Optimal Power Flow with Storage

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***Abstract*—Solution algorithms for the optimal power flow (OPF) problem are well established for traditional electricity networks. However, there is an increasing need for integrating renewable sources and energy storage into electricity networks. These newer devices have physical characteristics that require modification of standard OPF algorithms. In particular, energy storage devices introduce temporal coupling over the optimization horizon. Also, the modeling of non-unity storage efficiency requires complementarity conditions. This paper explores two algorithms that extend OPF methods to incorporate energy storage devices and wind generation. The first method is based on the well-known AC-LP OPF method, while the second is a quadratic program with DC power flow constraints. The algorithms are demonstrated using several test cases that consider a modified RTS-96 system. The performance of the two algorithms is compared in terms of convergence properties and quality/optimality of their respective solutions.**

I. INTRODUCTION

The optimal power flow (OPF) problem has been well-researched over the past few decades. The basic problem is to optimally schedule generation in a power system whilst ensuring power balance at all nodes, and satisfying network voltage and power flow constraints. Several metrics can be used to qualify an optimal solution, including minimal losses, minimal cost of generation, or minimal change in operating point from a baseline solution. Several solution methods are available, including gradient methods, Newton's method, linear program (LP) OPF, and interior point methods. Such methods are required as the standard OPF problem is nonlinear [1, pp. 514-559].

Another common practice is to use the DC power flow to cast the OPF problem as a quadratic program, as in [2]-[3]. The DC power flow introduces linear constraints that are an approximation of the true AC power flow constraints. However, it has been demonstrated that under normal operating conditions and with the traditional DC power flow assumptions of negligible network resistance, flat voltage profile and small angle differences, the DC power flow results generally fall within an acceptable error compared to the AC power flow solution [4]. Several variants of the DC OPF are available, including hot-start, cold-start and incremental models [5]. The linear network approximation ensures a simpler optimization problem than a nonlinear AC OPF.

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Recently, semidefinite programming (SDP) techniques have been applied to the OPF problem, as described in [6]. This approach is advantageous compared to the other methods as it is convex, and readily-available solvers exist for problems of moderate size. Under certain conditions, the solution obtained from this problem is globally optimal and satisfies the nonlinear AC power flow equations [7]-[8]. Whilst an AC-feasible solution is often obtained, there is no guarantee for arbitrary networks. Also, for larger networks, the sparsity requirements make this a much more complex optimization problem than other methods [9].

As described, methods of solving the standard OPF problem with traditional generation are well established. However, with growing demand for more environmentally friendly energy sources, there is an increasing need for integration of renewable generation in the current infrastructure. While presenting many benefits, such renewable generation also poses many challenges. In particular, renewable energy sources are inherently variable, complicating network reliability and control. This variability can be mitigated through the use of energy storage, motivating the formulation of OPF problems that incorporate storage [10]. The challenge with such problems lies in the temporal coupling inherent in the dynamics of storage devices. Additionally, when non-ideal efficiencies of the charging and discharging of these devices are considered, additional variables must be introduced and complementarity between charging and discharging variables must be ensured. Such a formulation for non-ideal efficiencies is given in [11]; a similar formulation to model storage is used in this paper.

The organization of this paper is as follows. In Section II, two formulations of the OPF problem with storage and wind are described; one is a DC OPF and the second is based on an AC-LP OPF method. Section III addresses the complementarity issues that arise when adding storage devices with non-ideal charging and discharging efficiencies. Two methods for enforcing this complementarity condition are given and compared with general forms of complementarity constraints. Section IV describes the test cases used. Section V documents the results of the methods on those test cases and assesses the results in terms of quality of solution and convergence of the algorithms. Conclusions are provided in Section VI.

II. OPF FORMULATION

The general problem being solved in OPF formulations is that of finding an optimal generation schedule, in terms of cheapest cost of traditional generation. Two OPF formulations follow, with both implementing storage devices and wind power. The first is based on a traditional DC OPF with an approximation for losses added. The second is based on the AC-LP OPF. The following nomenclature is used for both formulations.

Parameters:

$\mathcal{G}$	set of generation nodes
$C_i(P_{g,i})$	convex cost curve for each generator $i \in \mathcal{G}$
$\mathcal{D}$	set of demand nodes
$\mathcal{S}$	set of storage nodes
$\mathcal{W}$	set of wind nodes
$\mathcal{T}$	set of time periods
$\mathcal{N}$	set of nodes in the network
$d_j(t)$	active power demand at node $j \in \mathcal{D}$ at time $t \in \mathcal{T}$
B_i	battery energy limit at node $i \in \mathcal{S}$
Y	system/network admittance matrix
$W_i^{max}(t)$	available wind at node $i \in \mathcal{W}$ at time $t \in \mathcal{T}$
$\mathcal{L}_i$	set of lines out of and connected to node i
x_{ij}	reactance of line from node i to node j
R_{ij}	resistance of line from node i to node j
f_{i-j}^{max}	maximum power flow in line from node i to node j
$p_{g,i}^{min}$	min active power when generator at node $i \in \mathcal{G}$ in service
$p_{g,i}^{max}$	max active power when generator at node $i \in \mathcal{G}$ in service
R_i^{DWN}	maximum ramp down limit on generator at node $i \in \mathcal{G}$
R_i^{UP}	maximum ramp up limit on generator at node $i \in \mathcal{G}$
T_s	sampling time in storage dynamics model
η_c, η_d	charging, discharging efficiencies of storage devices
b_i^{term}	terminal value for storage at node $i \in \mathcal{S}$
R_c^{max}	maximum charging of storage at node $i \in \mathcal{S}$
R_d^{max}	maximum discharging of storage at node $i \in \mathcal{S}$
L	number of blocks in approximation of losses
$\Delta\theta$	length of each block in approximation of losses

Control Variables:

$P_{g,i}(t)$	active power generation at node $i \in \mathcal{G}$ at time $t \in \mathcal{T}$
$r_i(t)$	net battery active power at node $i \in \mathcal{S}$ at time $t \in \mathcal{T}$
$r_{c,i}(t)$	battery active power charging at node $i \in \mathcal{S}$ at time $t \in \mathcal{T}$
$r_{d,i}(t)$	battery active power discharging at node $i \in \mathcal{S}$ at time $t \in \mathcal{T}$
$b_i(t)$	battery energy at node $i \in \mathcal{S}$ at time $t \in \mathcal{T}$
$P_{w,i}(t)$	wind curtailment at node $i \in \mathcal{W}$ at time $t \in \mathcal{T}$
δ_i	angle in radians at node $i \in \mathcal{N}$
$\delta_{ij}^{pw}(l)$	angle block l in loss approximation in line from node i to j
$w_{ij}(l)$	binary variable indicating if block l is at its maximum
$y_{ij}(t)$	binary variable indicating the sign of the angle difference over line from node i to j at time $t \in \mathcal{T}$
p_{ij}^{loss}	active power loss in line from node i to node j
p_{ij}	active power flow in line from node i to node j

A. Method 1: DC OPF

The first method investigated is the DC OPF. This problem seeks to minimize the quadratic cost of traditional generation with the DC power flow embedded in the constraints. The advantage of this formulation is that it is a quadratic problem for which there exist many reliable solvers:

$$\min \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{G}} C_i(P_{g,i}, t) \tag{1a}$$

subject to ($\forall t \in \mathcal{T}$):

$$\begin{aligned} P_{g,i}(t) - r_i(t) + W_i^{max}(t) - P_{w,i}(t) & \\ = d_i(t) + \sum_{j\in\mathcal{L}_i} \frac{1}{x_{ij}}[\delta_i(t) - \delta_j(t)] & \\ + \sum_{j\in\mathcal{L}_i} \frac{1}{2} p_{ij}^{loss}(\delta_i(t), \delta_j(t)) & \\ \forall i \in \mathcal{N} & \end{aligned} \tag{1b}$$

$$P_{g,i}^{min} \leq P_{g,i}(t) \leq P_{g,i}^{max}, \forall i \in \mathcal{G} \tag{1c}$$

$$-R_i^{DWN} \leq P_{g,i}(t) - P_{g,i}(t-1) \leq R_i^{UP}, \forall i \in \mathcal{G} \tag{1d}$$

$$0 \leq P_{w,i}(t) \leq W_i^{max}(t), \forall i \in \mathcal{W}. \tag{1e}$$

Storage dynamics are modeled by the difference equations described in [11], giving the additional five constraints ($\forall i \in \mathcal{S}, t \in \mathcal{T}$):

$$b_i(t+1) = b_i(t) + T_s\eta_c r_{c,i}(t) - \frac{T_s}{\eta_d} r_{d,i}(t) \tag{2a}$$

$$b_i(T+1) = b_i^{term} \tag{2b}$$

$$r_i(t) = r_{c,i}(t) - r_{d,i}(t) \tag{2c}$$

$$r_{c,i} \in [0, R_c^{max}], r_{d,i} \in [0, R_d^{max}] \tag{2d}$$

$$0 \leq b_i(t) \leq B_i. \tag{2e}$$

Examining the storage constraints given in (2a)-(2e) reveals an important issue. As stated, simultaneous charging and discharging is possible. This gives rise to non-physical solutions if the charging and/or discharging efficiencies are not unity. For example, consider a solution that had $r_{c,i}(t) = r_{d,i}(t)$ for some $i \in \mathcal{S}$. Then $r_i(t) = 0$, which physically should imply that the state of charge of the device is unchanged. However, mathematically this does not hold: with $\eta_c < 1$ and/or $\eta_d < 1$, there will be a nonzero change in the state of charge of the device. Hence solutions to the problem stated above may be mathematically valid, yet not physically meaningful. Section III of this paper proposes two methods to avoid this issue, and hence ensure that solutions are both physically and mathematically valid.

Losses must also be taken into account to produce solutions that approximate AC solutions. To do this, the piecewise-linear approximation of losses developed in [2] is used, which is represented by the following constraints:

$$\|p_{ij}\| + \frac{1}{2}p_{ij}^{loss} \leq f_{i-j}^{max} \tag{3a}$$
$$\delta_i - \delta_j = \delta_{ij}^{+} - \delta_{ij}^{-} \tag{3b}$$
$$\delta_{ij}^{+} \geq 0, \delta_{ij}^{-} \geq 0 \tag{3c}$$
$$\delta_{ij}^{+} + \delta_{ij}^{-} = \sum_{l=1}^{L} \delta_{ij}^{pw}(l) \tag{3d}$$
$$p_{ij}^{loss}(\delta_i, \delta_j) = \frac{R_{ij}}{x_{ij}^2}\Delta\theta \sum_{l=1}^{L}(2l-1)\delta_{ij}^{pw}(l) \tag{3e}$$
$$\|p_{ij}\| = \frac{1}{x_{ij}} \sum_{l=1}^{L} \delta_{ij}^{pw}(l). \tag{3f}$$

In the previous set of constraints, equations (3b) and (3c) comprise a linear formulation of the absolute value of the difference in the angles of the sending and receiving ends of the line of interest. To avoid fictitious losses, the following constraints must be added to the piecewise linear approximation to explicitly enforce adjacency conditions [2]:

$$w_{ij}(l)\Delta\theta \leq \delta_{ij}^{pw}(l), \qquad \forall l = 1, ..., L-1 \tag{4a}$$
$$\delta_{ij}^{pw}(l) \leq w_{ij}(l-1)\Delta\theta, \quad \forall l = 2, ..., L \tag{4b}$$
$$w_{ij}(l) \leq w_{ij}(l-1), \qquad \forall l = 2, ..., L-1 \tag{4c}$$
$$\delta_{ij}^{pw}(l) \geq 0, \qquad \forall l = 1, ..., L \tag{4d}$$
$$w_{ij}(l) \in \{0, 1\}, \qquad \forall l = 1, ..., L-1. \tag{4e}$$

Similarly, binary variables must be added to explicitly enforce complementarity[1] between the δ_{ij}^{+} and δ_{ij}^{-} variables, $\forall ij \in \mathcal{L}$. To do so, the following constraints are added:

$$\delta_{ij}^{+} \leq L\Delta\theta y_{ij}(t), \qquad \forall ij \in \mathcal{L}, t \in \mathcal{T} \tag{5a}$$
$$\delta_{ij}^{-} \leq L\Delta\theta(1 - y_{ij}(t)), \quad \forall ij \in \mathcal{L}, t \in \mathcal{T} \tag{5b}$$
$$y_{ij}(t) \in \{0, 1\}, \qquad \forall ij \in \mathcal{L}, t \in \mathcal{T}. \tag{5c}$$

B. Method 2: AC-LP OPF

The next method investigated is based on the traditional AC-LP OPF. The optimization problem solved is a simple linear program that minimizes the total piecewise linear cost of traditional generation while satisfying power balance in the network. The objective function $\bar{C}_i(P_{g,i}, t)$ in this problem is the piecewise linear approximation of the quadratic cost curve in the DC OPF. The method iterates with a full AC power flow, updating the P_{loss} term in the problem at each iteration, to ensure the final solution is AC feasible. The linear program solved at each iteration is given by:

$$\min \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{G}} \bar{C}_i(P_{g,i}, t) \tag{6a}$$

[1]Complementarity of two variables a and b implies $a \times b = 0$, i.e., if one variable is non-zero, then the other one must be zero.

subject to ($\forall t \in \mathcal{T}$):

$$\sum_{i \in \mathcal{G}} P_{g,i}(t) - \sum_{i \in \mathcal{S}} r_i(t) + \sum_{i \in \mathcal{W}} (W_i^{max}(t) - P_{w,i}(t)) = \sum_{i \in \mathcal{D}} d_i(t) + P_{loss}(t) \tag{6b}$$
$$0 \leq P_{g,i}(t) \leq P_{g,i}^{max}, \quad \forall i \in \mathcal{G} \tag{6c}$$
$$-R_i^{DWN} \leq P_{g,i}(t) - P_{g,i}(t-1) \leq R_i^{UP}, \quad \forall i \in \mathcal{G} \tag{6d}$$
$$0 \leq P_{w,i}(t) \leq W_i^{max}(t), \quad \forall i \in \mathcal{W} \tag{6e}$$
$$b_i(0) \geq 0, \quad \forall i \in \mathcal{S} \tag{6f}$$
$$b_i(t+1) = b_i(t) + T_s\eta_c r_{c,i}(t) - \frac{T_s}{\eta_d} r_{d,i}(t), \quad \forall i \in \mathcal{S} \tag{6g}$$
$$b_i(T+1) = b_i^{term}, \quad \forall i \in \mathcal{S} \tag{6h}$$
$$r_i(t) = r_{c,i}(t) - r_{d,i}(t), \quad \forall i \in \mathcal{S} \tag{6i}$$
$$r_{c,i} \in [0, R_c^{max}], \quad \forall i \in \mathcal{S} \tag{6j}$$
$$r_{d,i} \in [0, R_d^{max}], \quad \forall i \in \mathcal{S} \tag{6k}$$
$$0 \leq b_i(t) \leq B_i, \quad \forall i \in \mathcal{S}, \tag{6l}$$

and the overall method is described by the algorithm shown in Figure 1.

Line limit constraints are nonlinear. However, to preserve the linearity of the optimization problem in the AC-LP OPF, the constraint,

$$f_{i-j}^0 + \sum_{k \in \mathcal{G}} a_{i-j,k}(P_{g,k} - r_k - P_{w,k} - P_{g,k}^0 + r_k^0 + P_{w,k}^0) \leq f_{i-j}^{max} \tag{7}$$

is added to the problem to enforce line-flow limits. This constraint is a successive linearization of the line flow constraints at the current operating point. Thus, a constraint of this form is added for every overloaded line after the initial LP-powerflow iteration. The quantities with superscript '0' are results from the AC power flow, and are updated at every iteration between the LP and AC power flow. The $a_{i-j,k}$ coefficients represent the sensitivity of the line flow to changes in generation (including conventional generation, wind power and storage device injected power), and are derived in [1].

III. Storage Complementarity Conditions

From an optimization perspective, the addition of wind power and storage devices into power system models present unique challenges that must be addressed. One issue of particular importance that is addressed in the formulations that follow is complementarity between charging and discharging of storage devices. From a purely mathematical point of view, producing solutions that result in devices simultaneously charging and discharging is not a problem. However, such solutions are not physically meaningful. Explicitly enforcing this complementarity in the OPF problem would require adding the constraint,

$$r_{c,i} \times r_{d,i} = 0, \quad \forall i \in \mathcal{S}, t \in \mathcal{T} \tag{8}$$

for every storage device in the network, which requires integer variables. Two methods for enforcing complementarity between storage charging and discharging are investigated.

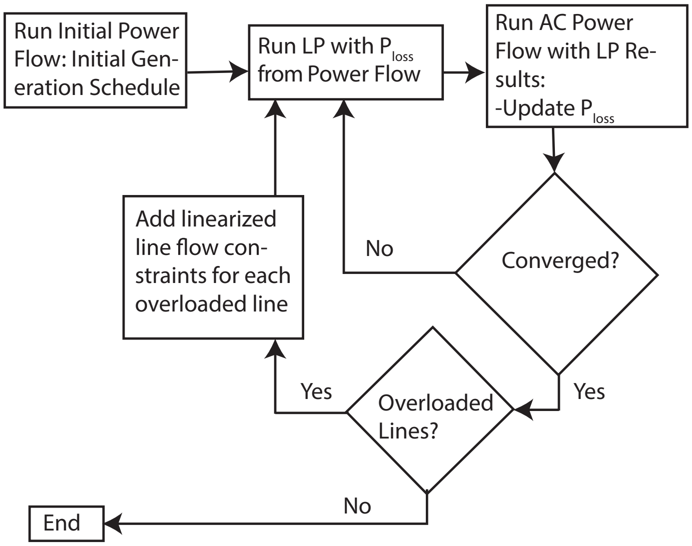

Fig. 1. AC-LP OPF Iterative Method.

The first is an iterative approach that does not require the use of integer variables; because the AC-LP OPF is already an iterative algorithm, this method lends itself well to the first approach. The second method utilizes binary variables; since the DC OPF already includes binary variables explicitly enforcing adjacency of angle difference blocks, this approach is implemented in the DC OPF. Because each approach does not change the type of problem being solved or nature of the method (iterative as compared to non-iterative), it is observed that the convergence of the AC-LP OPF and DC OPF are unchanged. The benefit of these additions is that the resulting solutions are physically meaningful, and reveal at optimality the charging/discharging patterns for storage devices.

Storage charging and discharging complementarity within the DC OPF is an example of a broader class of problems, namely quadratic programs with complementarity constraints (QPCCs) [12]. This set of problems has been widely researched, and many algorithms have been developed to solve them. If the problem has m complementarity constraints, a straightforward but inefficient way to solve the problem is through full enumeration of the possible constraint values satisfying complementarity. However, practical solution techniques seek to avoid such enumeration. One possibility is to use a semidefinite programming heuristic to find a suitable subset of the 2^m possible quadratic programs to solve, and select an optimal solution from the results of that subset [13]. Another broad class of solution methods often used for these problems include sequential quadratic programming [14]. Still others include a modified logical Benders' decomposition to generate cuts of the feasible region, sparsification to form a quadratic relaxation of the original QPCC, and penalty function methods that maintain convexity but satisfy complementarity at optimality [15]. This paper investigates two methods that ensure complementarity but avoid increasing the complexity of the original quadratic program (or linear program).

A. Approach 1: Iterative Updates

The AC-LP OPF problem includes upper and lower bounds on storage device charging and discharging. This method adds a simple additional step after each iteration of the algorithm. After the initial LP-powerflow iteration, the net of charging and discharging is checked for every device to determine its charging/discharging status. If the optimal solution (at that iteration) reveals that device $k \in \mathcal{S}$ is charging, the upper limit of discharging for that device is set to zero, so (6k) becomes,

$$r_{d,k} \in [0,0] \tag{9}$$

which forces that device to stay charging at the next iteration, and ensures complementarity between $r_{c,k}$ and $r_{d,k}$. Likewise, if device $k \in \mathcal{S}$ is discharging, (6j) is changed to,

$$r_{c,k} \in [0,0] \tag{10}$$

which ensures the device stays discharging during the next LP-powerflow iteration. Adding this check after the first iteration enforces complementarity between storage charging and discharging explicitly, ensuring physically meaningful solutions. However, this assignment of a device's charge/discharge status at the first iteration may not be optimal for subsequent iterations. To account for the fact that in subsequent iterations it may be optimal for a storage device to switch its status, additional checks are added.

At each subsequent iteration, if the device constraints enforce charging and the amount the device is charging is at its lower (zero) charging limit, this implies that it may be optimal for the status of the device to change to discharging. This is accomplished by reverting (6k) to its original form and changing (6j) to match (10). This sets the upper charging limit to zero and thus changes the device's status from charging to discharging. A similar switch is made if a discharging device encounters its lower (zero) discharging limit. The updated AC-LP OPF algorithm that enforces complementarity is shown in Figure 2.

This algorithm enforces complementarity explicitly, due to the fact that at every iteration exactly one of the device charging or discharging upper limits is modified to force the corresponding variable to zero. Additionally, the algorithm allows for device status switching between iterations. Therefore this algorithm produces optimal solutions to the OPF problem formulated in Section II-B but maintains linearity of the problem being solved.

B. Approach 2: Binary Variable Constraints

The second approach that has been implemented involves the addition of binary variables to the OPF problem to explicitly enforce charging/discharging complementarity, as in [11]. This approach has the advantage that it preserves the non-iterative nature of the DC OPF solution algorithm. Because the adjacency conditions already use binary variables, this

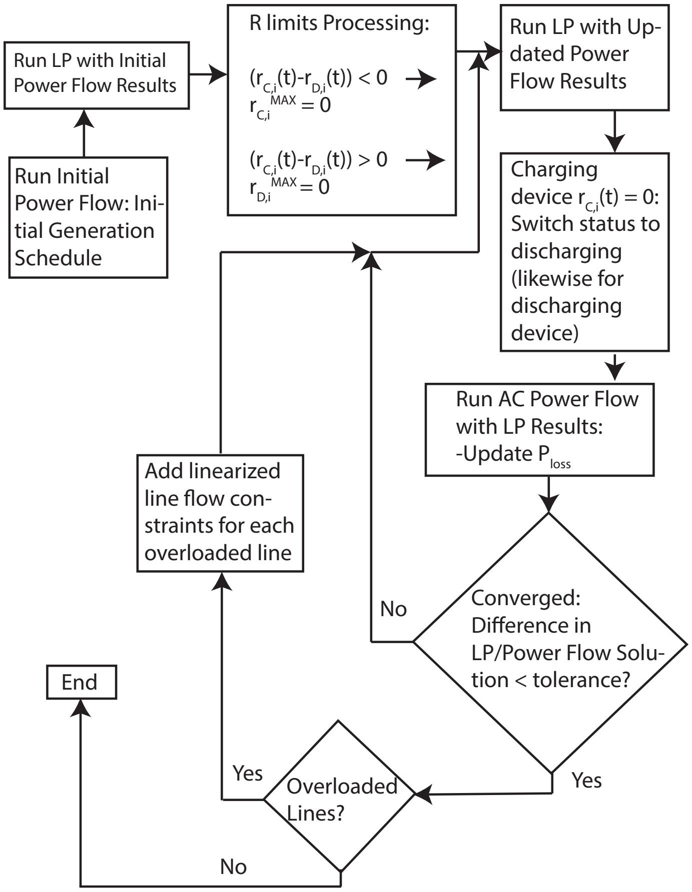

Fig. 2. Updated AC-LP OPF Iterative Method.

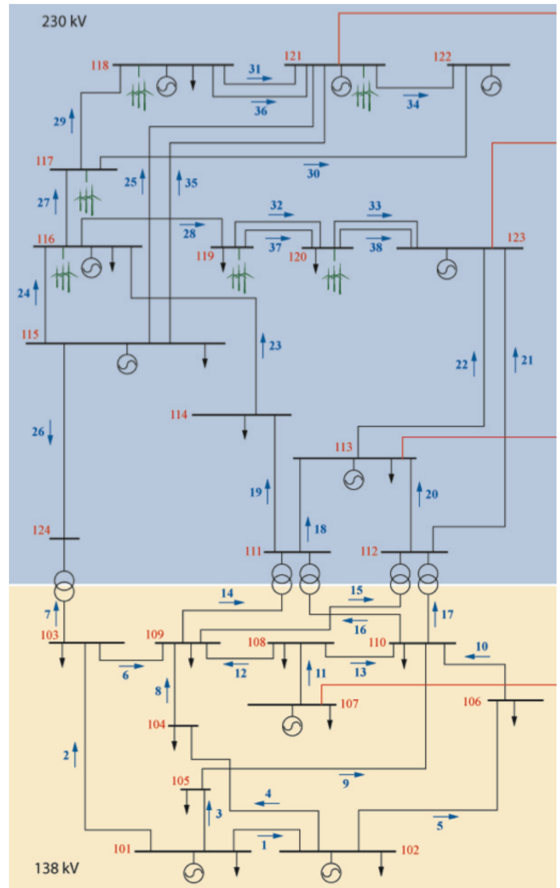

Fig. 3. RTS-96 System Topology.

approach does not significantly increase the overall complexity of the problem. The constraints,

$$0 \leq r_{c,i}(t) \leq R_c^{max}(S_{c,i}(t)), \qquad \forall i \in \mathcal{S}, t \in \mathcal{T} \qquad (11a)$$

$$0 \leq r_{d,i}(t) \leq R_d^{max}(1 - S_{c,i}(t)), \quad \forall i \in \mathcal{S}, t \in \mathcal{T} \qquad (11b)$$

$$S_{c,i}(t) \in \{0, 1\}, \qquad \forall i \in \mathcal{S}, t \in \mathcal{T}. \qquad (11c)$$

are added to the DC-OPF problem to explicitly enforce storage complementarity.

IV. Test Case

Both OPF algorithms, with and without complementarity enforced, were tested on a modified RTS-96 system [16]. The RTS-96 system is comprised of three symmetric areas with 73 nodes and 120 lines. Figure 3 shows the topology of area 1 of this system; areas 2 and 3 are identical. Five inter-area lines connect the three areas. Significant wind generation was added to area 1 of the system, with lesser wind added in areas 2 and 3. Correspondingly, traditional generators were removed from service in area 1 and to a lesser extent in area 2. In total, 19 wind locations were added in the system. Four storage devices were also added at locations throughout the system. This test case demonstrates the application of OPF algorithms in systems that include storage and wind generation. The example also highlights the potential cost benefit of scheduling renewable generation in conjunction with storage, and the challenges that must be addressed in doing so.

The test cases assume that a unit commitment (UC) has previously been run using day-ahead demand and wind forecasts. This UC produces a schedule of generators that are in service. The OPF then runs every 15 minutes or so, using up-to-date forecasts of demand and wind to establish the most economical real-time operating conditions.

V. Results

The OPF algorithms were run with and without storage complementarity being enforced, to explore how the solution quality might change. Two cases were considered. The first assumed that the forecast values from the unit commitment were correct, providing a baseline to which further results could be compared. The second demonstrates how the methods perform in the presence of forecast errors.

Figures 4 to 6 show the results for the base case, hours 10-13 of day 1 of the RTS-96 system data. Online generation is scheduled as shown in Figure 4, storage devices are scheduled as in Figures 5 and 6, and available wind generation is scheduled to maintain power balance. Figure 5 shows the total change in state-of-charge across all devices at each hour, and Figure 6 shows the total net power injection for

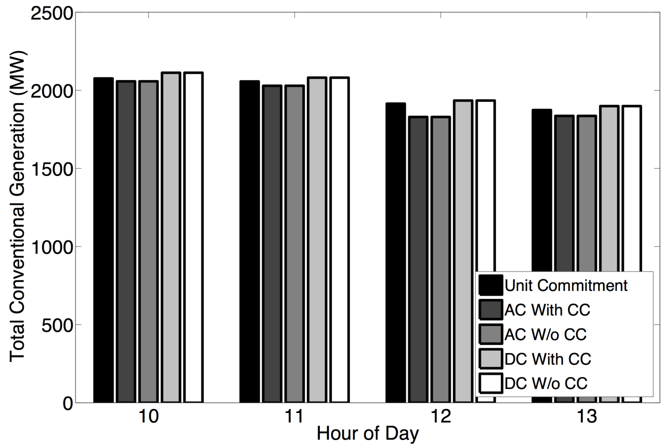

Fig. 4. Base Case: Total Conventional Generation.

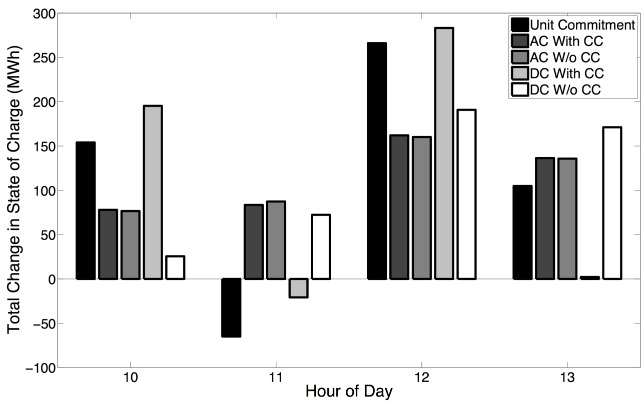

Fig. 5. Base Case: Total Change in SOC.

all storage devices at each hour. In both the DC OPF and AC-LP OPF methods, without complementarity enforced, the optimal solution may not be physically realizable. Figure 4 shows the effect on the scheduling of conventional generation, and therefore total cost, of adding storage complementarity constraints. The two methods of enforcing complementarity result in very similar generation schedules, and therefore the operating cost is almost equal.

Figures 5 and 6 highlight an important phenomenon that can occur when simultaneous charging and discharging is observed. Examining the AC-LP OPF results in Figure 5, with and without complementarity enforced, the change in state-of-charge at each hour is nearly equal. However, the corresponding AC results in Figure 6 indicate that for the same change in state-of-charge, storage devices appear to the network as much larger loads when simultaneous charging and discharging occur. In other words, when complementarity is not enforced, solutions that undergo simultaneous charging and discharging will always underestimate the state-of-charge, as proven in [17].

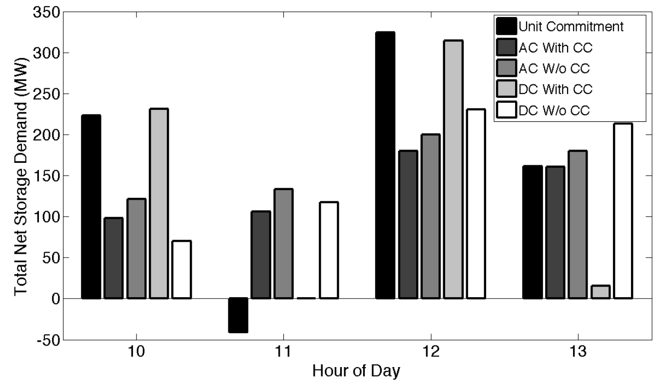

Fig. 6. Base Case: Total Storage Demand.

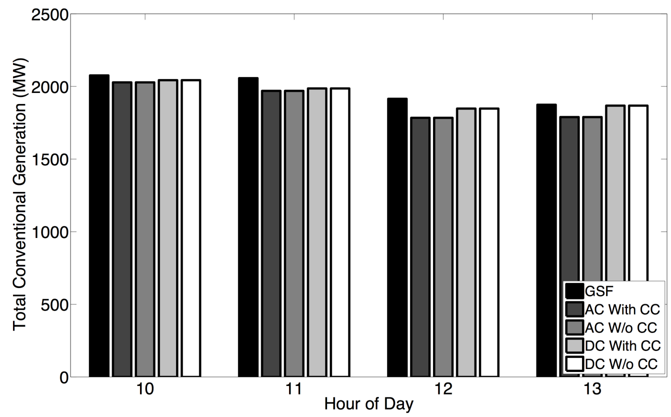

Fig. 7. Altered Wind Case: Total Conventional Generation.

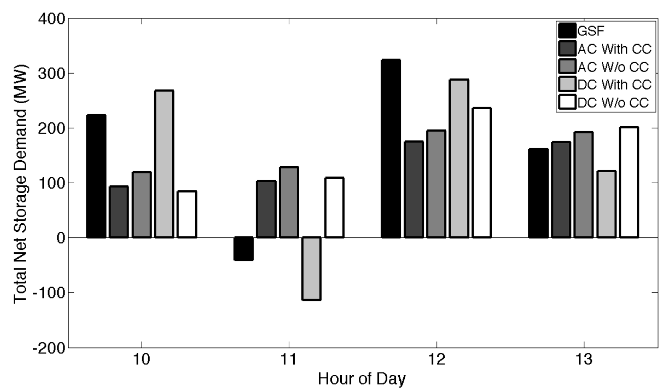

Fig. 8. Altered Wind Case: Total Storage Demand.

Figures 7 and 8 show the results of the OPF methods when the forecast and actual values of available wind differ. In this case actual wind is 10% greater than predicted in area 2 at hours 10 and 11, and 10% greater in area 3 at hours 12 and 13. Recall that the UC schedule is based on forecast values, and so is inconsistent with the altered wind availability. Therefore,

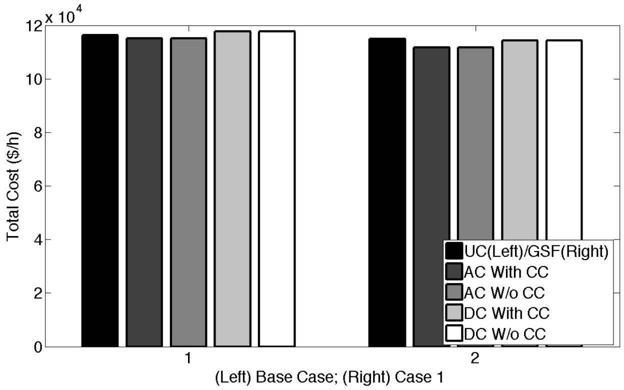

Fig. 9. Base Case (Left) and Altered Wind Case (Right) Total Cost.

in order to establish a meaningful comparison between the OPF results and the UC schedule, generation shift factors (GSFs) were used to adjust the UC schedule to eliminate line overloads [1]. This emulates operational practice.

Trends observed for the base case are again apparent in Figures 7 and 8. As in the base case, storage complementarity is not achieved without explicit constraints. The total change in state-of-charge across all devices is underestimated when complementarity is not enforced. However, as with the base case, the objective cost changes very little with the addition of explicit complementarity constraints. In fact, Figure 9 shows that both the DC OPF and AC-LP OPF achieve almost the same objective value with and without complementarity enforced. In this case, enforcing a physically meaningful solution does not incur additional cost due to the availability of excess (free) wind generation.

As there is greater wind in the network in the second case, Figure 9 shows that a slightly lower operating cost can be achieved relative to the base case, highlighting the benefits of wind generation availability. Finally, comparing the OPF and UC (with GSF) costs, both OPF algorithms achieve a more economical schedule in the presence of wind forecast errors.

VI. Conclusions

The DC OPF and AC-LP OPF problems have been formulated to include wind and storage, and to consider a multiple time-step optimization horizon. These formulations address the challenges associated with adding storage, namely including temporal coupling over the time horizon and complementarity between storage charging and discharging. Two methods of enforcing storage complementarity have been demonstrated. Compared with other solution methods used for quadratic programs with complementarity constraints (QPCCs), the proposed methods are simple to implement and do not increase the complexity of the original OPF problems. The two proposed methods maintain the convergence properties of the AC-LP and DC OPF problems.

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