Why Don't Back-to-Back Abrupt Junction Frequency Triplers Work?

R. F. Bradley and A. R. Kerr
National Radio Astronomy Observatory*
Charlottesville, VA 22903

N. R. Erickson
Five College Radio Astronomy Observatory
University of Massachusetts
Amherst, MA 01003

Abstract

The recent interest in direct frequency tripling, *i.e.*, tripling without idler circuits, is due primarily to advances in the fabrication of planar, two-terminal nonlinear circuit elements having a symmetrical capacitance-voltage characteristic. Over the past few years, several research groups have put considerable effort into the development of triplers based on uniformly doped (abrupt-junction) back-to-back varactors only to measure little if any power conversion despite an apparently reasonable C_{max}/C_{min} ratio, the common figure of merit. Although a symmetrical capacitance-voltage function is desirable for efficient direct tripling, the dynamics of direct tripling using symmetrical varactors is more complex than first expected.

We describe the nonlinear circuit behavior of back-to-back varactors using the elastance-charge function, and discuss the phenomenon of self-biasing. We conclude by showing that *it is physically impossible for a back-to-back uniformly doped varactor circuit to convert power from the pump frequency to any harmonic.*

---

*The National Radio Astronomy Observatory is a facility of the National Science Foundation operated under cooperative agreement by Associated Universities, Inc.

Introduction

The back-to-back varactor diode analyzed in this paper is shown schematically in Fig. *1(a)*. Physically, the device may consist of either a series connection of two identical varactors with no external connection to the center node, or it can be a monolithic semiconductor structure. In either case, it has often been assumed that this device should work as a frequency tripler, with efficiency dependent on the capacitance ratio C_{max}/C_{min}.

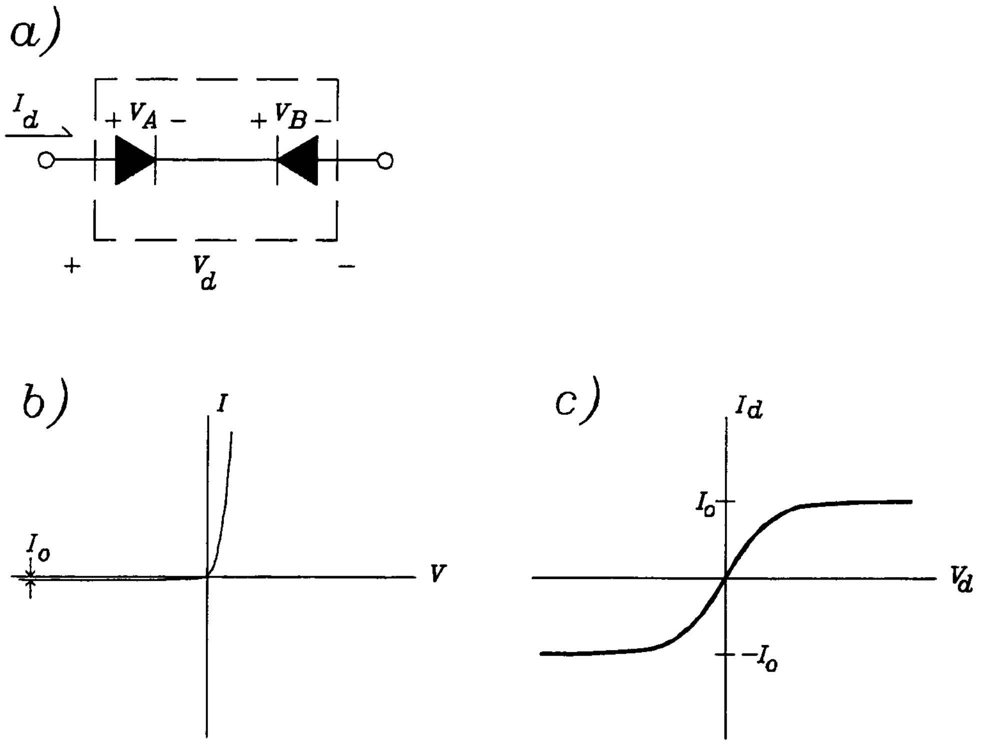

Figure 1 *a)* Schematic representation of the back-to-back diode varactor, *b)* I-V characteristic of a single diode, and *c)* I-V characteristic of the back-to-back varactor.

Figure *1(b)* shows the I-V curve of a single diode. The dc current in the pair of diodes is limited by the leakage current of the "off" diode. For two identical diodes in series,

$$i = i_o(e^{\alpha V_A} - 1) = -i_o(e^{-\alpha V_B} - 1) \quad , \tag{1}$$

and the voltage across the pair is

$$V_d = V_A + V_B = \frac{1}{\alpha}\ln\left[\frac{i}{i_o} + 1\right] - \frac{1}{\alpha}\ln\left[1 - \frac{i}{i_o}\right] \quad . \tag{2}$$

Combining terms and solving for i_d gives the dc I-V characteristic of the back-to-back varactor:

$$i_d = i_o \tanh\left(\frac{\alpha V_d}{2}\right) \quad . \tag{3}$$

An I-V curve according to eqn. 3 is sketched in Fig. *1(c)*.

The equivalent circuit of the back-to-back varactor, ignoring the series resistance, is shown in Fig. 2. The incremental impedance is

$$Z_d = \frac{1}{\left(g_A + j\omega C_A\right)} + \frac{1}{\left(g_B + j\omega C_B\right)} \quad . \tag{4}$$

For the individual diodes, $g = \alpha(i - i_o)$. Then, at zero bias,

$$Z_d = \left.\frac{2}{g_A + j\omega C_A}\right|_{V = 0} = \frac{2}{\alpha i_o + j\omega C_{jo}} \quad . \tag{5}$$

Using typical values for $i_o = 10^{-11}$ A, $\alpha = 39$ V^{-1}, and $C_{jo} = 10$ fF gives a corner frequency above which the back-to-back varactor looks primarily capacitive at zero bias:

$$f_x = \frac{1}{2\pi}\frac{\alpha i_o}{C_{jo}} \approx 400\ kHz \quad . \tag{6}$$

At large bias voltages, the current in the back-to-back pair is essentially the leakage current of one diode. Near the breakdown voltage V_{BR},

$$\begin{aligned} Z_d &= \frac{1}{\left[g_A \big|_{i=-i_o} + j\omega C_A \big|_{V_{BR}} \right]} + \frac{1}{\left[g_B + j\omega C_B \right] \big|_{i=i_o}} \\ &= \frac{1}{\left[0 + j\omega C_A \big|_{V_{BR}} \right]} + \frac{1}{\left[2\alpha i_o + j\omega C_B \big|_{i=i_o} \right]} \quad . \end{aligned} \tag{7}$$

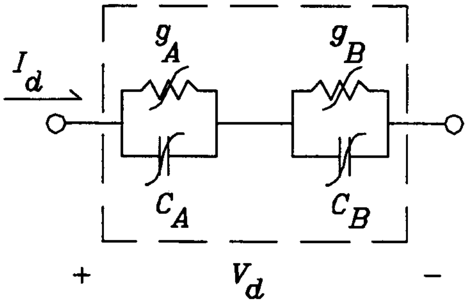

Figure 2 Equivalent circuit of the back-to-back varactor.

Since C_B at $i = i_o$ is approximately C_{jo}, it follows that at frequencies substantially above $2f_x$, the back-to-back diode is predominately capacitive at all bias voltages, with

$$C \approx \frac{C_A C_B}{C_A + C_B} \quad , \tag{8}$$

which is a function of bias. The C_{max}/C_{min} ratio can be measured as a function of the bias voltage using an RF bridge with a low-level test signal. A typical plot of C_{max}/C_{min} is given in Fig. 3.

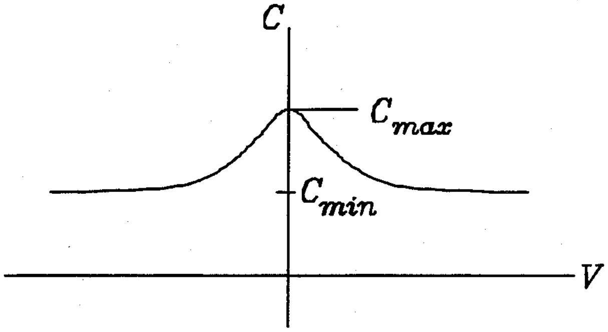

Figure 3 Sketch of typical C-V curve for the back-to-back varactor as measured using an RF bridge.

Among the many devices that exhibit a symmetrical C-V curve, the most popular candidates for millimeter-wave direct triplers have included 1) a pair of identical uniformly doped varactors, 2) a pair of identical hyperabrupt-junction varactors, 3) a quantum barrier varactor, 4) a pair of identical BIN or delta-doped varactors, and 5) a bbBNN varactor. The C_{max}/C_{min} ratio has been used in an attempt to rank the frequency tripling performance of each device. It was widely believed that the higher this ratio the better the tripling performance. However, this ratio, measured under small-signal conditions, is not an adequate indication of the large-signal behavior of the back-to-back diode in a tripler circuit. What is overlooked is the phenomenon of *self-biasing* caused by rectification at the diodes.

Self-Biasing

Self-biasing is well known in single-diode mixers when the dc bias source is replaced with a resistor R_b. The LO causes rectified current in the bias circuit, thus generating a (reverse) bias voltage across R_b. If R_b becomes infinite, the self-bias voltage increases (in the reverse direction) until the diode conducts no net charge over the pump cycle (if there is nowhere for the rectified current to flow, there can be no net current in the diode). This is illustrated in Fig. 4. The presence of a capacitor (usually the first element of a low-pass filter) in series with the RF circuit provides the necessary charge storage. Note that this circuit is simply that of a half-wave rectifier, which is the best example of self-biasing.

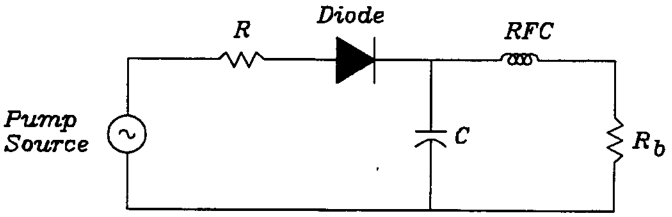

Figure 4 Self biasing in a single-diode mixer.

In the case of a pair of back-to-back diodes driven by a large-amplitude high frequency pump voltage, each diode presents a very high resistance path to the rectified current of the other. Both diodes self-bias, so there is no net current in either. The capacitance of each diode provides charge storage for the other. This is illustrated in Fig. 5.

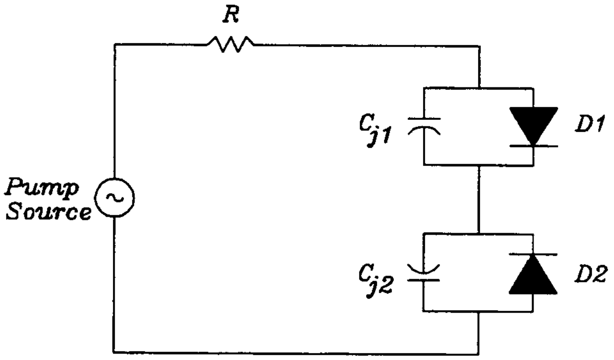

Figure 5 Simplified schematic diagram of a back-to-back varactor driven by a pump source.

Note the similarity between this circuit and a full-wave rectifier.

The effect of self-biasing is to clamp the peak of the diode voltage waveform at a small positive (forward) value, V_{CL} (which is zero for an ideal rectifier). For the back-to-back pair, the self-bias voltage adjusts itself so that during the part of the pump cycle in which the diode is reverse biased, the small amount of charge which leaks through the conductance of the diode in the reverse direction is equal to the charge which flows in the forward direction during the forward-biased part of the pump cycle.

Back-to-Back Uniformly Doped Varactor Tripler

We now examine the large-signal behavior of a pumped back-to-back varactor consisting of a pair of identical uniformly-doped diodes as shown in Fig. 6.

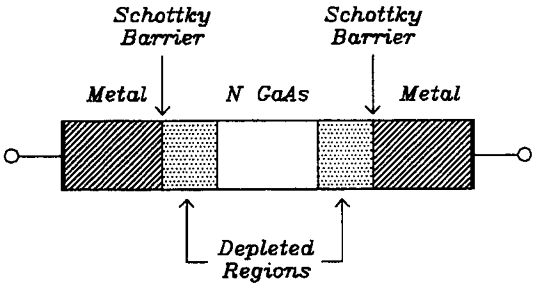

Figure 6 Diagram illustrating the physical structure of the back-to-back varactor.

Each of the twin diodes has a C-V curve which can be analyzed using Poisson's equation. Within the depletion region of each diode, $Q = N_d w A$, where N_d is the doping concentration, w is the depletion region width, and A is the cross-sectional area. Since we are considering a series combination of capacitances, it is easier to examine the *elastance*, S, the inverse of the capacitance, which for each diode is[**],

---

[**] Note that the elastance and capacitance used here are the incremental quantities $S = dV/dQ$ and $C = dQ/dV$, as appropriate for analysis of frequency multipliers.

$$S = \frac{1}{C} = \frac{w}{\epsilon A} = \frac{Q}{\epsilon N_d A^2} \tag{9}$$

At high frequencies, in the steady state, the large amplitude pump current in the back-to-back pair is almost entirely in the junction capacitance. This is a result of self-biasing which prevents either diode from going significantly into conduction and limits the forward voltage across each diode to the small value V_{CL}. Thus one can think of a fixed amount of charge oscillating between the two diodes with a corresponding modulation of their depletion layer widths, and, hence, of their elastances. The total elastance of the series pair $S_T = S_1 + S_2$. It is the time variation of this total elastance caused by the pump which determines the effectiveness of the back-to-back varactor as a frequency multiplier.

For a uniformly doped abrupt-junction diode, the elastance is proportional to the charge in the depletion layer. Hence, at the terminals of the back-to-back varactor there is no net elastance variation produced by the pump once steady-state operation is reached. The situation is analogous to two parallel plate capacitors in series, with the two internal plates connected by a rigid conducting rod; the total capacitance of the structure remains constant for any position of the rod.

The operation of the pumped back-to-back varactor in the steady-state condition can be represented graphically as shown in Fig. 7. Let $Q_1(t)$ and $Q_2(t)$ be the total depletion layer charges of the two diodes, and let $Q_P(t)$ be the charge waveform produced in the varactor by the pump. Since the leakage currents are negligible, $Q_P(t) = Q_1(t) - Q_2(t)$. If Q_{SB} is the depletion layer charge of the diode biased at the self-bias voltage, the charges on the individual diodes are $Q_1(t) = Q_{SB} + Q_P(t)$ and $Q_2(t) = Q_{SB} - Q_P(t)$. For purposes of illustration, it is assumed that the pump charge waveform is sinusoidal: $Q_P(t) = Q_{PO} \sin(\omega_P t)$.

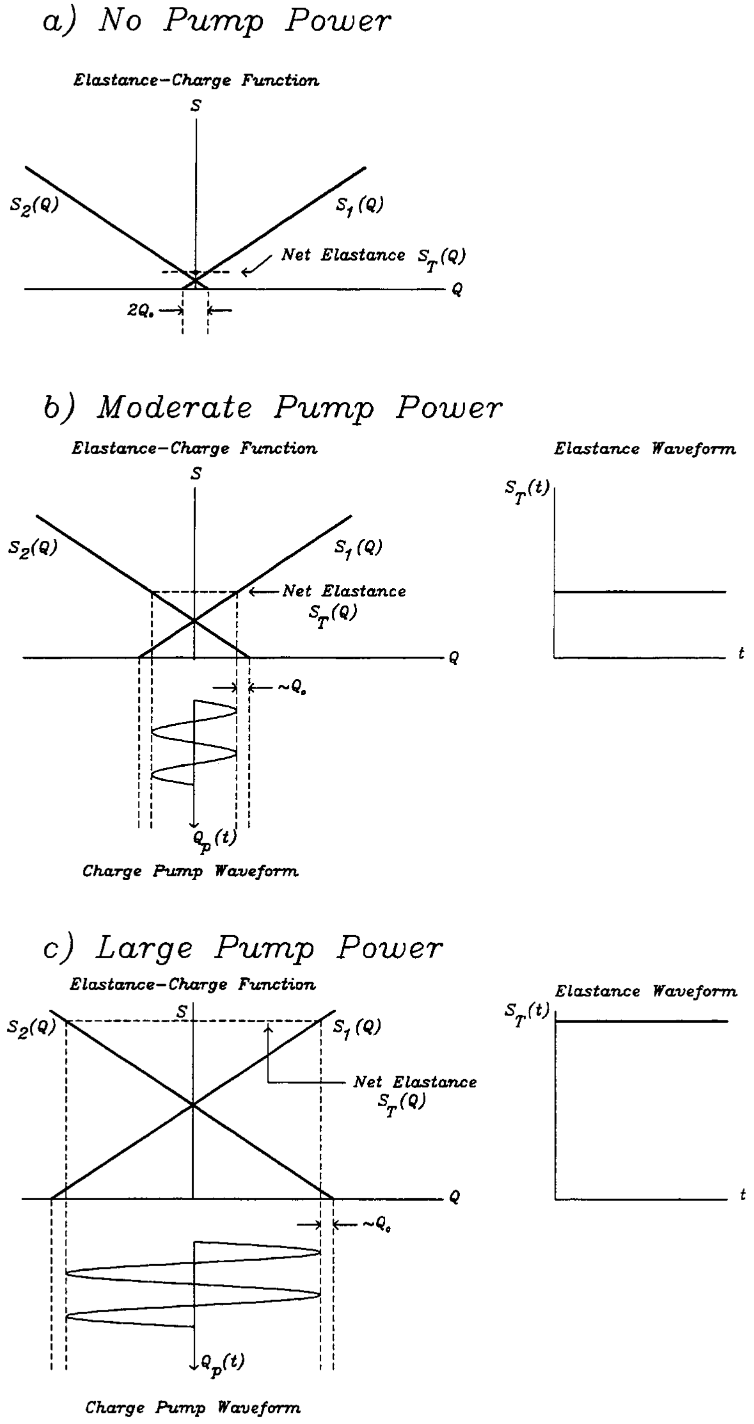

Figure 7 The steady-state elastance-charge functions and elastance waveforms for the back-to-back varactor under three pump amplitude conditions.

In Fig. *7(a)* no pump power is applied to the varactor. The depletion layer charges of both diodes have their zero-bias values Q_O. In Fig. *7(b)* pump power is applied to the back-to-back varactor, and each diode self-biases to a charge Q_{SB}. The charge on each diode now varies between $Q_{SB} + Q_{PO}$ and $Q_{SB} - Q_{PO}$. The self-bias voltage adjusts itself so that the voltage waveforms at the two diodes do not extend significantly into the region of forward conduction. Fig. *7(c)* is the same as *(b)* but with a larger pump amplitude. It is clear that in all cases the total elastance of the varactor remains constant at the value corresponding to the self-bias point.

Discussion

We have shown that a frequency multiplier cannot be made using back-to-back diodes if their doping profile is uniform. Although the pump causes a large-amplitude current to flow in the two diodes, self-biasing occurs, and the pump current flows almost entirely in the junction capacitance. If the doping is uniform, the pump produces no modulation of the overall elastance of the pair of diodes.

If the doping is non-uniform, application of the analysis described here shows that the pump modulates the overall elastance of the pair of diodes, and frequency multiplication is possible. The choice of an optimum doping profile for tripling (or other-order multiplication) is outside the scope of the present paper.

The omission of the series resistance of the diodes in the analysis does not affect the above conclusions. The pump current flows through the series connection of the series resistance and junction elastance, so self-biasing still occurs, and the overall elastance of a back-to-back pair of uniformly doped diodes remains fixed.