

# On Influence of Load Modelling for Undervoltage Load Shedding Studies

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**Abstract:** This paper explores the influence of load models on decisions of undervoltage load shedding in power systems. A controlled load rejection can be used as an emergency countermeasure to avoid widespread blackout when system voltages are unstable. In this paper, dynamic simulations of a small power system using both static and dynamic load models are presented. When using a static load model, the system includes an explicit model of a transformer with load tap changer. The aim is to demonstrate how different load models influence the analysis and calculation of the amount of load needed to be shed to stabilise the system voltage.

**Keywords:** Load modelling, Load shedding, Long-term dynamics.

## I. INTRODUCTION

There is a tendency that power transmission systems of today are operating closer and closer to their limits, and it is not uncommon that the limiting factor for power transfers in systems is the risk of voltage instability. As a consequence, at least some 15 major incidents of voltage collapses occurred worldwide during the 1970s and 1980s [1]. In the event of an approaching blackout, the disconnection of loads under controlled conditions and/or blocking of tap changers on transformers can minimise the disruption [2].

There is an obvious need to get most benefit from such a drastic control action as load shedding, so the goal is to optimize the benefit of where, how much and when to shed load. When studying voltage stability, the choice of load model may play a significant role in the outcome of the analysis. The load models are usually classified as time dependent or not, i.e. dynamic or static. When a decision of shedding load is taken to reduce the system loading, a correct load modelling is important. The present paper makes a step in this direction by devel-

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oping a systematic procedure for showing how much load to shed and what influence different load models have on the simulation result as well as on the analysis.

The paper is organised in the following way. Section II deals with the special topic of load modelling in voltage stability studies, including the systems and models used. Three different load models will be discussed. Section III presents the different results for load shedding analysis which these load models give rise to. In Section IV the conclusions that can be drawn from the work are presented.

## II. SYSTEM AND MODELS

The general form of load modelling equations can be written as [3]

$$\dot{x} = a(x, V) \quad (1)$$

$$P_d = b_p(x, V) \quad (2)$$

$$Q_d = b_q(x, V) \quad (3)$$

In the following, only the equations for active load power are given but similar equations hold for reactive load power. The static load characteristic is

$$P_d = b_p(x(\infty), V) \quad (4)$$

where  $x(\infty)$  solves  $\dot{x} = 0$ , i.e.  $a(x, V) = 0$ , and the transient load characteristic is

$$P_d = b_p(x(0), V) \quad (5)$$

where  $x(0)$  is the value of the state when the initial change occurs. The three different load models that will be studied can be written as special cases of the general form given. The load models are:

- #1. Dynamic load model with exponential recovery, proposed in [4]. Load characteristics are constant power in steady state and constant impedance during transients. This load model will be referred to as load model #1.
- #2. Dynamic load model proposed in [5]. Load characteristics correspond to those used for load model #1. This load model will be referred to as load model #2. Other variations of this model are proposed in [6, 7].
- #3. Static load model connected to the system via a transformer with load tap changer, LTC. The static load used

has a constant impedance characteristic. This load model will be referred to as load model #3.

The test system studied has one generator bus and one load bus, see Figure 1. The load is connected to the load bus via a transformer. At the load bus there is also a shunt capacitor for load compensation. There are two identical lines in parallel between the buses. This simple system is used to develop and illustrate the basic ideas.

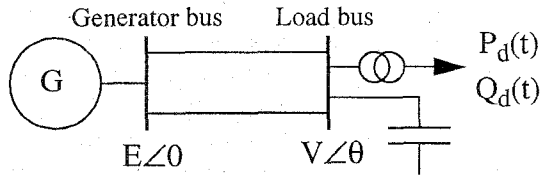


Figure 1 System for load shedding.

To force the system into a voltage unstable state, one of the parallel lines is disconnected. The system is always exposed to the same initial disturbance. The load recovery contributes to force the system into a collapse. In the cases using aggregate dynamic load models, the load recovery is fully modelled in the load model itself. When using a static load model, the transformer is assumed to be equipped with tap changing which provides effective load restoration.

The generator is modelled as a synchronous machine with one field winding, one damper winding in d-axis and one damper winding in q-axis. The exciter model used describes an alternator with controlled rectifier. A PSS is included in the AVR loop. The turbine is modelled as a hydro turbine with a speed-governor. Delayed rotor and stator current limiters in the generator are modelled. System data can be found in the appendix.

In the cases using a dynamic load model, the LTC dynamics are not explicitly modelled. But the dynamic load models which are used, with monotone recovery, can be used to model the aggregate effect of several LTCs seen from a high voltage level [4, 7].

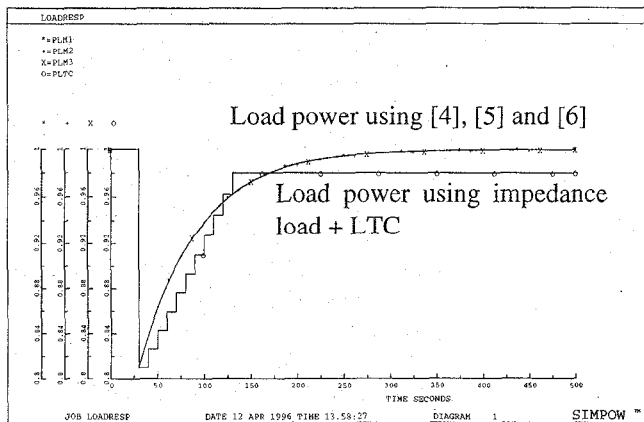


Figure 2 Load response to a voltage step of 10%.

To be able to compare the system responses when using different load models, the load parameters are tuned in the following way. The case using an impedance load (static load) has been chosen as a reference case. The LTC has a constant time delay of 10 seconds. In Figure 2 the different load models are disturbed by a voltage step of 10% and the load powers are given as a function of time. The case using the LTC is the curve showing a step wise recovery in load power.

The curve showing a smooth monotone recovery is actually three different curves on top of one another showing the load recovery for the dynamic load models proposed by [4], [5] and [6], respectively. The difference in steady state load power after the disturbance is due to the voltage dead band in the LTC. The load voltage is not restored to the pre-disturbance level which cause a slightly lower load power. The fact that the step responses for the dynamic load models closely agree does not imply that the load response agree for other disturbances. The different dynamic load models discussed have different levels of generality.

A. Dynamic load model #1

In looking for a simple dynamic load model based on the response to a voltage step, it is a useful approximation that the recovery is exponential [4]. A differential equation which captures that behaviour is

$$T_p \dot{P}_d + P_d = P_s(V) + k_p(V)\dot{V} \quad (6)$$

This equation can be rewritten in first-order normal form as

$$\dot{x}_p = P_s(V) - P_d \quad (7)$$

$$P_d = \frac{x_p}{T_p} + P_t(V) \quad (8)$$

where  $T_p$  is the time constant,  $x_p$  is a state variable,  $P_s(V)$  is the static load function and  $P_t(V)$  is the transient load function. By using this form of differential equation for active as well as reactive load power, the system voltages after a line trip are shown in Figure 3.

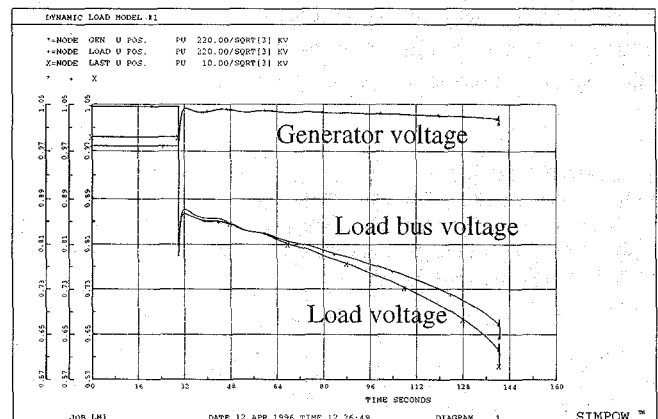


Figure 3 System voltages when using load model #1.

In this figure, the voltages shown are the generator terminal voltage together with the voltages on either side of the load transformer. In this case, the system voltages are collapsing two minutes after line trip.

### B. Dynamic load model #2

The load model is given in general form in [5] by the following equations

$$\dot{x}_p = \frac{1}{T_p}(P_s(V) - P_d) \quad (9)$$

$$P_d = x_p P_t(V) \quad (10)$$

where the notation is common with the notation used for load model #1. If the load model proposed by [5] is used, the system voltages after the disturbance will be as shown in Figure 4.

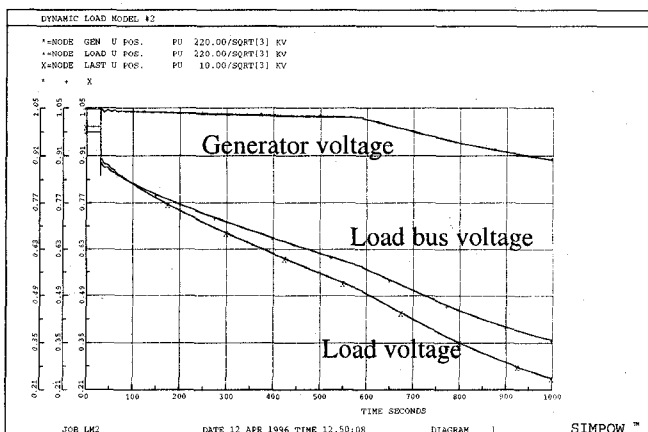


Figure 4 System voltages when using load model #2.

This case does not end in an abrupt collapse but the system voltages become extremely low. Just before  $t=600$  seconds, the delayed rotor current limiter goes into operation and lowers the generator voltage continuously. The simulation is manually aborted after 1000 seconds because the system voltages are now decreasing more slowly.

The special case of load model #2 [6] can be described by the following equation

$$T_p \dot{g} = P_0 - V^2 g \quad (11)$$

where  $g$  is the load conductance and  $P_0$  is the constant load demand. The simulation result with this special case of load model #2 is identical to the result shown in Figure 4.

### C. Static load model and LTC, #3

For a discrete-time, discrete-tap dynamic model of an LTC with a fixed time delay  $h$ , the equation for the tap ratio  $n$  can be written as [8]

$$n(t) = n(t-h) + \Delta n \quad (12)$$

where  $\Delta n$  is the tap increment if the voltage on the secondary

side of the transformer is outside the dead band settings for a period equal to the time delay  $h$ . The load model is only dependent on load bus voltage

$$P_d = P_d(V) \quad (13)$$

If an explicit LTC model together with a constant impedance load model are used, the system voltages after a line disconnection are as shown in Figure 5.

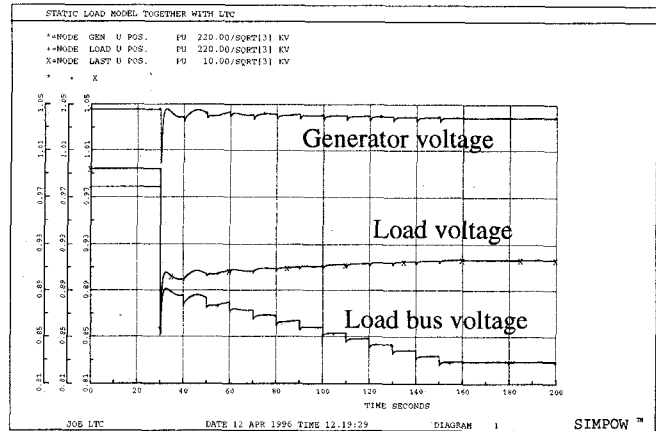


Figure 5 System voltages when using static load model together with an LTC, i.e. load model #3.

The system voltages will settle down at low values after the LTC has stepped all 12 steps available. The generator terminal voltage stays around the set point while the load bus voltage (the high voltage side of the load transformer) is sagging for every step made by the LTC. The voltage on the low voltage side of the transformer is not fully restored since the number of taps is limited, in addition, the high side voltage is decreasing at every attempt to restore the load voltage.

If the load voltage, for the three different load models, is plotted in the same figure, the result is shown in Figure 6.

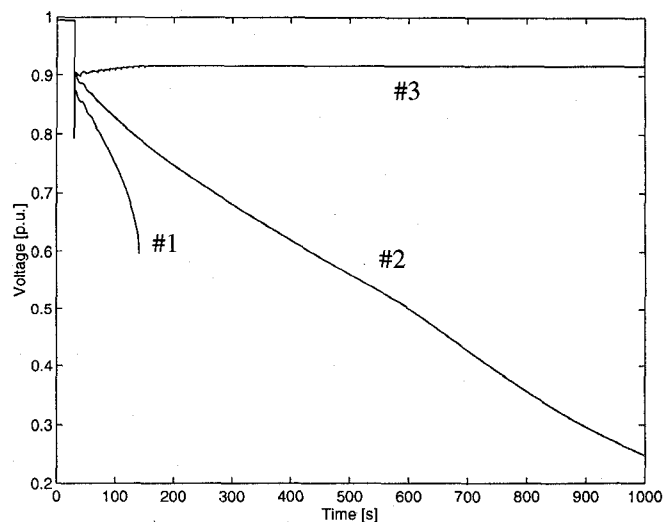


Figure 6 Load voltage for model #1, #2 and #3.

### III. UNDERVOLTAGE LOAD SHEDDING ANALYSIS

The three different load models give rise to different formulations of the algorithm to determine the proper action needed to stop the voltage decay. During the state of unstable voltages, the system transmission capability does not match the system loading. To bring the system back into a steady state, an emergency control action must be taken. The action of interest in this paper is load shedding based on undervoltages in the system. The general aim of the load shedding is to bring the major part of the system back to steady state by disconnecting some (minor) part of the system, namely load areas [9].

To be able to bring the system back to a steady state, it is important to know the load characteristic. In this case, all three load models can be said to have the same static load characteristic, viz. constant power, if the movements of the LTC is taken into account. However, there is a difference between the dynamic load models that are time continuous and the static load model which includes a time discrete component, i.e. the LTC. When using a static load model together with an LTC model, the voltage dead band of the tap changer is of interest. In this dead band, the LTC is insensitive to voltage fluctuations. This implies that a narrow voltage band of transmission voltages will not give rise to any stepping of the LTC.

In terms of load shedding, the load characteristics will influence the analysis. Since the load characteristics for dynamic load models give a specified, unique load power at any instant, the amount of load to shed to stabilise the system voltages can be determined uniquely. In the case of the voltage dead band in LTCs there will be a band of different load powers that will correspond to a steady state.

An important step in the calculations of the amount of load to shed, is the calculation of the transient load response to a voltage step. The load power after the step is equal to the initial load power minus the transient response of the load

$$P_{d,new} = P_d - \Delta P_d \quad (14)$$

If the voltage step originates from load shedding, then the remaining fraction of the load,  $z_p$ , must fulfil the mentioned relationship,

$$P_{d,new} = z_p (P_d - \Delta P_d) \quad (15)$$

The same equations apply for reactive power. The transient change in load power due to a voltage step from  $V_0$  to  $V_1$ , is for load model #1

$$\Delta P_d = \frac{k_p}{2T_p} (V_0^2 - V_1^2) \quad (16)$$

for load model #2

$$\Delta P_d = x_p (V_0^2 - V_1^2) \quad (17)$$

for the static load model, #3

$$\Delta P_d = \frac{P_{nom}}{V_{nom}^2} (V_0^2 - V_1^2) \quad (18)$$

If the purpose of the load shedding is to stabilise the system voltage at a voltage level higher than the voltage at the time of load shedding, the amount to shed can be calculated. Note that it is a *stable* voltage that is of interest for the algorithm and not the post-shedding voltage level obtained. The algorithm gives a stable voltage and if it is found to be too low, another load shedding or countermeasure can be performed.

If a dynamic load model is used, the following main structure of an iterative algorithm can be used [9]:

1. Use the actual load powers,  $P_d$  and  $Q_d$ , at the instant when load shedding is going to be performed, solve a load flow for the current state.
2. Choose new values of the load powers,  $P_{d,new}$  and  $Q_{d,new}$ , to the location where load is going to be shed.
3. Calculate the new voltage,  $V_{new}$ , at the load. This new voltage must be higher than the current voltage, else choose new load values, i.e. go back to step 2.
4. Calculate the transient change in load powers,  $\Delta P_d$  and  $\Delta Q_d$ , according to the transient characteristic of the load.
5. Calculate how much load that needs to be shed so that voltages and load powers after load shedding correspond to the chosen load values in step 2. The fraction of load power to shed,  $f_p$ , is calculated as  $1 - z_p$ . The value of  $f_p$  is between zero and unity. The same applies to reactive load power.
6. Scale the nominal load powers by the calculated factors.
7. Calculate the new steady-state load powers,  $P_{s,new}$  and  $Q_{s,new}$  at the new load voltage.
8. If  $P_{d,new} > P_{s,new}$  and  $Q_{d,new} > Q_{s,new}$  the load shedding will result in an increased load voltage. If the relationships are not fulfilled, return to step 2 and choose new load values.

An analytical justification of this algorithm is provided in [9]. The load dynamics are taken into account in the algorithm since condition #8 checks whether the system state is within the region of attraction of a steady-state operating point or not [9]. The algorithm can very easily be adapted to large systems where load may be shed at different locations [10].

The algorithm seems to be robust to parameter uncertainty. If the transient voltage dependence of the load is assumed to have an index of  $\alpha$ , the outcome of the algorithm will end up with a little bit too much of load shedding for loads having an index  $< \alpha$ . If the actual voltage dependence is  $> \alpha$ , then the amount of load shed may be too small to stabilise the system voltage at a voltage level higher than the voltage at the time of load shedding. Note that the system may find a new steady state operating point but at a lower system voltage than the voltage at the time of load shedding.

When the static load model is used, the iterative algorithm will look like the following:

1. Use the actual load powers,  $P_d$  and  $Q_d$ , at the instant when load shedding is going to be performed, solve a load flow for the current state.
2. Choose new values of the load powers,  $P_{d,new}$  and  $Q_{d,new}$ , of the location where load is going to be shed.
3. Calculate the new voltage,  $V_{new}$ , at the load. This new voltage must be inside or above the voltage dead band of the tap changing transformer. If the new voltage is outside the dead band, then the chosen load powers are either too low or too high. Return to step 2.
4. Calculate the transient change in load powers,  $\Delta P_d$  and  $\Delta Q_d$ , according to the characteristic of the load.
5. Calculate how much load that needs to be shed so that voltages and load powers after load shedding correspond to the chosen load values in step 2. The fraction of load power to shed,  $f_p$ , is calculated as  $1 - z_p$ . The same applies to reactive power.

To demonstrate these iterative algorithms, two examples will be given using the system given in Figure 1 together with load model #2 and #3, respectively. In [9], more detailed examples using load model #1 in single and multi load bus systems are presented.

#### Example 1

When using load model #2, the load voltage is slightly below 0.8 p.u. at  $t=131$  seconds, see Figure 4. Assume that a decision of shedding load is taken at that time. A load flow is solved for the present state of the system, i.e. the load powers are  $1.195+j1.110$  p.u. and the load voltage is 0.7998 p.u. New load powers chosen are  $1.11+j1.14$  p.u. A new load flow for the disturbed system and the new load power gives that the load voltage is now changed to 0.8694 p.u. which is higher than before. As the fourth step, the transient load change is calculated according to (17). As the fifth step, the amount of load to shed is calculated by using (15). This load shed gives that

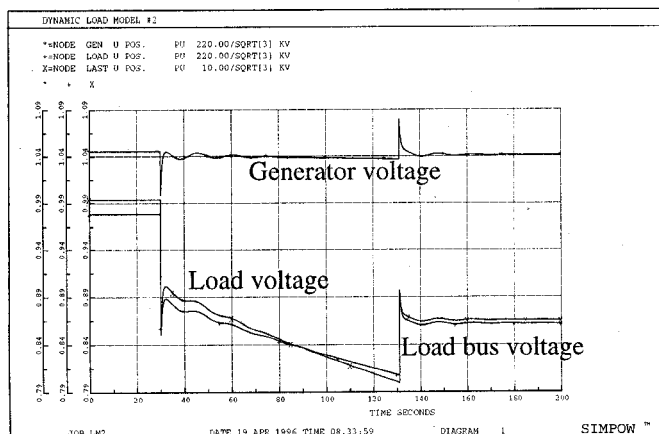


Figure 7 Verification of result in example 1.

the new steady state load powers are  $1.1022+j1.1320$  p.u. which are slightly lower than the new chosen load demands and therefore fulfils the relationships in step 8 of the algorithm. This means that from the original load of  $1.40+j1.30$  p.u.,  $0.2978+j0.1680$  p.u. should be shed.

To verify the result, a time simulation is performed where the calculated amount of load is tripped at  $t=131$  seconds, see Figure 7. Since the system voltages settle quite quickly without drifting upwards nor downwards, the amount of load shed is quite well tuned. □

#### Example 2

The initial (pre-fault) state of the system gives that the load voltage is 0.9942 p.u. which therefore is the set point of the LTC. The dead band of the LTC is  $\pm 1.5\%$  which means that the load voltage should be within 0.9792 - 1.0092 p.u. When using load model #3, the load voltage recovery ends at  $t=150$  seconds since the LTC run out of taps, see Figure 5. The fact that the transformer has run out of taps is of no importance for the algorithm. At  $t=160$ , the load voltage is only 0.9167 p.u. and a decision of shedding load is taken. A load flow is solved to find the system state at  $t=160$  when the load powers are  $1.190+j1.105$  p.u. owing to the squared voltage dependence. New load powers chosen are  $1.10+j1.13$  p.u. A new load flow solution gives that the load voltage is now changed to 0.9810 p.u. which is inside the dead band of the transformer. As the fourth step, the transient load change is calculated according to (18). As the fifth step, the amount of load to shed is calculated by using (15) which results in a load shed of  $0.2702+j0.1394$  p.u.

A time simulation of this load shedding operation is shown in Figure 8.

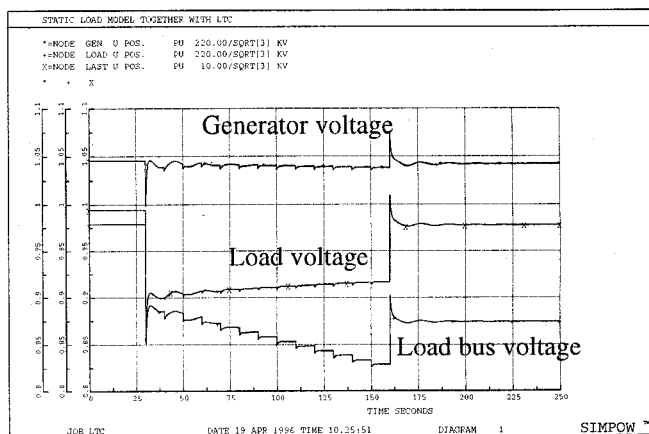


Figure 8 Verification of result in example 2.

It can be seen that the load voltage after load shedding, is now back to a value close to the dead band settings of the LTC. □

These two examples show that the algorithm for determining the amount of load to shed to stabilise the system voltage, is applicable for different load models.

#### IV. CONCLUSIONS

The importance of choosing an accurate load model has been highlighted. Even if the load model has a step response matching the desired one, the system behaviour for other types of disturbances can vary significantly.

An algorithm has been presented which robustly determines the amount of load that needs to be shed to stabilise the system voltage after a severe disturbance. The application of this algorithm is independent of the load model used.

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#### APPENDIX

Static data for 2-bus test system:

$S_{\text{base}} = 100 \text{ MVA}$ ,  $V_{\text{base}} = 220 \text{ kV}$ ,  $Z_{\text{line}} = 0.044 + j0.176 \text{ p.u.}$ ,  
 $B_{\text{line}} = 0.30855 \text{ p.u.}$ ,  $Q_{\text{nom}}$  for shunt = 95 Mvar.  $S_{\text{nom}}$  for load  
 $= 140 + j130 \text{ MVA}$ ,  $V_{\text{set}}$  for generator = 221 kV.

Dynamic data for 2-bus test system:

Synchronous machine  $S_{\text{nom}} = 280 \text{ MVA}$ , saturation excluded,  
 $V_{\text{nom}} = 220 \text{ kV}$ ,  $H = 4.0 \text{ MWs/MVA}$ ,  $R_T = 0 \text{ p.u.}$ ,  $X_T = 0.10 \text{ p.u.}$ ,  
 $R_a = 0 \text{ p.u.}$ ,  $X_a = 0.18 \text{ p.u.}$ ,  $X_d = 1.0 \text{ p.u.}$ ,  $X_d' = 0.4 \text{ p.u.}$ ,  
 $X_d'' = 0.249 \text{ p.u.}$ ,  $T_{d0}' = 5.0 \text{ s}$ ,  $T_{d0}'' = 0.06 \text{ s}$ ,  $X_q = 0.65 \text{ p.u.}$ ,  
 $X_q' = 0.249 \text{ p.u.}$ ,  $T_{q0}' = 0.125 \text{ s}$ .

Load model #1:  $T_p = T_q = 62$ ,  $k_p = k_q = 124$ .

Load model #2:  $T_p = T_q = 50$ ,  $\alpha = \beta = 2$ ,  $a = b = 0$ .

Load model #3: impedance load, tap changer with 12 steps, constant time delay = 10 s., dead band  $\pm 1.5\%$ .

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