Coordinated PEV Charging and its Effect on Distribution System Dynamics

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***Abstract*—Increased market penetration of plug-in electric vehicles (PEVs) will enable a shift in the transportation sector towards more sustainable energy options. However, PEV battery charging represents a significant additional burden on power systems, especially at the distribution level where the radial structure of the network may accentuate the effects of load variations. Furthermore, uncoordinated PEV charging could cause transformers to regularly operate beyond their thermal limits, increasing their likelihood of failure. Coordinated charging, based on distributed control methods, has been proposed as a means of mitigating voltage fluctuations and transformer overloads. Recent research can be divided into two categories: studies into the effects of static PEV loads on distribution networks, and development of control algorithms that vary PEV charging to accomplish specific goals. This paper combines these two ideas by analyzing distribution network load flow dynamics in response to a large population of coordinated PEVs. An IEEE 34-node distribution test feeder is simulated in conjunction with a fleet of PEVs under Additive-Increase Multiplicative-Decrease (AIMD) control. The resulting scheme ensures that all loads are satisfied, while controlling PEV demand to meet secondary considerations such as voltage regulation and transformer capacity limits. However, detrimental oscillations may develop under certain conditions. The paper investigates the cause of these unwanted variations.**

I. Introduction

Sales of plug-in electric vehicles (PEVs) are on the rise, and will comprise a significant portion of the light-vehicle fleet in the coming decades. Through the use of on-board batteries charged by connections to the terrestrial power system, PEVs can shift a portion of the transportation sector's energy use off oil products, potentially reducing greenhouse gas emissions. However, placing a larger burden on the electrical grid is not without consequences, as the system is already operated close to limits and issues of stability and power quality may emerge with even moderate PEV penetration levels.

At the transmission level, the near-term consequences of large-scale PEV adoption may include higher generation capacity requirements if aggregate vehicle charging coincides with peaks in background demand [1]. Although the PEV portion may be only a fraction of the total system load, any increase in the maximum system demand necessitates investment in costly peaker plants. Research efforts [2]–[4] in this area have attempted to shift PEV loading to off-peak hours, often termed valley-filling, using distributed control techniques, while others [5], use real-time pricing strategies to accomplish a similar objective.

At the distribution level, charging PEVs on the grid can have adverse effects on localized portions of the circuit. Existing electric vehicle supply equipment (EVSE) can already provide battery charging power in excess of the per-household peak in most residential areas. This unplanned additional load can cause voltage dips and reactive power imbalances across the network [6], [7]. Furthermore, the additional power requirements may decrease the lifetime of distribution transformers servicing the load [8], and these effects tend to be exponential as the transformer exceeds its rated power capacity [9].

Recent research on charging algorithms has focused on mitigating undesirable effects at the distribution level, with several papers combining analysis of charging scheme effectiveness from a controls perspective with simulations of the resulting power dynamics. Work in [10] applied a utility function method to prevent voltage dips and line overloading on realistic low-voltage networks. Other researchers have used queuing formulations [11] to dispatch PEV charging while maintaining grid stability. Another approach detailed in [12] proposes utilizing vehicle chargers for reactive power balance in order to bolster voltage sags in high PEV penetration networks. On-line linear programming techniques have also been applied [13].

This paper extends previous work by examining a low-voltage test circuit, the IEEE 34-node network, in conjunction with a fleet of electric vehicles under additive-increase multiplicative-decrease (AIMD) control. Unlike similar charging schemes, AIMD has the ability to provide decentralized, coordinated control with a minimal investment in computational and communication equipment. While others [14] have focused on power regulation at the substation level, the emphasis of this work is on preventing thermal overload of individual distribution transformers using temperature-based AIMD control, and the distribution-level power system dynamics that result when PEVs are charged in this manner.

The remainder of the paper is organized as follows. Section II discusses the theory behind the AIMD algorithm and its application to the PEV charging problem. Section III explains the parameters used in simulations. Section IV examines simulation results including voltage quality and transformer temperature. Section V looks at the conditions under which

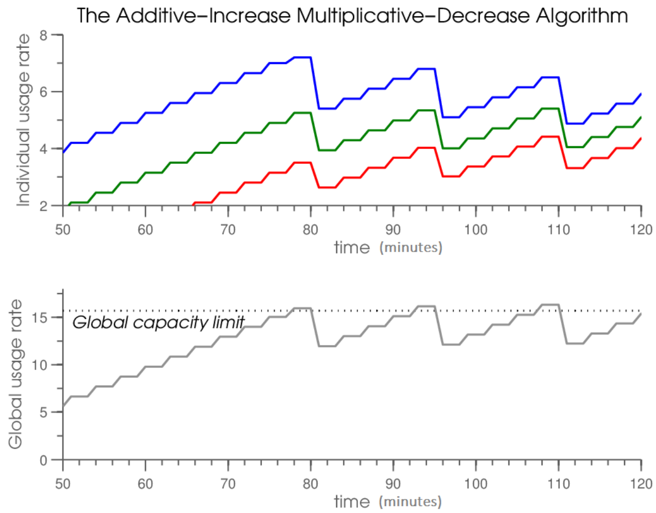

Fig. 1. Schema of the AIMD algorithm in practice. As participants join the network, their rate of use evolves additively until a global limit is reach, at which point a capacity event is triggered, reducing the usage rate of all participants multiplicatively. In perpetuity, all participants converge to the same utilization pattern.

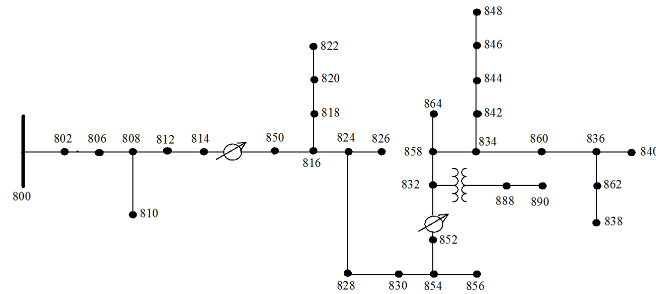

Fig. 2. The IEEE 34-node test feeder.

thermal overload can arise, and Section VI concludes the paper.

II. Additive-Increase Multiplicative-Decrease Control

The AIMD algorithm was first proposed by Chiu and Jain [15] as a means of maximizing throughput while managing congestion in constrained networks, originally applied to communication links. The implementation is as follows. A group of participants shares a common, constrained network resource. At each time-step, participants increase their rate of use of the global resource by a fixed additive amount α. This process continues unabated until the common resource exceeds its maximum limit. At this time a congestion event signal is broadcast out, and each participant decreases its consumption rate by a multiplicative factor β. This process is illustrated in Figure 1.

The AIMD algorithm is appealing for several reasons. Through adjustment of the parameters α and β, trade-offs can be made between fairness (the equitable sharing of resources among participants) and efficiency (the fraction of the total available throughput that is being utilized), as well as between responsiveness (the time it takes the system to achieve equilibrium) and smoothness (the size of the oscillations that develop in steady state). Given the simple implementation and adaptable operation, AIMD control has long been used for regulating packet congestion on TCP/IP links.

More recently, investigators [16] proposed using AIMD as a novel means of controlling PEV charging on a power constrained network. Under this scheme, vehicles start charging at a low rate when first plugged into the power grid. The vehicles on a particular feeder increase their individual charging rates until they reach a local constraint dictated by the EVSE, or a global power constraint on the substation feeder is reached, at which point the vehicles decrease their power multiplicatively. The advantage of an AIMD implementation is the reduced communication and computation requirements, as the feeder can simply monitor its power locally. The only information broadcast to the participating PEVs is the occurrence of a capacity event.

This work focuses on distribution level effects of AIMD charging, specifically the adjustment of PEV charging rates to limit aging in distribution transformers, which experience rapid degradation when hot-spot temperatures exceed rated values. Vehicles on a particular transformer increase their charging rate additively until the transformer's temperature reaches a set limit, at which point PEVs are commanded to decrease charging in a multiplicative fashion. The choice of additive and multiplicative parameters affects coordinated charging performance, as discussed in Section V.

III. Simulation Specifications

All simulations were undertaken using the IEEE 34-node distribution test feeder, which represents a realistic rural distribution circuit [17]. This feeder layout is shown in Figure 2. Current and voltage dynamics were calculated using the three-phase radial power flow method developed in [18], which uses a forward/backward iterative technique. This method incorporates full three-phase dynamics, including inductive coupling between lines and the existence of single phase branching feeders that extend from the main feeder.

In order to streamline the simulation, the distributed loads on the test feeder were converted to spot loads by dividing the real and reactive power requirements equally between the two ends of a given line. Capacitor banks were modeled as a shunt capacitance at the relevant nodes. The two voltage regulators present in the IEEE 34-node system are programmed to step tap positions to regulate voltages. It is quite plausible that the voltage variations induced by load control schemes may interact with such voltage regulators, possibly resulting in excessive tapping. However, to better understand the influence of charger controls, it has been found helpful to decouple regulator effects by holding tap positions constant for fifteen minute intervals. Further work is required to determine a tap update strategy that achieves good voltage regulation yet prevents excessive tap-change operations.

The electric vehicles on the network represent a fleet of heterogeneous medium-range plug-in hybrids similar to the Chevy Volt. Though the Volt has a listed battery capacity of 16kWh [19], the effective portion that can be repeatedly charged and discharged is only about half this total. As a result, the simulations use battery sizes that vary randomly between 6 and 10 kWh. For the remained of the paper, effective state-of-charge (SoC) refers to the capacity of the portion of the battery which can be charged/discharged, and can vary from 0-100%. Vehicles arrive for nighttime charging with between 1-25% effective SoC. Existing Level II charging capabilities for the EVSE servicing the car are a 240 V, 15 A connection for a rated maximum power of 7.2 kW, with an efficiency of $\eta = 90\%$.

TABLE I
ELECTRIC VEHICLE CHARGING SPECIFICATIONS.

Battery size	Between 6 and 10 kWh
Initial state-of-charge	1-25%
Maximum charging rate	7.2 kW
Charging efficiency (η)	90%
Power factor	1.0 (unity)

These simulations investigate an overnight charging scenario. The specified background (non-PEV) load on the system wanes overnight, with load values scaled proportional to a sample Midwest ISO demand curve. The given IEEE 34-node values for system load were matched to the peak of the demand curve, and nighttime values were adjusted accordingly. Electric vehicles were assumed to arrive randomly with a uniform distribution between 9pm and 12am. In order to capture the full overnight charging profile of the PEVs, the total simulation runs in 1 minute time-steps for the ten hour period from 9pm to 7am to reflect typical charging patterns.

The total IEEE-34 load is divided by an assumed mean peak household load of 1.75 kW, for a total of 1294 individual residences. PEV penetration levels listed for some of the simulations are based off of this figure, ex. 25% PEV penetration corresponds to 323 vehicles on the network.

Each node has a number of 25 kVA single-phase transformers apportioned appropriately to meet normal daytime demand. For instance, a node with 60 kVA aggregate load on its A-phase would be assigned three 25 kVA transformers, and each would serve a background load of 20 kVA. The amount of spare capacity on each transformer for PEV charging therefore varies randomly from node to node. The total vehicle population was then assigned randomly to the transformers on the network, with a fixed maximum number of PEVs per transformer to avoid unrealistic buildup on a single piece of equipment.

The temperature dynamics are derived using a first-order differential equation model of a thermal mass, where the transformer is heated by i^2R losses and cooled by the ambient temperature, as in [21]. Thermal dynamics are specified in Kelvin, and the AIMD algorithm engages whenever the total current through the transformer would result in a steady-state temperature above 120°C (or 393°K).

The IEEE 34-node network is long and lightly loaded, which can on occasion lead to convergence issues with some power flow solvers, although this concern did not arise during the loading scenarios studied.

IV. Simulation Results

A. Sample Case

The main goal of temperature-based AIMD control is to keep the 25 kVA transformers that service residential loads from exceeding their hot-spot temperature, while providing as much power as possible to the connected electric vehicles in a fair and efficient manner. Simulations were undertaken to assess control performance, with these objectives met in most cases. As an example, Figures 3 and 4 show respectively the charging profile of all PEVs attached to Transformer #1 at node 860 and the temperature of this transformer. Initially, only one PEV is connected to the transformer, and even though it charges at the maximum rate of 7.2 kW, the transformer temperature actually drops due to the significant nighttime drop-off in background load. However, as more PEVs begin to charge, increasing the current through the transformer, its temperature begins to increase towards its specified maximum. AIMD control sends out the first congestion event at the 68th minute to reduce the charging rate of all attached vehicles.

The average charging rate of each PEV continues to decrease until all vehicles have arrived. Shortly after this point the charging rates converge to a common pattern and enter a period of regular, repeated oscillations. This persists until vehicles begin to complete charging. As the PEVs disconnect, the average charging rate per vehicle increases, causing the remaining cars to finish even faster. Eventually, so few vehicles are connected that they can all charge at full power once again, and the temperature of the transformer begins to drop. The final result is a group of fully charged vehicles and reduced thermal stress on the distribution transformer.

In this example, all the vehicles achieve unity state-of-charge well before the given deadline. However, situations can arise where this is not feasible. The initial background load for the A-phase of node 860 is 51 kVA, which is split equally among three 25 kVA transformers at 17 kVA each, leaving 8 kVA of available charging capacity. Available capacity increases further as the nighttime background demand wanes. However, if background peak demand at this node was slightly below 50 kVA, for instance, and only two transformers were assigned to service the load, then the power available to charge the electric vehicles could be inadequate for the given charging-time constraint. In such cases, no form of control could charge all of the vehicles in time without violating transformer hot-spot temperature limits. The distribution utility would have no option other than to install a new transformer. In these situations, AIMD gives priority to distribution transformer health, but still provides a way maximize throughput to the vehicle loads within this constraint.

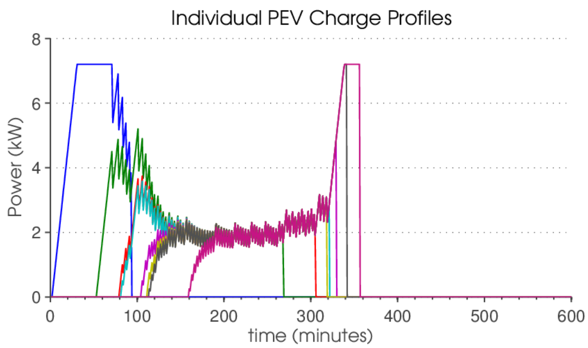

Fig. 3. Charging rates of each electric vehicle attached to Transformer #1 on node 860 A-phase. Vehicles arrive at randomly determined times and draw power until their batteries are fully charged. The AIMD algorithm engages whenever the current through Transformer #1 exceeds rated value.

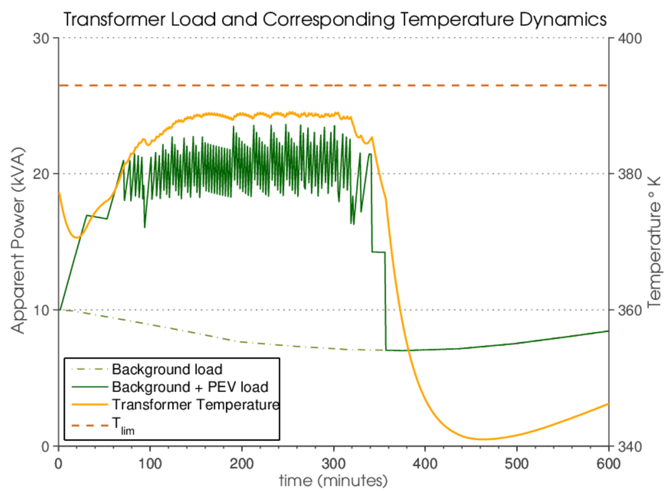

Fig. 4. Load and temperature data for Transformer #1 temperature at node 860 A-phase. Total load (solid line) includes background demand (dash-dot line) and the sum of the charging profiles from Figure 3. Temperature dynamics are derived from this load and the equations in [21]. The AIMD algorithm succeeds in maintaining transformer temperature below the imposed thermal limit.

B. Voltage Dynamics

In addition to the car charging dynamics, it is important to ensure that AIMD-induced variations in power levels do not cause unacceptable voltage oscillations. AIMD control is designed to synchronize vehicle charging rates in a pattern of slow increases in power (the additive stage) followed by a rapid decrease (the multiplicative stage), and these dynamics have corresponding effects on voltage levels, which in a radial distribution network may already be close to their limits.

A sample voltage profile for node 860 is shown in Figure 5, which displays the three phase voltages over the entirety of nighttime charging. Since control occurs in one minute intervals, voltage oscillations vary on the same time scale. In this example, there are a total of 24 vehicles charging on the transformers attached to the three phases of node 860, and with a PEV penetration level of 15%, 194 total cars on the network.

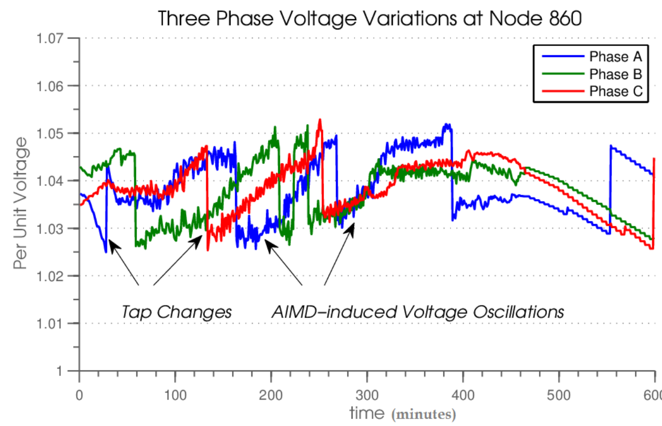

Fig. 5. Voltage oscillations under AIMD regulation with a 15% PEV penetration level. Tap changes occur as nighttime demand falls, independent of vehicle loading.

Despite the size of the vehicle population, the amplitude of the resulting voltage variations is minimal. Ignoring tap changes, the largest minute-to-minute voltage variation in this simulation is less than 0.005 per unit. Moreover, there is no discernible pattern in the variations, as the minute-to-minute voltage profile fluctuates arbitrarily. Over hour long timescales, there is a general upward trend in voltage due to decreasing background load, but this phenomena occurs regardless of coordinated vehicle charging.

The lack of more problematic voltage variations is due to several factors. First, although all of the vehicles on one transformer eventually synchronize in a shared pattern of charging, the other transformers on the network have different available power capacities, randomized vehicle arrival times and charging requirements. The variations in load capability along with the heterogeneous vehicle fleet lead to different charging patterns at each transformer, and the net effect is a canceling out of much of the variations in power (and corresponding changes in voltage) from one time step to the next.

Furthermore, each transformer has only a fixed amount of available power for vehicle charging, and although this capacity increases somewhat as the nighttime background load decreases, this nonetheless limits the amount of power that can oscillate on the system. Consequently, charging variations have less of an effect than might be expected, especially at lower vehicle population levels.

To illustrate this point, a series of simulations was run to determine the largest difference in voltage between two successive steps at any node (excluding tap change events). The results are shown in Table II, which records the worst-case voltage fluctuations after 100 simulations for several PEV penetration levels.

This sensitivity analysis shows that as the PEV population increases, both average and absolute worst-case voltage variations tend to increase in magnitude, likely because there are more cars per transformer and correspondingly larger

TABLE II
MAXIMUM NODAL VOLTAGE VARIATIONS FOR RANDOMIZED PEV POPULATIONS OVER 100 SIMULATIONS.

PEV %	Average worst-case	Absolute worst-case	Node
10	0.0083 p.u.	0.0145 p.u.	846, Aϕ
25	0.0089 p.u.	0.0163 p.u.	840, Aϕ
40	0.0109 p.u.	0.0167 p.u.	840, Aϕ
55	0.0133 p.u.	0.0230 p.u.	840, Aϕ

multiplicative power drops (and voltage rises). Furthermore, voltage deviations become increasingly acute as vehicle penetrations levels rise. This non-linear relationship is due to the presence of background demand. At low vehicle populations, the continually-varying PEV charging load represents a small proportion of aggregate system demand, and variability is minimal. However, larger vehicle populations result in a significant proportion of load being adjustable, leading to a more erratic voltage profile.

V. DETRIMENTAL VARIATIONS LEADING TO THERMAL OVERLOAD

Using AIMD-based control allows electric vehicles to receive their required energy while allocating the load in such a manner that the transformers servicing the vehicles are prevented from overheating. However, under certain conditions, the transformer temperature can actually rise past the specified hot-spot limit. This section examines the causes of thermal overload and possible solutions to mitigate this phenomenon.

For simplicity, assume that background demand on a particular transformer is constant and thus the total power available for PEV charging is also constant. In this situation, every additional vehicle that plugs into the network lowers the average power delivered to each individual charger.

Given n vehicles charging on one transformer and a maximum charge rate P, the total increase in power during each additive step is $n \times \alpha$, and the amplitude of the subsequent multiplicative drop in power is $P \times (1 - \beta)$. Eventually, if enough vehicles charge simultaneously, the aggregate power increase during the additive stage of load control exceeds the decrease during the multiplicative stage. For a given P, α, and β, this occurs when the number of vehicles exceeds the constraint

$$n > P(1 - \beta)/\alpha$$

An example of this charging situation is shown in Figure 6, in which twelve PEVs charge on Transformer #2 at node 820 A-phase. The corresponding transformer temperature profile is shown in Figure 7. Using $\alpha = 0.25$ kW and $\beta = 0.7$, and given that Transformer #2 has 8 kW of spare capacity, the equipment is only capable of handling nine cars before thermal overload will occur. As can be seen from the detailed inset, once all of the vehicles charge simultaneously, the power increase from a single additive step surpasses the power decrease made during the multiplicative step, and eventually two multiplicative steps are called in a row to keep the power below its limit. When this occurs, the average power delivered to the transformer is greater than its rated kVA limit, and the transformer overheats as a result. Additionally, given the thermal inertia of the transformer, even once the power drops below its rated capacity, the transformer temperature will take an extended period to fall to an acceptable level. Therefore it is important to prevent excessive power draw over the entire time-frame.

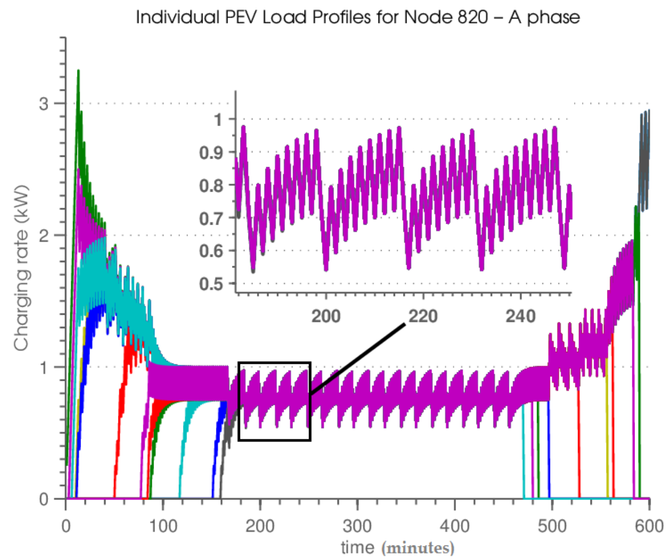

Fig. 6. With enough electric vehicles on transformer #2 of node 820 A-phase, the additive increase step exceeds the magnitude of the corresponding multiplicative decrease step, leading to an average aggregate PEV charging level that exceeds the transformer rating.

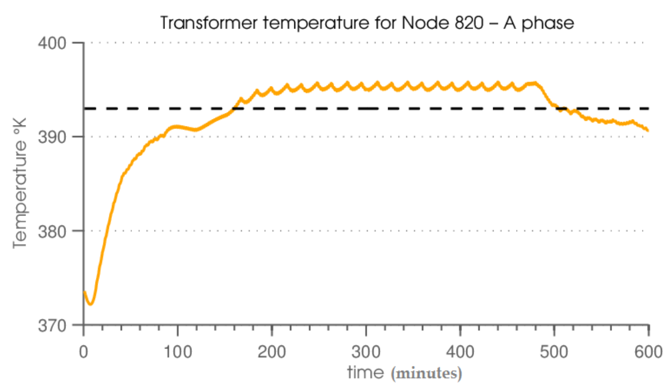

Fig. 7. The excess power through the transformer causes thermal overload. Due to thermal inertia, this elevated temperature persists until several cars complete charging around 475 minutes into the simulation.

There are several possibilities to address this issue. First, one could simply limit the number of vehicles charging on any one transformer. However, the limit may be relatively small. Consider the transformers attached to node 820 A-phase. Each has a spare capacity of 8 kW. If the AIMD parameters $\alpha = 0.25$ kW and $\beta = 0.9$ are used, then no more than three vehicles could charge at this transformer. This method would vastly under-utilize the charging capabilities of individual transformers.

A second approach would be to adjust the AIMD parameters in real-time in accordance with the number of vehicles connected to a transformer. Decreasing the β parameter allows more vehicles to plug in, but decreases the overall throughput of power delivered to the cars. On the other hand, decreasing the α parameter also accommodates more vehicles without affecting overall throughput, but increases the time needed for PEVs to initially reach full charging rate, and increases the time taken for multiple cars to reach fair, synchronized charging. Given that thermal overload conditions only occur during synchronized charging, it makes sense to adjust the additive parameter according to the number of vehicles on a transformer.

This method would require additional communication and computational infrastructure to assess the number of PEVs at a particular transformer. Solving the optimization algorithm could be accomplished off-line and then implemented as a simple lookup table, where values of α and β were stored for a particular number of vehicles and available power. This would ensure that all the vehicles use the largest percentage of available power possible without danger of violating transformer thermal limits.

VI. Conclusion

Plug-in electric vehicles (PEVs) represent a sizeable additional load on power distribution systems. Due to the radial structure of these networks, this additional load could potentially lead to voltage quality issues. Also, overloaded distribution transformers will be vulnerable to thermal damage. It is therefore imperative that large-scale PEV charging be accompanied by some form of coordinated control. The additive-increase multiplicative-decrease (AIMD) algorithm has been proposed as a possible method of regulating charging due to its minimal computational and communication requirements.

This paper illustratese that AIMD control can be used to limit the temperature of distribution transformers, thereby protecting the health of these expensive devices. This control method induces oscillations in the power consumption of PEV chargers and hence in nodal voltages. It was therefore important to assess the amplitude of those variations. This was considered through simulation of a fleet of PEV charges connected to the IEEE 34-node network during a nighttime loading scenario.

Given randomized vehicle arrival times and initial battery state-of-charge, the AIMD control algorithm was generally capable of charging vehicles by their scheduled completion times. The exception was when a transformer had very little spare capacity due to excessive background load. It should be noted though that in such cases no form of control would have accomplished full charging without overheating the transformer. Under certain circumstances, transformer temperature limits were exceeded when a large number of vehicles charged with poorly adjusted AIMD control parameters. Modifying these parameters to account for the number of vehicles guarantees that thermal constraints are never violated.

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