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# RANKING LOADS IN POWER SYSTEMS - COMPARISON OF DIFFERENT APPROACHES

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## Abstract

The paper addresses the issue of determining the most influential loads in power systems. Two recently developed techniques are used for load ranking in a real power system. The results obtained by sensitivity based and optimisation based technique are analysed and compared. It is shown that both approaches give valuable information regarding influential loads in a power system. Further, it is shown that for reliable ranking it is necessary to consider all modes of interest, all load parameters and various operating conditions of the power system. The limitations and application of the two techniques are discussed.

## 1 Introduction

The issue of power system load modelling has come into the focus in recent years. The majority of the other power system components (e.g., generators and associated controls, FACTS devices) are sufficiently accurately modelled for power system control and stability studies. However there are still discrepancies between predicted and measured behaviour of power systems following small and/or large disturbances. So, load modelling is an important and relevant issue despite all the difficulties involved [1].

In some recent studies it has been clearly established [2, 3] that the dynamic response of loads can greatly influence the damping of electromechanical modes, and in particular inter-area modes. It has also been shown that ignoring dynamics, and treating loads as statically dependent on voltage, can result in quite misleading predictions of damping.

However detailed modelling of all loads in a power system would be a very expensive and time consuming process. Utilities are wary of devoting significant funds and manpower to such a demanding venture. Particularly as not all loads in the power system have the same influence on its steady state and transient behaviour. Therefore an important issue is to identify loads which should be modelled in detail, as against those loads where a less detailed models would suffice. Knowing which are the most influential loads in the system, the funds, manpower and time can be dispatched to achieve appropriate modelling.

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In previous work [4] it was shown that load dynamics can be thought of as a feedback mechanism which influences system behaviour. This representation motivates the use of linear system ideas which have proved useful in the assessment of the most suitable positions for placement of power system stabilisers, static VAr compensators and variable series capacitors [5] for damping power system oscillations and improving stability. Therefore, the use of these ideas/techniques for locating dynamic loads which significantly influence damping was proposed in [6]. These techniques allow for fast scanning of all loads in large systems. They can be used to identify not only locations where load modelling is important, but also loads which have a significant influence on damping. After significant loads have been identified, other more detailed analysis techniques could be used for further investigations. Such techniques are usually more computationally intensive, so they are not generally suited to the problem of scanning all loads of large power systems.

However one should be aware that the results obtained using sensitivity ideas are generally valid for small parameter perturbations only. Therefore techniques that allows large variation of load model parameters is needed. One such technique, based on optimisation of a specific cost function, is proposed in [7]. By applying this technique it is possible to establish a more detailed view of load importance in power systems. Such information can be useful for establishing load importance under various diverse operating conditions. Also it is possible to establish the importance of load dynamics for ranking loads, as well as the importance of particular load parameters.

This paper presents and compares results obtained using these two techniques. The techniques are tested and compared on an actual power system (an Australian state power system).

## 2 Load models

The generic load model used in this study was developed in [1] on the basis of field measurements [8]. It was motivated by the general form of response of dynamic loads such as induction motors, heating loads and tap changing transformers. The general form of the model is:

$$\dot{P}_d + f_1(P_d, V) = f_2(P_d, V)\dot{V} \tag{1}$$

Similar equations relate $Q_d$ to $V$. In the sequel only active power - voltage dependence will be discussed, though similar ideas apply for reactive power.

If exponential recovery is assumed, specific forms can be obtained for functions $f_1$ and $f_2$. The state space form of the load model is:

$$T_p \dot{x}_p = -x_p + P_s(V) - P_t(V) \quad (2)$$

$$P_d = x_p + P_t(V) \quad (3)$$

The functions $P_s(V)$ and $P_t(V)$ can be conveniently defined as:

$$P_s(V) = P_o(\frac{V}{V_o})^{n_{ps}} \quad (4)$$

$$P_t(V) = P_o(\frac{V}{V_o})^{n_{pt}} \quad (5)$$

where $V_o$ and $P_o$ are the nominal voltage of the bus and corresponding active power of the load respectively, and $n_{ps}$ and $n_{pt}$ are steady state and transient voltage exponents. Time constants $T_p$, which characterise the recovery response of the load, can be chosen to represent different types of loads [3].

Field tests which are reported in [9] show the response of loads with recovery times of less than a second. High voltage aggregate bus loads in this case were predominantly residential, though there was a small participation of industrial and commercial loads [9]. The fast load response has been influenced by a large presence of air-conditioners in residential loads. Apart from this type of load, a similar fast response to voltage variations is exhibited by industrial and agricultural loads where there are generally large numbers of induction machines, and by loads with rectifiers, such as aluminium smelters [3].

This type of load model exhibits exponential response to a step in voltage. That form of response corresponds quite closely to measured results given in [8, 9]. The emphasis in this study is on loads that exibit fast exponential recovery, i.e., loads with time constants around 1s, and the parameters of the load model are chosen accordingly.

This model is linearised and incorporated in the power system model in [2, 4]. The linearised form of the model is:

$$\Delta P_d = \frac{P_o}{V_o} n_{ps} \frac{(\frac{n_{pt}}{n_{ps}} T_p s + 1)}{(T_p s + 1)} \Delta V \quad (6)$$

It can be seen that there are six parameters per load; namely, real and reactive power time constants $(T_p, T_q)$, and real and reactive power steady-state and transient voltage exponents $(n_{ps}, n_{qs}, n_{pt}, n_{qt})$. The usual ranges of these parameters are given [3].

In some cases in Section 4 loads were modelled statically. Then the classical exponential load model (4) was used in its linearised form.

# 3 Ranking methods

## 3.1 Sensitivity based method

The first method used in this study for load ranking was a sensitivity based technique. It was based on analysis of the linearised power system model.

Assume that **A** is the system matrix of the linearised power system model [3]. For the power system electromechanical mode, (i.e. eigenvalue of matrix **A**) of the form $\lambda_i = \sigma_i + j\omega_i$, $i = 1, \ldots, n$ with damping $\sigma_i$ and angular frequency $\omega_i$ sensitivity of the mode to the change in parameter $K_i$, $i = 1, \ldots, n$ is given as [10]:

$$\frac{\partial \lambda_i}{\partial K_i} = \frac{\mathbf{w}_i^T \frac{\partial \mathbf{A}}{\partial K_i} \mathbf{v}_i}{\mathbf{w}_i^T \mathbf{v}_i} \quad (7)$$

where $\mathbf{w}_i^T$ and $\mathbf{v}_i$ are the left and right eigenvectors of **A** corresponding to $\lambda_i$. In this particular case parameter $K_i$, $i = 1, \ldots, 6$ in (7) is a parameter of the load model.

In the sensitivity based ranking method the rank of loads is established as the following index:

$$r = \sum_{(c)} \sum_{\lambda_j} \sum_{K_i} \mid \frac{\partial \sigma_j}{\partial K_i} k_i \mid \quad (8)$$

where $(c)$ are selected operating conditions of the power system, and $k_i$ weights the relative importance of parameter $K_i$. Weighting coefficients $k_i$ are based on the expected range of $K_i$ variation $({K_i}^{max} - {K_i}^{min})$, where ${K_i}^{max}$ and ${K_i}^{min}$ are the upper and lower limits of parameter variation respectively, adopted on the basis of reported results of field measurements and summarised in [3].

The rank is therefore established on the basis of the sensitivity of the real parts $(\sigma)$ of all modes of interest to some/all load parameters and for a variety of operating conditions.

## 3.2 Optimisation based method

The second method for load ranking developed in [7] uses an optimisation procedure of [11] and load ranking function of [7]. The procedure optimises the cost function

$$f = \sum_{(c)} \sum_{i=1}^{n} (\beta_i^{(c)} - \sigma_i^{(c)})^2 \quad (9)$$

where $(c)$ is the operating condition of the power system, $\sigma_i$ is the real part of the $i$-th eigenvalue (oscillation mode) and $\beta_i$ is a fixed reference value for $\sigma_i$. The optimisation is over the specified load parameter ranges. The absolute value of the increment of the cost function due to changes in each parameter of each load is used as a measure of the load's contribution to damping. The function used for ranking loads in this case is:

$$r = \sum_{j}^{m} \sum_{(l)} \mid \Delta f_j \mid \quad (10)$$

where $(l)$ is number of iterations, $m$ is a number of parameters of load $i$, and $\Delta f_j$ is defined [7] as:

$$\Delta f_j = \frac{\partial f}{\partial K_j} \Delta K_j \quad (11)$$

By applying both minimisation and maximisation of the cost function, the maximal range of damping of selected modes can be obtained for load parameters $K_i$ varying within their ranges. The optimisation uses the steepest descent procedure. The gradient of the cost function gives a direction of change of load parameter $K_i$ at each iteration.

The advantage of this procedure is that it allows large parameter variation in order to achieve the best and worst possible damping of the systems electromechanical oscillations. Details are given in [7].

The software POISK [11] was used in analysis.

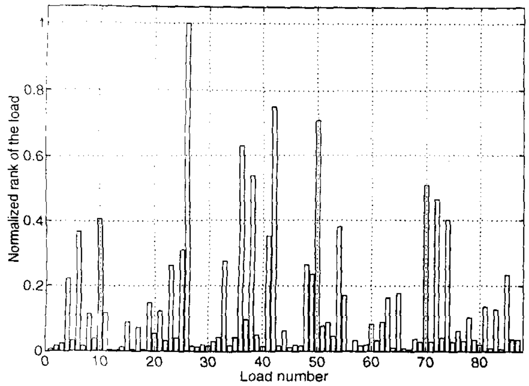


Figure 1: Rank of the dynamic loads - Case A

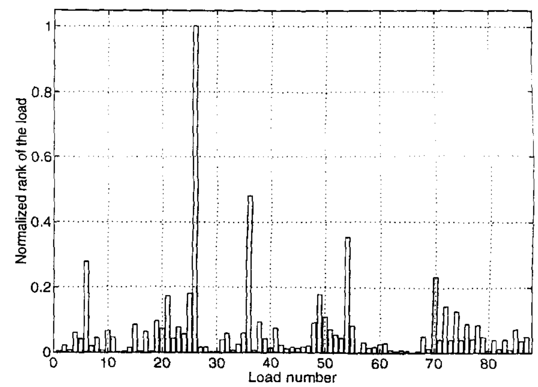


Figure 2: Rank of the dynamic loads - Case A - 1st mode

# 4 Load ranking - case study

The ranking methods described in Section 3 were tested and their results were compared on a large power system. This system consisted of 154 buses, and included 10 equivalent generators (with associated AVRs, PSSs and governors), 87 loads and 1 SVC. Connections were made by 289 lines and transformers. (This system was motivated by an Australian state system.) With all loads modelled statically, the system has two inter-area modes, with frequencies of 0.31Hz and 1.05Hz.

Two operating conditions of the power system were considered; a normal peak load condition and a light load condition.

## 4.1 Sensitivity based load ranking

Firstly, the ranking of the loads was undertaken assuming the system was operating under normal peak load conditions. The input data for this sensitivity analysis was obtained from a standard load flow solution. This operating condition will be referred to as Case A in the sequel.

Some of the results obtained by applying the sensitivity based method are given in Figures 1 to 3.

Consider Figure 1. It shows the normalised rank of all 87 loads. (The load with the largest value of $r$, $r = r_{max}$ is used as the base for normalisation, i.e., ranks $r$ of all loads are divided by $r_{max}$.) Ranking based on sensitivities to all load parameters and for both eigenvalues, is shown in this figure. ( Each of the 87 loads had six parameters and ranking is based on sensitivity of the mode of interest to all load parameters.) In producing this ranking, the third summation in (8) was over $K_i \in \{n_{ps}, n_{qs}, n_{pt}, n_{qt}, T_p, T_q\}$. It can be seen that the most influential loads include loads 10, 26, 36, 38, 42, 50, 70, 72, and 74.

This method of ranking can be very useful in identifying loads that have the largest impact on particular oscillation modes. This is illustrated by Figures 2 and 3, which correspond to different modes. These figures clearly show the different involvement of loads in the different system modes. It is also clear that the importance of a load should be judged on the basis of its influence on all modes of interest. Figure 2 relates to the least damped electromechanical mode in the system. If for example, a decision about load importance was made on the basis of the results shown in this figure, loads 50 and particularly load 42 would be missed as not important at all, or at least not among the loads ranked in the first ten. Similarly from Figure 3, load 26 would be considered as only the 8th most important load. However from Figure 1, where both

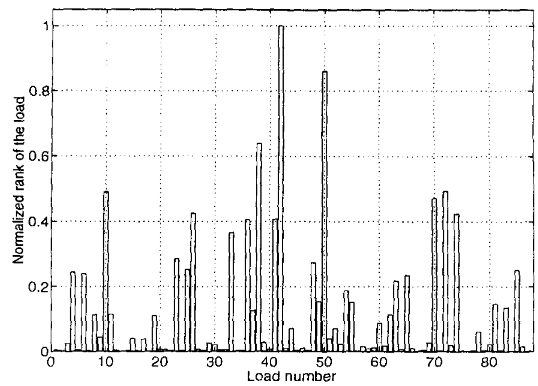


Figure 3: Rank of the dynamic loads - Case A - 2nd mode

modes are considered simultaneously these three loads, in order 26, 42, 50, appear to be the most influential loads in the system.

The significance of the correct choice of loads to be modelled dynamically is further illustrated by Figures 4 and 5. Figure 4 shows two root loci of the 1.05Hz mode. The solid line corresponds to five dynamic loads which have most influence on the 0.31Hz mode (loads 6, 26, 36, 54, and 70). The dashed line is for five dynamic loads which have most influence on the 1.05Hz mode (loads 10, 38, 42, 50, and 72). It can be seen that the influence on the damping of this mode is two times larger if the appropriate loads are modelled as dynamic. Similarly in Figure 5, the root loci are given for the 0.31Hz mode. In this case the dashed line corresponds to five dynamic loads which have most influence on the 0.31Hz mode. The solid line is for five dynamic loads which have most influence on the 1.05Hz mode. From this figure, the importance of correct modelling of the right loads is even more evident. In this case the sensitivity to damping (size of the root locus) is around six times larger when the correct loads are modelled. The root loci in previous figures were produced by simultaneous variation of all the time constants of the loads involved (ten time constants, corresponding to real and reactive power at five buses) from 0s to 100s. The arrows denote the direction of movement of the electromechanical modes as a result of time constant variation.

Figure 4 also shows variation in damping, $\Delta\sigma = 0.55\ 1/s$, and frequency, $\Delta\omega = 0.57\ rad/s$, of 1.05Hz mode when influential loads in the system are modeled dynam-

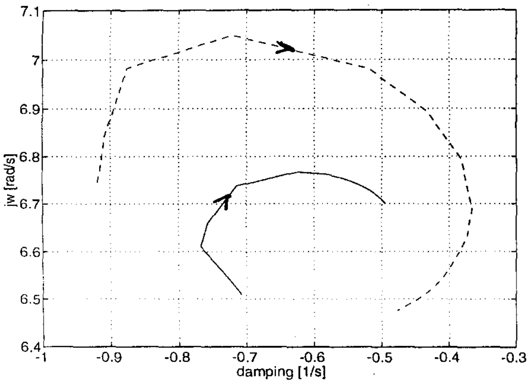


Figure 4: Root loci for the 1.05Hz mode for different influential groups of loads

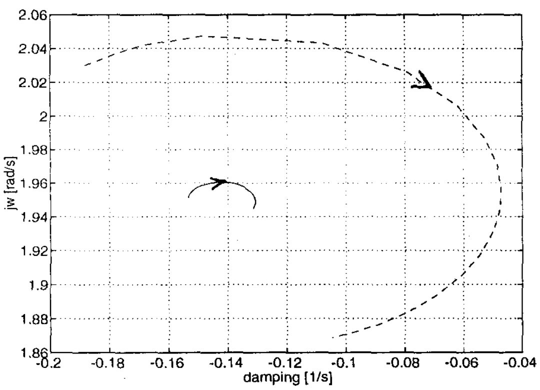


Figure 5: Root loci for the 0.31Hz mode for different influential groups of loads

ically. Similar effects can be seen in Figure 5 for 0.31Hz mode where variation in damping is $\Delta\sigma = 0.14\ 1/s$, and variation in frequency $\Delta\omega = 0.18\ rad/s$.

Figures 6 further illustrate the significance of appropriate load modelling. It can be seen that for a large disturbance, namely a single-phase-to-ground fault, the power system experiences different damping depending on the type of load dynamics. In producing the results shown in Figure 6, the 10 most influential loads determined above were modelled as dynamic. The results show that the power system with some of the loads modelled as dynamic can be more (load time constants, T=10s) or less (load time constants, T=0.3s) damped than the power system with all loads modelled as static (load time constants, T=0s). The damping of the system depends on the dynamics of its loads.

It is interesting to consider whether there is any significant correlation between the rank of the load and its magnitude. Figure 7 addresses that question. This figure is based on the case shown in Figure 1. From Figure 7, it seems that there is a group of loads with high magnitudes that are not ranked as very important, and alternatively there are loads with comparatively small magnitudes that are ranked as being very important. However load 26, which has the highest rank also has the largest magnitude. Loads 21, 76 and 20, which are 2nd, 3rd and 4th in the order of magnitude have a very small influence on damping and therefore a small ranking. On the other hand loads

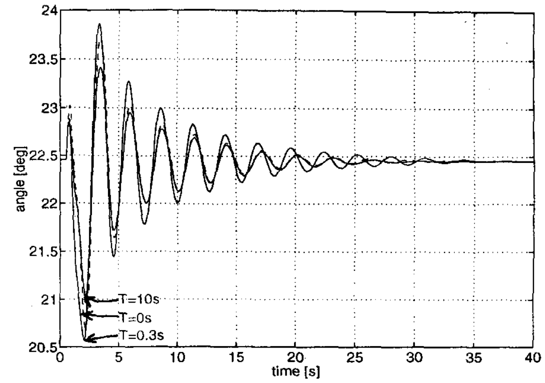


Figure 6: Dynamic response for different load dynamics with 10 significant loads

42 and 50, which are ranked 2nd and 3rd in their effect on damping, are 17th and 18th in the order of magnitude.

These results confirm the conclusion from [6] that a load's ranking depends not only on its magnitude, but that its position in the system is also very important. In [6] it was suggested that eigenvalue residues could be used to identify locations where loads would have an important influence on system damping. (These ideas followed from the use of residues in tuning PSSs.) In this example it was found that loads 26, 42 and 50 have high values of residues, and so their final rank was largely influenced by their location in the system. On the other hand load 21 has a very small value of residue. That indicates that its location has low importance. Therefore, despite having a large load magnitude, the rank of the load is not high.

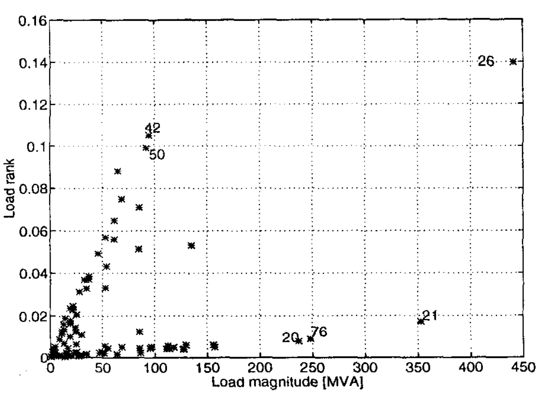


Figure 7: Load magnitude versus its rank

The next point of interest in establishing the usefulness of this form of load ranking, is to consider the relative load importance under different operating conditions. To do so, a light load case was considered. The total generation in the system was reduced to approximately half of its previous value by disconnecting certain generators and switching some hydro units to pumping mode. All loads except for load 26 (an aluminium smelter) were scaled down by a factor of two (approximately). This light load operating condition, and the results obtained from it, will be referred to as Case B in the sequel.

The load ranking for Case B when all 87 loads were modelled dynamically, and all load parameters and both

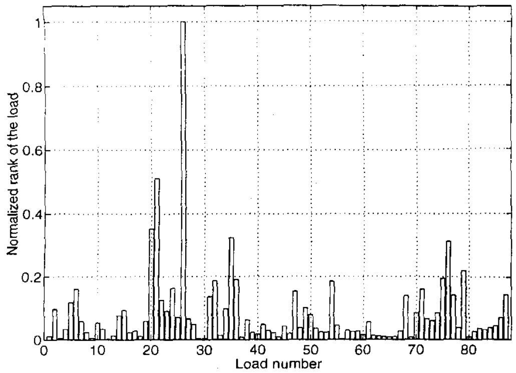


Figure 8: Rank of the dynamic loads - Case B

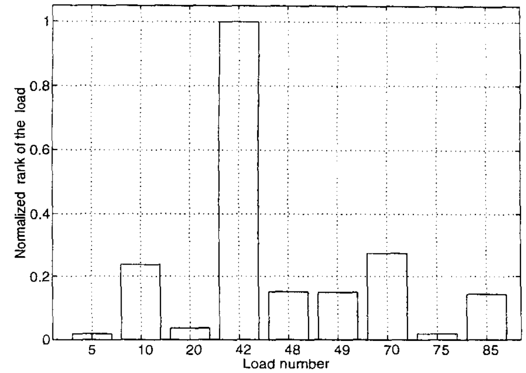


Figure 9: Rank based on optimisation procedure - Case A

inter-area modes were taken into account is shown in Figure 8. It is interesting to compare this new load ranking with the ranking obtained for Case A, shown in Figure 1. Notice that load 26 remained most important, as in Case A. This is not unexpected considering the fact that its magnitude, (which remained unchanged while all others were halved), makes it far more important in the system and even more dominant than in Case A. As far as the rank of the other loads from the most important group in Case A is concerned, only load 36 (ranked 9th) remained among the fifteen most important in Case B. This indicates different load involvement under the different operating conditions of the power system. So, to have better confidence in the rankings, various operating conditions should be considered.

## 4.2 Optimisation based load ranking

For the optimisation based load ranking the POISK software [11] was used. The optimisation procedure was performed over 54 parameters simultaneously. The following example was established to rank loads in test power system. From the set of 87 loads of Case A, 27 were modelled as dynamic. These included the 20 loads that were identified by sensitivity analysis as being the most important. Of the 27 dynamic loads, 9 were chosen for parameter optimisation. Therefore 18 dynamic loads had fixed parameters, and the other 9 had a total of 54 variable parameters. The loads with variable parameters were 5, 10, 20, 42, 48, 49, 70, 75 and 85. These 9 loads were chosen to be representative of loads of different importance (different rank), different magnitude and different locations in the power system. After applying the optimisation procedure the rank of these 9 loads was established. This is shown in Figure 9.

## 4.3 Comparison of the results

Comparing the ranking of the loads given in Figure 9 with their relative ranking given in Figure 1, it can be seen that their relative correlation remains the same as in the sensitivity based procedure.

In order to compare the global ranking of loads obtained when both operating points are considered simultaneously in the cost function with ranking based on sensitivities a new example was established.

Firstly, the results of Figures 1 and 8, (which showed the sensitivity based ranking of loads for Cases A and B respectively), were plotted in non-normalised form so that the absolute values of sensitivities could be compared. On

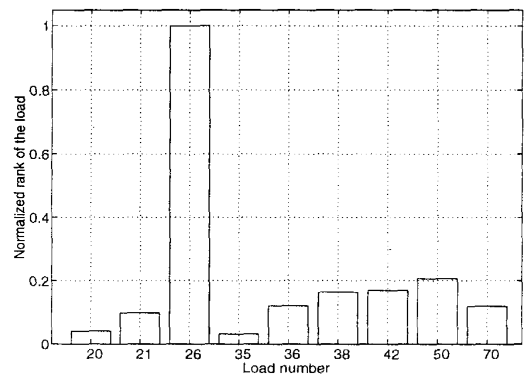


Figure 10: Rank based on optimisation procedure - Case A+B

the basis of those absolute values, the 9 most important loads (6 from Case A and 3 from Case B) were chosen for optimisation. The remaining 78 loads were modelled as static. The selected loads were 20, 21, 26, 35, 36, 38, 42, 50 and 70. The normalised ranks, given by the optimisation based technique, are presented in Figure 10.

This ranking is more representative than the previous one, as it considers both operating conditions. It clearly shows that there are three groups of loads based on their importance. The most important one is load 26. The second group of importance consists of loads 36, 38, 42, 50 and 70. The least important among the selected loads are loads 20, 21 and 35.

If this ranking is now compared with that obtained using sensitivities, for the same arrangement of static and dynamic loads (shown in Figure 11), exactly the same groups of load importance can be noticed. Slight differences occur in the ranking within the second group. Also loads from this group have relatively higher importance compared to load 26 than was the case for the optimisation based ranking. In the optimisation based ranking, load 26 has far more importance in the system than the loads from the second group.

# 5 Limitations and applications

The case studies of Section 4 show good agreement between results obtained by sensitivity and optimisation based techniques. It should be noted though that such good agreement may not necessarily occur for every sys-

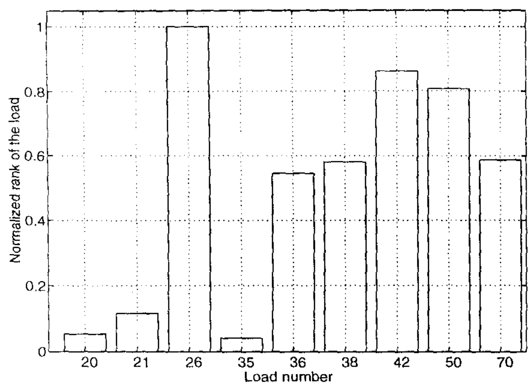


Figure 11: Rank based on sensitivities - Case A+B

tem. These techniques can give different results under certain circumstances (e.g., for a power system with unrealistic parameters). However for real power systems, and for realistic ranges of load parameter variation, both techniques appear to give qualitatively similar results. If the ranges of load parameter variation were extended beyond realistic values, the differences would tend to increase though.

The sensitivity based load information is local, but can be obtained for very large systems. Therefore this approach is suitable for fast scanning of large system.

Generally, the optimisation approach is valid for large parameter variation. However this technique is more computationally intensive and therefore more suited to smaller systems.

# 6 Conclusions

Two different approaches for determining important loads in power system are presented in the paper. It is shown that both approaches give valuable information. Both identify the same groups of influential loads, though rankings within groups may be slightly different.

Further it is shown that for reliable ranking it is necessary to consider all poorly damped electromechanical modes in the system, all load parameters and various operating conditions.

Limitations of both approaches are also considered. The sensitivity based technique should be treated as approximate, and suitable for fast scanning of high order systems, i.e., systems with a numerous states. The optimisation based technique is more reliable, as it considers large parameter variation but more computationally expensive.

By using either of these approaches the order of the system to be studied may be significantly reduced. Even for the simplest dynamic load model there are two new differential and two algebraic equations per load to be added to the power system model. Considering this together with the large number of loads in power systems, it is easy to see how significant "savings" in the order of the system model and computational time can be accomplish. Further it means resources can be devoted to obtaining good models of important loads, rather than trying to obtain adequate models of many extra loads.

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