
              'network' [] [1 0     0 0 ... % Infinite bus
                            1 0 -0.4 0 ... % Bus 1
                            1 0  0.4 0 ... % Bus 2
                            1 0 -0.4 0]... % Bus 3
                          [1 2 0 0.65 0;    % Line data: f t R X B
                           3 4 0 0.80 0] ;
              'infinite_bus' [] [1.05 0] [1.05] ;
              'switched_line' [0] [1 0 0 0 ... % From bus
                                   1 0 0 0 -1] ... % To bus
                              [0 0.40625 0 10] ;
              'out_Vmag' [] [1 0 1] [] };
connections = {[1 2 3 -13] ...
               [1 3 3 -14] ...
               [1 4 3 -15] ...
               [1 5 3 -16] ...
               [2 7 3 -5] ...
               [2 8 3 -6] ...
               [2 9 3 -9] ...
               [2 10 3 -10] ...
               [2 11 3 -7 5 7] ...
               [2 12 3 -8 5 8] ...
               [2 13 3 -11] ...
               [2 14 3 -12] ...
               [4 1 3 -1] ...
               [4 2 3 -2] ...
               [5 1 3 -1] ...
               [5 2 3 -2] ...
               [5 5 3 -5] ...
               [5 6 3 -6] ...
               [6 1 3 -13] ...
               [6 2 3 -14] };
```

Figure 3: Data file for transformer-load system.

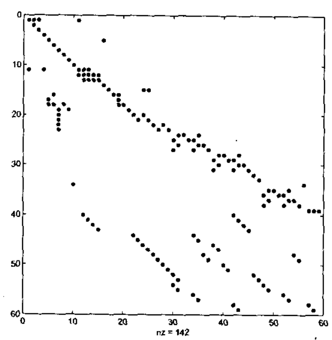


Figure 4: Sparsity structure for the two machine system.

The Jacobian of the continuous dynamics

$$\mathcal{J} = \begin{bmatrix} f_{\underline{x}} & f_y \\ g_{\underline{x}} & g_y \end{bmatrix} \tag{16}$$

is useful for illustrating the interconnections between models. The structure of $\mathcal{J}$ for this example is shown in Figure 4. It is clearly sparse. The matrix is composed of blocks down the diagonal, together with the cross couplings $f_y$ and $g_{\underline{x}}$ and the connection equations. The matrix $f_{\underline{x}}$, which occupies the top left corner of $\mathcal{J}$, has dimension $\mathbb{R}^{10\times 10}$ in this example. The last 20 rows correspond to the connection equations. They interconnect the diagonal blocks of $g_y$.

A disturbance was applied to the power system to illustrate the interactions between continuous dynamics (due to the load) and discrete event dynamics (resulting from the tap-changing transformer). At $t = 10$s, the feeder with impedance $jX_2$ was tripped. The behaviour of the voltage at bus 3 is shown in Figure 5, along with the load demand $P_d$. The non-smooth nature of the trajectory is clearly evident. □

In general, components and subsystems of any form can be modelled, provided they are structured with input/output algebraic variables that can be linked to other components. For example a subsystem representing market dynamics could be linked to appropriate inputs of the physical power system. Noise and/or random disturbances can be added to the model by linking components that generate random signals.

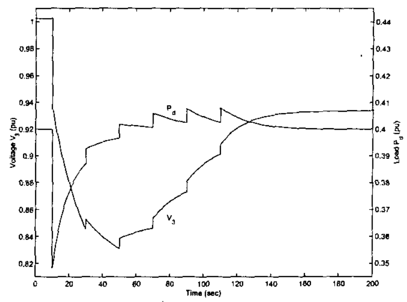


Figure 5: System response to disturbance.

## 5. CONCLUSIONS

Power systems often exhibit complex behaviour in response to large disturbances. Such behaviour frequently involves interactions between continuous dynamics and discrete events. Power systems are therefore an important example of hybrid systems. Their behaviour can be captured by a model which has a differential-algebraic-discrete (DAD) structure. It is shown in the paper that the DAD model is a useful realization of a more general representation of hybrid dynamical systems.

Models of large systems are most effectively constructed using a hierarchical, object-oriented approach. However the interactions inherent in hybrid systems make that difficult to achieve. The paper shows that the desired modularity can be achieved with the DAD model. Components and/or subsystems are modelled autonomously, with inputs and outputs linked via simple algebraic equations. The resulting models have a sparse structure.

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